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THESIS

DEPTH AND PITCH CONTROL SYSTEM FOR A NEAR SURFACE SUBMARINE

by

Panos E. Vassiliadis

September 1976

Thesis Advisor G. J. Thaler

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forces are applied to the submarine and operation of the combined controller is tested for various values of the parameter. Further analysis and evaluation of the controller is also included.

Depth and Pitch Control
System for A Near Surface
Submarine

by

Panos E. Vassiliadis
Lieutenant Commander, Hellenic Navy
B.S., Naval Postgraduate School, 1975

Submitted in partial fulfillment of
the requirements for the degree of

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The problem of the near surface submarine depth and pitch controller is studied. Optimal unbounded automatic depth and pitch controllers are designed (a) for a submarine with stern plane only control and (b) for a submarine with stern and fairwater plane control. The two controllers are combined by a variable parameter function for use under a seaway. Submarine motion is simulated using an existing program, modified for the special cases. Step and pulse forces are applied to the submarine and operation of the combined controller is tested for various values of the parameter. Further analysis and evaluation of the controller is also included.

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INTRODUCTION

Operational experience has shown that there is a definite need for a new, or at least improved, method for control of a near surface submarine.

"Near surface" generally refers to a submarine which is close enough to the surface to be affected by surface waves.

The purpose of this paper was to devise a depth and pitch controller that could improve the sea state capability and attain more accurate depth control. In order to decide on the proper control strategy, gaining some knowledge of the nature of the near surface problem was required.

In Chapter I, some of the reasons for a submarine to be near surface are listed. This is followed by a section presenting elements of wave theory. The next section then considers the effects of the surface excitation, on the submarine. Due to these excitations the submarine operating in the near surface environment may be deflected far enough from its ordered trajectory to cause position saturation of the forward planes. The resulting situation is essentially a stern planes only control system. This configuration may be unstable.

To prevent serious problems in this area, the controller must be able to perform the following functions:

1. Identification of plant dynamics.

2. Evaluation of present position in relation to an optimum defined and action to drive the system toward the optimum by modification of controller parameters.

Closed loop optimization is used for plants which exhibit substantial changes in their dynamic characteristics and where random disturbances may have considerable influence on the plant state.

Aiming in that direction, two control systems were designed. The one is based on stern planes only control and is presented in Chapter II. The other is based on having both planes available and is presented in Chapter III, in three slightly different forms.

The two separate controllers were designed using optimal control theory and their combination led to a configuration capable of improving system performance. The submarine was subjected to step and pulse forces and the controller tested by simulation on the computer. In Chapter IV, the new scheme of controller is introduced and the results of the test are presented and analyzed. In Chapter V, further investigation of the combined scheme was attempted in an effort to better understand its function and test the generality of its parameters.

Conclusions by the author conlude the main body of the paper. However, seven Appendices are included. Appendix A describes the linearization process of equations of motion. In Appendix B a derivation of equations of motion from first principles is repeated from Ref. 9. A method for relating classical feedback to optimal theory is presented in Appendix C and a simplified trim analysis in Appendix D. In Appendix E, computer programs are described, setting limits on values

of parameters in the combined controller of Chapter IV. Lastly, in Appendix F the equations of motion for a submarine in six degrees of freedom are repeated from Ref.1 and in Appendix G all drawings are kept.

I. THE NEAR SURFACE PROBLEM

A. MISSIONS OF NEAR SURFACE SUBMARINES

Knowledge of the reason for a submarine to be near surface helps us to gain some insight into control requirements. Some noteworthy operations that a near surface submarine in straight motion is expected to be capable of are:

1. Periscope depth for navigational aiding observations.
2. Pretorpedo run optical sightings.
3. Transmission.
4. Missile launching.

The demand for each one in a combat environment varies and satisfactory operation is directly connected with relative convenience in choosing speed, particular course with respect to the direction of prevailing seas and accepting limits on pitch and heave amplitudes.

The near surface submarine cannot neglect the effects of surface waves. Furthermore, the problem is compounded in comparison with that encountered by the surface ship, since the operator cannot visually detect the approaching disturbance.

The outstanding characteristic of the open ocean is its irregularity. The irregular sea can be described by statistical mathematics on the basis of the assumption that a large number of regular waves having different lengths, directions and amplitudes are superimposed. Also the irregular motions of a ship in a seaway can be described as the superposition

of the responses of the ship to all the wave components of the seaway.

B. WAVE THEORY

In the theory of surface waves, simple two-dimensional waves are considered, which can be described completely by the motions, pressures, and so on, in any plane perpendicular to the crest lines. It is assumed that (a) the crests are straight, infinitely long, parallel and equally spaced and that wave heights are constant, (b) water has zero viscosity and is incompressible.

To facilitate the theoretical treatment of ship motions in regular waves and the use of the statistical approach to the study of behavior in actual sea states, the simple harmonic (sinusoidal) wave was introduced.

The surface wave is the visible manifestation of pressure changes and water particle motions, affecting the entire body of fluid, theoretically to infinite depth.

In Fig. I-1, the parameters and properties applicable for a sinusoidal wave on the free surface are displayed.

Φ, the velocity potential is defined as a function whose negative derivatives yield the velocity components of the fluid, and from this, all of the desired wave characteristics can be derived.

For a two-dimensional wave in any depth of water

$$\phi = \zeta_o V_\omega \frac{\cosh K(-z+h)}{\sinh Kh} \cdot \sin K(x - V_\omega t)$$

where

x = horizontal coordinate, positive in the direction of wave propagation.

z = vertical coordinate, positive downward.

ζa = surface wave amplitude

Lw = wave length

h = depth of water

k = wave number 2π/Lw

Vw = wave velocity

t = time.

For the case of deep water, the ratio

$$\frac{\cosh k(-z+h)}{\sinh kh} = \frac{e^{k(-z+h)} + e^{-k(-z+h)}}{e^{kh} - e^{-kh}} \quad \text{approaches} \quad e^{-kz}$$

and then $\phi = \zeta_a V_\omega e^{-kz} \sin k(x - V_\omega t)$

Then $\dfrac{-\partial \phi}{\partial x} = u$ = horizontal component of water velocity

$$= -k\zeta_a V_\omega e^{-kz} \cos k(x - V_\omega t)$$

$\dfrac{-\partial \phi}{\partial z}$ = w = vertical component of water velocity

$$= k\zeta_a V_\omega e^{-kz} \sin k(x - V_\omega t)$$

In deep water all particles describe circular paths having radii ζ_a at the surface and decreasing with depth in proportion to e^{-kz}

It can be shown that $V_\omega^2 = \frac{g}{k} \tanh kh$ which defines the the velocity of wave in any depth. In deep water $V_\omega^2 = \frac{g}{k} = \frac{g L_\omega}{2\pi}$
For many problems, the most important aspect of waves is the distribution of pressure below the surface.

The elevation at lines of equal pressure in a wave relative to the still water pressure lines is shown to be

$$\zeta = \zeta_a \frac{\cosh(-z+h)}{\cosh kh} \cos k(x - V_\omega t) \text{ and for}$$

deep water $\zeta = \zeta_a e^{-kz} \cos k(x - V_\omega t)$

Since e^{-kz} decreases as z increases, the contours of equal pressure are attenuated with depth.

To obtain the surface wave profile, z is taken equal to zero. Then

$$\zeta_0 = \zeta_a \cos k(x - V_\omega t)$$

$$= \zeta_a \cos(kx - \omega t) \qquad \text{where} \quad \omega = 2\pi/T_\omega$$

Most of the characteristics of the simple harmonic wave--except the precise profile--are the same as those of the more exact wave formulation.

The contours at constant pressure that have been presented also indicate the increase or decrease in pressure relative to still water at any point in terms of depth or head. That is, $p = \rho \cdot g(z - \zeta)$ and in deep water $p = \rho g z - \zeta_a \rho g e^{-kz} \cos(kx - \omega t)$. z should be measured to the center of the circular path described by the particle at the point of question.

Application of the formula shows that under the crest the pressures are decreased and under the hollow are increased.

The energy in a train of regular waves consists of kinetic energy associated with the orbital motion of water particles and potential energy resulting from the change of water level in wave hollows and crests.

It can be shown that for a simple cosine wave the wave energy is half potential and half kinetic when averaged over a wavelength. The total energy is:

$E = \frac{1}{2} \rho g \zeta_a^2 L\omega$ or if the average energy per unit area of surface is considered,

$$E = \frac{1}{2} \rho g \zeta_a^2$$

Stokes and others formulated the hydrodynamic theory of waves of finite amplitude. It corresponds with the observed fact that actual waves have sharper crests and flatter hollows from the simple cosine wave. The approximate result is that the surface profile simple cosine curve is modified by a harmonic which is half the length of the fundamental. This formulation leads to a limiting wave height from crest to trough of 0.14Lw or about 1/7Lw. Real waves will break as this height is approached.

C. FACTORS AFFECTING THE CONTROL OF THE NEAR SURFACE SUBMARINE

The forces on a submerged body may be expressed as the integral of the pressure taken over the surface of the body. Under the assumptions already made (ideal fluid, etc.),

$$p = \rho \left\{ \frac{\partial \phi}{\partial t} + \frac{1}{2} V^2 + gz \right\}$$

The last term above is connected with the contours of constant pressure. From $\zeta = \zeta_a e^{-kz} \cos(kx - \omega t)$

$$\zeta(z) = \zeta_a \cdot e^{-kz}, \ (\zeta_a = \zeta_{surface})$$

it is seen for example, that for a submarine mean depth of 50 ft. there can still be a significant wave amplitude, especially in moderate or high sea states. The problem

examined as one of hydrostatics gives $\bar{F}_{hs} = -\rho g V \cdot \hat{k}$ where V is the volume of the body and $\hat{k}$ is the unit vector in the Z direction. Ideally, this passage of a pure pressure wave without the accompanying particles velocity would create essentially no force, since the integral of the pressure would not change but remain constant.

Next there is a component of the pressure equation which results in the submarine being sucked towards the surface. Part of this force is usually referred to as a venturi effect, although the last is applicable for rigid boundaries and steady flow.

Assuming infinite depth beneath the submarine, there will be no constriction of flow around the lower half of the boat. Assuming also that the submarine is in a uniform flow field with no disturbance, there will be a slight reduction in the cross sectional area of flow around the upper half, resulting in a higher velocity and thus a lower pressure.

The reduction in cross sectional area is lessened due to the nonrigidity of the boundaries and the wave that the submarine will create on the surface. In reality, the water particles, due to existing surface irregularities, will have orbital velocities, another discrepancy of the venturi effect steady flow assumption.

Consideration of a regular sinusoidal component of the irregular seaway shows that the velocities of the orbital particles increase the magnitude of the V^2 term in the pressure equation. So:

$$V_{relative} = V_{ship} + u_0 \cos(kx - \omega t)$$

$$= V_{ship} + u_0 \cos(\omega t + \varepsilon) \quad \text{where } \varepsilon \text{ is arbitrary}$$

$$\text{and } u_0 = K \zeta_a V_\omega \cdot e^{-KZ} = U_{surf.max.} \cdot e^{-KZ_0}$$

Squaring and averaging over time

$$V_{rel}^2 = V_{ship}^2 + 1/2\, U_{surf.max}^2 \cdot e^{-2KZ_0}$$

This effect coupled with the previously mentioned venturi effect can create as much as ten to twenty tons of lift on a submarine.

Provided that the sea state remains unchanged, this is a time averaged force which remains relatively constant in time. Even if the essential irregularity of the sea has been recognized, it is a fact that over a wide area and often for a period of several hours, the sea may maintain a characteristic appearance which defies precise description in terms of average or typical wavelength and height but which nevertheless is constant or steady. At other times or places, the sea will be quite different and yet there will again be a characteristic appearance. These observations suggested the possibility of statistical description of the sea and this was found to be quite feasible. The stochastic case can be modeled using energy density analysis. The spectrum can provide a mean value for the frequency and the wave height. The expected height and frequency can be combined to form an average or expected wave. This wave can be used to predict the average suck due to particle velocities.

Upon encountering this force, the normally stable submarine becomes unstable and seeks a new equilibrium position

by ascending to the surface unless proper precautions are taken to prevent "broaching."

There are basically two ways to offset suck. Adding ballast presents the difficulty in determining the proper amount for various sea states without making the submarine dangerously heavy and sluggish. Using an angle on the sail planes to create a downward force presents the problem of reduction of control authority, saturation of planes and even losing the downward force if the orbital particle velocity is such that the angle of attack on the planes is reduced to zero. In addition, the particles also induce time dependent forces and moments, pitch and heave in particular.

In Ref. 2, a qualitative example of the happenings as a submarine passes under a regular sinusoidal wave is given. The additional $\partial\phi/\partial t$ term in the pressure equation is usually neglected.

An overall survey of the excitations experienced by the near surface submarine reveals that they may be broken into two basic categories: The first is a d.c. component in the form of a suck and the second is an a.c. component which is a combination of the various oscillatory forces. When the body is subjected to a stochastic seaway, accurate simulation and prediction becomes quite involved if not impossible. For a submarine under a seaway, the following must be noticed also:

1. While at great depths, some of the partial derivatives are intuitively zero, Z_z for instance. Near the surface, the Z force varies with depth even in calm seas. The mere

presence of Z_zZ alone is sufficient to make the system unstable in the vertical plane.

2. Under a seaway, the frequencies of heave and pitch will be identical to those at the encountered excitation. Then

$$Z_{excit} = Z_{max} \sin(\omega_e t + \varepsilon)$$
$$M_{excit} = M_{max} \cos(\omega_e t + \varepsilon) \qquad (\omega_e = \text{frequency of encounter})$$

for a regular crested wave. When a stochastic excitation is introduced and assuming linear superposition,

$$Z_{excit} = \sum_{i=1}^{\infty} z_i \sin(\omega_{ei} t + \varepsilon_i)$$

$$M_{excit} = \sum_{i=1}^{\infty} M_i \cos(\omega_{ei} t + \varepsilon_i)$$

Model tests reveal, $\left(Z/h\right)_i = \text{constant}$

$\left(M/h\right)_i = \text{constant}$

and

$$Z_{excit} = \sum_{i=1}^{\infty} C_{1i} h_i \sin(\omega_{ei} t + \varepsilon_i)$$

$$M_{excit} = \sum_{i=1}^{\infty} C_{2i} h_i \cos(\omega_{ei} t + \varepsilon_i)$$

These infinite sums are impossible to work, so they have to be replaced, if simplification is desired, by some known frequency and wave height statistical averages for the examined sea state.

II. STERN PLANES ONLY DEPTH AND PITCH CONTROL SYSTEM

In order to keep the simulated stern planes only submarine on ordered depth and pitch in the presence of disturbances and to effect depth changes, a depth and pitch controller is designed.

Figure II-1 illustrates the assumed structure for the controller optimal design. The application of the optimal control is very well served by the linearization of the equations of motion. For motions in the vertical plane, roll, pitch, heave and surge are applicable. The method reduces the problem to four equations.

For a completely submerged submarine, the effects of surge are small enough to be neglected and roll is not considered. For a submarine near surface, the problem of roll is worthy of an entire paper by itself. To simplify matters, it was neglected. For a submarine in head or following seas this is a very valid step. The state vector for the controller design was therefore $\underline{x} = \begin{bmatrix} z \\ \dot{z} \\ \theta \\ \dot{\theta} \end{bmatrix}$

and the model for determining controller gains

$$\underline{\dot{x}} = \begin{bmatrix} 0 & 1 & -u & 0 \\ 0 & A_{11} & 0 & A_{21} \\ 0 & 0 & 0 & 1 \\ 0 & A_{12} & 0 & A_{22} \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ B_{11} \\ 0 \\ B_{12} \end{bmatrix} D_S$$

Details on linearization of the equations of motion are given in Appendix A.

Using $\underline{r} = \begin{bmatrix} Z_{ord} \\ 0 \\ \theta_{ord} \\ 0 \end{bmatrix}$ for the command inputs, then for small perturbations $\underline{x}$ can be replaced by $\underline{E}$, the state error vector and the problem solved as a linear regulator one, with states and control unbounded.

The cost function to be minimized will be

$$J = 1/2 \int_{t_0}^{t_f} [\underline{E}^T \underline{Q}\, \underline{E} + u^2 R]\, dt$$

where

$$\underline{Q} = \begin{bmatrix} E & 0 & 0 & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & D & 0 \\ 0 & 0 & 0 & B \end{bmatrix} , \quad R = C$$

Forming the Hamiltonian

$$\mathcal{H} = 1/2\, \underline{E}^T \underline{Q}\, \underline{E} + 1/2\, u^2 R + \underline{p}^T \underline{A}\, \underline{E} + \underline{p}^T \underline{B}\, u$$

Necessary conditions for optimality are:

$$\dot{\underline{E}}^* = \frac{\partial H}{\partial \underline{p}} = \underline{A}\, \underline{E}^* + \underline{B}\, \underline{u}^*$$

$$\dot{\underline{p}}^* = -\frac{\partial H}{\partial \underline{E}} = -\underline{Q}\, \underline{E}^* - \underline{A}^T \underline{p}^*$$

and $\frac{\partial H}{\partial u} = u^* R + \underline{B}^T \underline{p}^* = 0$ which gives

$$u^* = -\frac{1}{R} \underline{B}^T \underline{p}^*$$

Solution of the system

$$\begin{bmatrix} \dot{\underline{E}}^* \\ \dot{\underline{p}}^* \end{bmatrix} = \left[\begin{array}{c|c} \underline{A} & -\underline{B}\, 1/R\, \underline{B}^T \\ \hline -\underline{Q} & -\underline{A}^T \end{array}\right] \begin{bmatrix} \underline{E}^* \\ \underline{p}^* \end{bmatrix}$$

will finally give

$$u^* = OSAD = -1/R\, \underline{B}^T \underline{K}(t)\, \underline{E}$$

It can be shown that the matrix K satisfies the matrix differential equation

$$\dot{\underline{K}}(t) = -\underline{K}\,\underline{A} - \underline{A}^T\underline{K} - \underline{Q} + \underline{K}\,\underline{B}\,{}^1\!/_R\,\underline{B}^T\underline{K}$$

with the boundary conditions:

$\underline{K}(t_f) = \underline{O}$ Note that K is symmetric.

For a plant completely controllable, with A, B, R and Q constant matrices, $K(t_f) \longrightarrow K$ (a constant matrix) as $t_f \rightarrow \infty$. That is for an infinite duration process, the optimal control is stationary. From a practical point of view it is feasible in some cases, to use the fixed control law even for processes of finite duration. Then the K matrix can be determined also by solving the nonlinear algebraic equations $(\dot{\underline{K}} = \underline{O})$:

$$\underline{O} = -\underline{K}\,\underline{A} - \underline{A}^T\underline{K} - \underline{Q} + \underline{K}\,\underline{B}\,R^{-1}\underline{B}^T\underline{K}$$

Using the method followed in Ref. 1.

$$\dot{\underline{K}} = \begin{bmatrix} \dot K_{11} & \dot K_{21} & \dot K_{31} & \dot K_{41} \\ \dot K_{12} & \dot K_{22} & \dot K_{32} & \dot K_{42} \\ \dot K_{13} & \dot K_{23} & \dot K_{33} & \dot K_{43} \\ \dot K_{14} & \dot K_{24} & \dot K_{34} & \dot K_{44} \end{bmatrix} \qquad \underline{K} = \begin{bmatrix} K_{11} & K_{21} & K_{31} & K_{41} \\ K_{12} & K_{22} & K_{32} & K_{42} \\ K_{13} & K_{23} & K_{33} & K_{43} \\ K_{14} & K_{24} & K_{34} & K_{44} \end{bmatrix}$$

$$-\underline{K}\,\underline{A} = \begin{bmatrix} 0 & K_{11} + A_{11}K_{21} + K_{41}A_{12} & -uK_{11} & A_{21}K_{21} + K_{31} + A_{22}K_{41} \\ 0 & K_{12} + A_{11}K_{22} + K_{42}A_{12} & -uK_{12} & A_{21}K_{22} + K_{32} + A_{22}K_{42} \\ 0 & K_{13} + A_{11}K_{23} + K_{43}A_{12} & -uK_{13} & A_{21}K_{23} + K_{33} + A_{22}K_{43} \\ 0 & K_{14} + A_{11}K_{24} + K_{44}A_{12} & -uK_{14} & A_{21}K_{24} + K_{34} + A_{22}K_{44} \end{bmatrix}$$

$$-A^T \underline{K} =$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ K_{11} + K_{12}A_{11} + K_{14}A_{12} & K_{21} + A_{11}K_{22} + K_{24}A_{12} & K_{31} + A_{11}K_{32} + K_{34}A_{12} & K_{41} + A_{11}K_{42} + K_{44}A_{12} \\ -uK_{11} & -uK_{21} & -uK_{31} & -uK_{41} \\ K_{12}A_{21} + K_{13} + K_{14}A_{22} & A_{21}K_{22} + K_{23} + A_{22}K_{24} & A_{21}K_{32} + K_{33} + A_{22}K_{34} & A_{21}K_{42} + K_{43} + A_{22}K_{44} \end{bmatrix}$$

$$\underline{K}\,\underline{B} = \begin{bmatrix} B_{11}K_{21} + B_{12}K_{41} \\ B_{11}K_{22} + B_{12}K_{42} \\ B_{11}K_{23} + B_{12}K_{43} \\ B_{11}K_{24} + B_{12}K_{44} \end{bmatrix} = \begin{bmatrix} F_{11} \\ F_{21} \\ F_{31} \\ F_{41} \end{bmatrix}$$

$$\underline{B}^T \underline{K} = \begin{bmatrix} F_{11} & F_{21} & F_{31} & F_{41} \end{bmatrix}$$

$$\underline{K}\,\underline{B}\,\underline{B}^T \underline{K} = \begin{bmatrix} F_{11}^2 & F_{21}F_{11} & F_{31}F_{11} & F_{41}F_{11} \\ F_{11}F_{21} & F_{21}^2 & F_{31}F_{21} & F_{41}F_{21} \\ F_{11}F_{31} & F_{21}F_{31} & F_{31}^2 & F_{41}F_{31} \\ F_{11}F_{41} & F_{21}F_{41} & F_{31}F_{41} & F_{41}^2 \end{bmatrix}$$

Finally

$$\dot{K}_{11} = F_{11}^2 \cdot CI - E$$

$$\dot{K}_{12} = -(K_{11} + K_{12} A_{11} + K_{14} A_{12}) + F_{11} F_{21} \cdot CI$$

$$\dot{K}_{13} = F_{11} F_{31} \cdot CI + U K_{11}$$

$$\dot{K}_{14} = -(K_{12} A_{21} + K_{13} + K_{14} A_{22}) + F_{11} F_{41} \cdot CI$$

$$\dot{K}_{22} = -2(K_{12} + A_{11} K_{22} + K_{42} A_{12}) + F_{21}^2 \cdot CI - A$$

$$\dot{K}_{23} = -(K_{13} + A_{11} K_{23} + K_{43} A_{12}) + F_{21} F_{31} \cdot CI + U K_{21}$$

$$\dot{K}_{24} = -(K_{14} + A_{11} K_{24} + K_{44} A_{12}) - (A_{21} K_{22} + K_{23} + A_{22} K_{24}) + F_{21} F_{41} \cdot CI$$

$$\dot{K}_{33} = F_{31}^2 \cdot CI - D + 2 \cdot U K_{13}$$

$$\dot{K}_{34} = -(A_{21} K_{32} + K_{33} + A_{22} K_{34}) + F_{31} F_{41} \cdot CI + U \cdot K_{14}$$

$$\dot{K}_{44} = -2(K_{24} A_{21} + K_{34} + K_{44} A_{22}) + F_{41}^2 \cdot CI - B$$

$$\text{where} \quad CI = 1/C$$

Integration backwards in time is executed since $\underline{K}(t_f) = \underline{0}$, until a steady state solution is obtained.

Setting $\tau = t_f - t$, $d\tau = -dt$, when $t = t_f$ $\tau = 0$

and $\dot{\underline{K}} = \frac{d\underline{K}}{dt} = -\frac{dK}{d\tau}$

The signs of all equations are reversed and the initial conditions are those at t_f. Then

$$u = -1/R\ \underline{B}^T \underline{K}\ \underline{E} = -1/C \begin{bmatrix} F_{11} & F_{21} & F_{31} & F_{41} \end{bmatrix} \underline{E}$$

or

$$DSAD = \begin{bmatrix} Y_{11} & Y_{21}/u & Y_{31} & Y_{41}/u \end{bmatrix} \underline{E}$$

For the above, small changes are required in the original program (program 2 of Ref. 1). This is also due to the fact that the approximate proportionalities observed in Ref. 1 between the K's and powers of U (speed) were again found valid. K_{12}, K_{14}, K_{23} and K_{34} were inversely proportional to U^2. K_{22}, K_{24}, and K_{44} were inversely proportional to U^3.

The controller gains depend on the selection of weighting factors. It became apparent that a method was required that would indicate at least the region from which the factors could be selected. Such a method and the reasons that led to it are described in Appendix C. Using this method, two sets of weighting factors were found, the one with C=10, B=800, E=1, and the other with C=0.001, B=164, E=0.001, which gave to the resulting controllers the desired characteristics.

The weighting factors C=10, E=1, D=3000 (from Ref. 1) were also tested with the submarine at a speed of 15 knots and with no ballast added or removed. The controller failed to keep the submarine at the ordered depth. It was found that if the submarine were brought "in trim," then the depth keeping ability of the controller was considerably improved.

In Appendix D, a simplified trim analysis is presented to help in the calculations for the required ballast for the "in trim" condition, at different speeds.

For the stern planes only submarine, small changes were required in the rest of the simulation program. Db was set equal to zero and the bow plane angle generator was removed.

A short description, connecting the methods of solution of the nonlinear equations of motion followed in Ref. 1 and 5, is given below.

First step is the matrix formulation of the system equations. In Ref. 5

$$\begin{bmatrix} m-X_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & m-Y_6 & 0 & -Y_4 & 0 & -Y_5 \\ 0 & 0 & m-Z_6 & 0 & -Z_4 & 0 \\ 0 & -K_6 & 0 & I_x-K_4 & 0 & -K_5 \\ 0 & 0 & -M_6 & 0 & I_y-M_4 & 0 \\ 0 & -N_6 & 0 & -N_4 & 0 & I_z-N_5 \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} a+m(vr-mq) \\ CHAS' \\ ABLE \\ ECHO \\ BAKER \\ FOY \end{bmatrix}$$

where the matrix coefficient of dot vector corresponds in Ref. 1 to:

AAA	ABA	ACA	ADA	AEA	AFA
AAB	ABB	ACB	ADB	AEB	AFB
AAC	ABC	ACC	ADC	AEC	AFC
AAD	ABD	ACD	ADD	AED	AFD
AAE	ABE	ACE	ADE	AEE	AFE
AAF	ABF	ACF	ADF	AEF	AFF

Note that X_6 corresponds to COEFX(6) or $X_{\dot{u}}$ etc.

The above matrix, being a matrix of constants, is inversed and multiplied by the vector in the right hand side of the matrix equations, and the resulting six differential, non-linear equations are solved simultaneously.

III. BOTH PLANES DEPTH AND PITCH CONTROL SYSTEM

A. OPTIMAL SOLUTION, UNBOUNDED CONTROLS

The approach selected for the design of the controller, to maintain automatically depth and pitch, was to let the planes individually achieve whatever angle was required to meet an optimal control based on the minimization of the cost function

$$J = 1/2 \int_{t_0}^{t_f} [\underline{E}^T \underline{Q} \underline{E} + \underline{u}^T \underline{R} \underline{u}] dt$$

where $\underline{E}$ = state error vector

$\underline{u}$ = control vector

$\underline{Q}, \underline{R}$ = weighting matrices

That is, it was desired to maintain the state vector close to the ordered trajectory, without excessive expenditure of control effort. It was assumed that admissible controls and states are not constrained by any boundaries.

This approach was used also in Ref. 1 and for this reason only the required changes after the selected linear model for stern planes only submarine will be presented.

The changes originated in the inclusion of term "$-U \cdot \theta$" in the state equations and resulted in the following altered form of the gain derivative equations:

$$\dot{K}_{11} = -1/C_1 F_{11}^2 - 1/C_2 F_{12}^2 + E$$

$$\dot{K}_{12} = K_{11} + A_{11} K_{12} + A_{12} K_{14} - 1/C_1 F_{11} F_{21} - 1/C_2 F_{12} F_{22}$$

$$\dot{K}_{13} = -U \cdot K_{11} - 1/C_1 F_{11} F_{31} - 1/C_2 F_{12} F_{32}$$

$$\dot{K}_{14} = A_{21} K_{12} + K_{13} + A_{22} K_{14} - 1/C_1 F_{11} F_{41} - 1/C_2 F_{12} F_{42}$$

$$\dot{K}_{22} = 2(K_{12} + A_{11} K_{22} + K_{24} A_{12}) - 1/C_1 F_{21}^2 - 1/C_2 F_{22}^2 + A$$

$$\dot{K}_{23} = K_{13} + A_{11}K_{23} + K_{34}A_{12} - uK_{12} - 1/C_1\, F_{21}F_{31} - 1/C_2\, F_{22}F_{32}$$

$$\dot{K}_{24} = K_{14} + A_{11}K_{24} + A_{12}K_{44} + A_{21}K_{22} + A_{22}K_{24} + K_{23} - 1/C_1\, F_{21}F_{41} - 1/C_2\, F_{22}F_{42}$$

$$\dot{K}_{33} = -2\,u\,K_{13} - 1/C_1\, F_{31}^2 - 1/C_2\, F_{32}^2 + D$$

$$\dot{K}_{34} = -u\,K_{14} + A_{21}K_{23} + K_{33} + A_{22}K_{34} - 1/C_1\, F_{31}F_{41} - 1/C_2\, F_{32}F_{42}$$

$$\dot{K}_{44} = 2\,(A_{21}\cdot K_{24} + K_{34} + A_{22}K_{44}) - 1/C_1\, F_{41}^2 - 1/C_2\, F_{42}^2 + B$$

C_1, C_2 were included for the case that different weighting on the planes effort would be desired. As used in the simulation runs of this paper, C was set equal to C_1, C_2.

B. CONSTRAINTS ON ADMISSIBLE CONTROLS

The same controller in an alternate form can result if bounds are defined on the controls and Pontryagin's minimum principle applied.

From $\mathcal{H} = 1/2\left[\underline{E}^{*T}\underline{Q}\,\underline{E}^{*} + \underline{u}^{*T}\underline{R}\,\underline{u}^{*}\right] + \underline{p}^{*T}\underline{A}\,\underline{E} + \underline{p}^{*T}\underline{B}\,\underline{u}^{*}$

separating the terms that contain $\underline{u}$

$$1/2\ \underline{u}^T\underline{R}\,\underline{u} + \underline{p}^T\underline{B}\,\underline{u} \quad \text{or}$$

$$1/2\begin{bmatrix} D_s & D_b \end{bmatrix}\begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}\begin{bmatrix} D_s \\ D_b \end{bmatrix} + \begin{bmatrix} p_1 & p_2 & p_3 & p_4 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ B_{11} & B_{21} \\ 0 & 0 \\ B_{12} & B_{22} \end{bmatrix}\begin{bmatrix} D_s \\ D_b \end{bmatrix} =$$

$$1/2\left(C_1 D_s^2 + C_2 D_b^2\right) + \begin{bmatrix} p_2 B_{11} + p_4 B_{12} & p_2 B_{21} + p_4 B_{22} \end{bmatrix}\begin{bmatrix} D_s \\ D_b \end{bmatrix}$$

Under the assumption of independent controls, the quantities to be minimized are:

$$\tfrac{1}{2} C_1 D_s^2 + (P_2 B_{11} + p_4 B_{12})\, D_s$$

$$\tfrac{1}{2} C_2 D_b^2 + (p_2 B_{21} + p_4 B_{22})\, D_b$$

with

$$|D_s| < 0.436 \text{ rad } (\sim 25^\circ)$$

$$|D_b| < 0.436 \text{ rad } (\sim 25^\circ)$$

For the time that the controls are not saturated:

$$Ds = -\frac{P_2 B_{11} + P_4 B_{12}}{C_1}$$

(equal to DSAD in the original program of reference (1)

$$Db = -\frac{P_2 B_{21} + p_4 B_{22}}{C_2}$$

(equal to DBAD in the original program of reference (1)

For $\dfrac{P_2 B_{11} + P_4 B_{12}}{C_1} = \text{REGS}$

$$\frac{P_2 B_{21} + P_4 B_{22}}{C_2} = \text{REGB}$$

finally:

If $-0.436 \leq \text{REGS} \leq 0.436$ then $D_s = -\text{REGS}$

If $\text{REGS} > 0.436$ then $D_s = -0.436$

If $\text{REGS} < -0.436$ then $D_s = 0.436$

If $-0.436 \leq \text{REGB} \leq 0.436$ then $D_b = -\text{REGB}$

If $\text{REGB} > 0.436$ then $D_b = -0.436$

If $\text{REGB} < -0.436$ then $D_b = 0.436$

The two controllers are equivalent. The one will be referred to as unbounded and the other as bounded.

C. LINEAR TRACKING CONTROLLER, EXACT DESIGN

Until now the procedure of Ref. 1 was followed and the initial tracking problem was converted by use of simplifying approximations to a linear regulator one, where it was desired to maintain the state error vector close to the origin.

From an attempted more exact examination of the linear tracking problem, a modified controller resulted whose capabilities are considered greater. This is because approximations such as use of $\dot{\underline{e}} = \underline{A}\,\underline{e} + \underline{B}\,\underline{u}$ instead of the correct $\dot{\underline{e}} = \underline{A}\,\underline{e} + \underline{B}\,\underline{u} - \underline{A}\,\underline{r}$ are avoided.

Forming the Hamiltonian [Ref. 5]

$$\mathcal{H} = 1/2\,\|\underline{x} - \underline{r}\|_{Q}^{2} + 1/2\,\|\underline{u}\|_{R}^{2} + \underline{p}^{T}\underline{A}\,\underline{x} + \underline{p}^{T}\underline{B}\,\underline{u}$$

$$\dot{\underline{p}}^{*} = -\frac{\partial \mathcal{H}}{\partial \underline{x}} \qquad \underline{u}^{*} = -\underline{R}^{-1}\,\underline{B}^{T}\,\underline{p}^{*}$$

and for $\underline{p}^{*} = \underline{K}\,\underline{x}^{*} + \underline{s}$

$$\underline{u}^{*} = -\underline{R}^{-1}\,\underline{B}^{T}\,\underline{K}\,\underline{x}^{*} - \underline{R}^{-1}\,\underline{B}^{T}\,\underline{s} \quad \text{where}$$

$$\dot{\underline{K}} = -\underline{K}\,\underline{A} - \underline{A}^{T}\underline{K} - \underline{Q} + \underline{K}\,\underline{B}\,\underline{R}^{-1}\,\underline{B}^{T}\,\underline{K}$$

$$\dot{\underline{s}} = -\left[\underline{A}^{T} - \underline{K}\,\underline{B}\,\underline{R}^{-1}\,\underline{B}^{T}\right]\underline{s} + \underline{Q}\,\underline{r}$$

with $\underline{K}(t_f) = \underline{H}$, $s(t_f) = -\underline{H}\,\underline{r}$

Integration from t_f to t_o gives $\underline{K}$ and $\underline{S}$. With $\underline{H} = \underline{O}$ $\underline{K}$ results as before.

Using the symbols and notations of Ref. 1 then,

$\dot{s} = (\underline{A}^{T} - \underline{K}\,\underline{B}\,\underline{R}^{-1}\,\underline{B}^{T})\,\underline{s} - \underline{Q}\,\underline{r}$, where once again

$$\underline{Q} = \begin{bmatrix} E & 0 & 0 & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & D & 0 \\ 0 & 0 & 0 & B \end{bmatrix}$$

and $\underline{r} = \begin{bmatrix} ZODR \\ 0 \\ PORD \\ 0 \end{bmatrix}$

In explicit form

$$\dot{S1} = -\left(1/C_1\, B_{11} F_{11} + 1/C_2\, B_{21} F_{12}\right) S2 - \left(1/C_1\, B_{12} F_{12} + 1/C_2\, B_{22} F_{12}\right) S4 - E \cdot ZODR$$

$$\dot{S2} = S1 + \left(A_{11} - 1/C_1\, B_{11} F_{21} - 1/C_2\, B_{21} F_{22}\right) S2 + \left(A_{12} - 1/C_1\, B_{12} F_{21} - 1/C_2\, B_{22} F_{22}\right) S4$$

$$\dot{S3} = -U \cdot S1 - \left(1/C_1\, B_{11} F_{31} + 1/C_2\, B_{21} F_{32}\right) S2 + \left(-1/C_1\, B_{12} F_{31} - 1/C_2\, B_{22} F_{32}\right) S4 - D \cdot PORD$$

$$\dot{S4} = \left(A_{21} - 1/C_1 \cdot B_{11} F_4 - 1/C_2\, B_{21} F_{42}\right) S2 + S3 + \left(A_{22} - 1/C_1\, B_{12} F_{41} - 1/C_2\, B_{22} F_{42}\right) \cdot S4$$

In the expression for $\underline{U}^*$ the part $-\underline{R}^{-1}\, \underline{B}^T\, \underline{K}\, \underline{X}$ is the same as in the regulator case with $\underline{X}$ the state vector instead of $\underline{E}$ the error vector. So, finally

$$DSAD = Y_{11} \cdot DEPTH + Y_{21} \cdot ZODOT/U + Y_{31} \cdot PITCH + Y_{41} \cdot PIDOT/U$$
$$-1/C_1 \cdot \left(B_{11} \cdot S_2 + B_{12} \cdot S_4\right)$$

$$DBAD = Y_{12} \cdot DEPTH + Y_{22} \cdot ZODOT/U + Y_{32} \cdot PITCH + Y_{42} \cdot PIDOT/U$$
$$-1/C_2 \cdot \left(B_{21} \cdot S_2 + B_{22} \cdot S_4\right)$$

For bounded controls and

(1) controls unsaturated

$$\begin{bmatrix} DSAD \\ DBAD \end{bmatrix} = - \begin{bmatrix} 1/C_1 \left(P_2 \cdot B_{11} + P_4 \cdot B_{12}\right) \\ 1/C_2 \left(P_2 \cdot B_{21} + P_4 \cdot B_{22}\right) \end{bmatrix} = - \begin{bmatrix} REGS \\ REGB \end{bmatrix}$$

This is easily shown from

$$\frac{\partial H}{\partial \underline{u}} = \underline{R}\,\underline{u} + \frac{\partial(\underline{p}^T \underline{B}\,\underline{u})}{\partial \underline{u}}$$

$$= \underline{R}\,\underline{u} + \underline{B}^T \underline{p}$$ which implies

$$\underline{u}^* = -\underline{R}^{-1}\,\underline{B}^T\,\underline{p}$$

(2) controls saturated

For REGS $>$ 0.436 DSAD = -0.436

" REGS $<$ -0.436 DSAD = 0.436

" REGB $>$ 0.436 DBAD = -0.436

" REGB $<$ -0.436 DBAD = 0.436

The controller now becomes:

P2 = K12 · DEPTH + K22 · ZODOT + K23 · PITCH + K24 · PIDOT + S2

P4 = K14 · DEPTH + K24 · ZODOT + K34 · PITCH + K44 · PIDOT + S4

REGS = (P2 · B11 + P4 · B21)/C1

REGB = (P2 · B21 + P4 · B22)/C2

etc.

D. TESTING THE CONTROLLERS

All tests were run at the forward speed of 15 knots and with the values of the weighting factors C=10, D=3000, E=1, to provide results for easy comparison with that of Ref. 1.

To check the depth keeping capabilities of the bounded form of controller under the presence of external disturbances, a ramp input AU = -2.E-05 · TIME, was applied at the auxiliary trim tank (Fig. IV-4). The results are shown in Fig. III-1 for the otherwise unballasted submarine.

The controller kept the submarine at near zero depth until the fairwater planes became saturated. Then the "light"

submarine started losing depth quickly. The following must also be pointed out at this point: In the unbounded controller of Ref. 1 and in order to limit the effect of initial depth error (corresponding to a large step order) resulting in large plane angles, unachievable on a real ship, the depth and pitch errors were limited as shown in Fig. III-2. The limiting values tested were $\pm$ 2 feet for depth and $\pm 10^{\circ}$ for pitch errors. Also to reduce the noise level in the plane positions, the plane angle ordered was filtered with a simple low pass filter.

In the test of Fig. III-1, the limiters were preserved in the bounded controller but the filters were removed. Thus as seen in Fig. III-1c., although the submarine reaches a depth of -80 feet, the reaction of the unsaturated stern planes is very slow.

From Fig. III-3 and III-4, the effect of the low pass filters can be seen. Fig. III-3 shows the response of the submarine with bounded controller and without filters to a sinusoidal input, $AU=2.0 \times 10^{-4} \times \sin 0.8 \cdot t$.

Figure III-4 presents the response to the same input when the unbounded controller of Ref. 1 as modified in this section is used. Except for noise components, the results are otherwise similar.

To compare the effectiveness of the controllers to depth changes, an ordered depth of 10 feet was used. Figure III-5 shows the response of the submarine with the unbounded controllers, with limiters and filters.

Figure III-6 presents the response of the submarine with the bounded controller, with limiters but no filters. Again, no appreciable difference is detected, as was expected since the controllers are in fact the same in that region of operation. Both responses were acceptable.

In Fig. III-7, the submarine was ballasted with AU= $8.8X10^{-5}$, FT=$6.4X10^{-5}$, AT=$-6.4X10^{-5}$. The bounded controller was tested for its ability to keep depth. To check this, the ordered depth was put to zero. The controller maintained a steady state depth error of 0.05 feet and a steady state pitch error of 0.0002 rad. A steady state fairwater plane angle of about -0.7° was required to maintain depth. This small value implies that the simulated submarine was almost "in trim."

The next test was again with the submarine unballasted. A depth of 10 feet and a pitch of -0.150 rad (-8.6°) were ordered with the tracking controller in use. The error limiters and noise filters were removed. Responses are shown in Fig. III-8. Although the resultant steady state values of 10.4 feet and -9.8° could be considered acceptable, the submarine overshoots to a depth of 24 feet before achieving its final depth. The negative pitch was ordered in order to speed up the depth change but on the other hand this created difficulties for the planes in their action to stop the pitching submarine when it reached the ordered depth for the first time.

In Fig. III-9, the same maneuver was repeated while a force AU=$2X10^{-4}$ sin 0.8t was also applied on the boat. The results were similar. The depth overshoot was not acceptable.

Since it was desired to be able to use suitable pitch of the submarine when ordering depth changes, the method of presenting the orders to the tracking controller was modified and the following statements were added to the program:

```
PI1 = -0.052
PI2 =  0.052
ADCG= ABS(DCHNG)
DY    = 20. + ADCG/RATE
XD    = (DCHNG)· PI2/100
Y1     = RAMP(20.)
Y2     = RAMP (DY)
ZODR  = RATE·(Y1-Y2)·DCHNG/ADCG
PORD  = LIMIT (PI1 ,PI2, XD)
```

The inputs were now the desired depth change and the rate of depth change (DCHNG and RATE). Suitable pitch between PI_1 and PI_2 is automatically ordered during the depth transition. The values of RATE, PI_1, PI_2 and the form of XD can easily be modified to achieve desired depth change characteristics.

In Fig. III-9, results are presented for the ballasted submarine, the sinusoidal input $AU=2X10^{-4}$ sin 0.8t superimposed and an ordered depth change of 50 ft. The final depth, achieved without significant overshooting, was 51.7 feet and the pitch 0.019 rad $(\sim 1^{\circ})$. The values of the parameters RATE, PI_1, PI_2, as appear above, were selected to give a response similar to that of the submarine in Ref. 1 with the unbounded controller, the error limiters and noise filters. This was verified by the results.

Although setting bounds on the planes may lead the boat to unadmissible limit cycles, the nature of the near surface problem prohibits the use of the unbounded controller with error limiters when improved performance is desired.

The tracking controller, due to its more exact design, is expected to have better performance. With the introduced method of ordering depth changes, the error limiters are no longer necessary. Further investigation is required on the operational characteristics of the tracking controller.

In the rest of this work, the bounded controller is mainly used due to the fact that it was developed earlier. Lack of time prevent duplication of the tests with the tracking controller in use.

In Fig. III-10, the block diagrams of plant and controller in the linear regulator problem and in the linear tracking problem are shown together to depict differences.

In Appendix C, the method developed to connect linear feedback with optimal control theory is examined for the possibility of its application in the case of the both planes submarine. It may be repeated here that the results are rather prohibitive.

IV. CONTROLLER FOR THE NEAR SURFACE SUBMARINE

A. FORMULATION OF THE CONTROLLER. COMBINED MODE CONTROLLER (CMC)

The need for an improved depth keeping performance for the near surface submarine under a seaway led to an investigation for an "optimal" near surface controller.

The inadequacy of the already designed optimal unbounded and bounded ones is easily and simply demonstrated. A 70-ton pulse input is applied at the auxiliary trim tank and the unacceptable results of submarine response shown in Fig. IV-5 and 6 are discussed in the simulation section of this chapter.

The phenomena governing submarines response to submerged turbulence are not completely understood and much of the existing experimental data have been classified. Thus many simplifications were accepted in the formulation of the problem and a great effort was initially devoted in the direction of finding a mathematical proof that would suggest an optimal solution to the simplified problem. Even this task was proven very difficult to the author and a mathematical solution was unattainable in the time allocated for this work.

The next step was toward the examination of a proposed controller scheme, for which existing test results were known to be encouraging. In that scheme, two separate control systems are designed, one based on having both planes available and the other based on stern planes only control. The two stern planes commands are then computed and combined as a function of the forward plane command.

Defining δ_{s1} = f1 (Depth Error, Depth Rate, Pitch Error, Pitch Rate)

= stern plane command in both planes mode

δ_{s2} = f2 (Depth Error, Depth Rate, Pitch Error, Pitch Rate)

= stern plane command in stern only mode

$\delta_{fcom} = \delta_{f1}$ = f3 (Depth Error, Depth Rate, Pitch Error, Pitch Rate)

= bow plane command in both planes mode.

then the actual stern plane command formed of a linear combination of δ_{s1}, δ_{s2} is

$$\delta_{scom} = K \cdot \delta_{s1} + (1-K)\,\delta_{s2} \quad \text{where}$$

$$K = 1 \quad \text{if} \quad |\delta_{fcom}| < \delta_{f\max}$$

(i.e. bow planes not saturated)

$K = f_4(\delta_{fcom}, \delta_{f\max})$ otherwise

The function $K = \left(\frac{\delta_{f\max}}{\delta_{fcom}}\right)^x$ was used with $\delta_{f\max} = 0.436 (\sim 25^\circ)$ and x a variable parameter.

The problem now was to try to find a "best" value for X for most acceptable controller performance, test the generality of this value in various kinds of controller designs and, if possible, also find a mathematical proof of its uniqueness or at least a proof of the existence of a restricted region of permissible values. For convenience, the following abbreviations will be used in the next.

SOPOC. = stern only planes optimal controller

SOPC = stern only planes controller (not optimal)

BPOC = both planes optimal controller

C M C = combined mode controller (meaning the proposed scheme)

It is again noted here, that the S O P O C. and B P O C designed in Chapters II and III are in fact optimal only under the assumptions of unbounded states and controls.

The synthesis of the C M C. was realized at first by the use of the already designed S O P O C. and B P O C. Then the S O P O C was replaced in the C M C by a S O P C. corresponding to a characteristic equation, same with that resulting from the B P O C. The formulation of the C M C. from the independently designed S O P O C or S O P C and B P O C ones is indicated in Fig. IV-1 to 3.

In the next section of this chapter simulation results, mainly of the use of C M C on a submarine excited by external forces, are presented. Parallel analysis of the data on individual and comparative bases is attempted.

Some conclusions appear proper at the end of the chapter. The part concerning the question of the generality of a unique "best" value or region of values for the C M C. parameter is left for the next chapter.

B. SIMULATION RESULTS

The computer used for the simulation was the I.B.M. 360/67 located at the W. R. Church Computer Facility of the Naval Postgraduate School.

Program 2, developed in Ref. 1 as a modification in D.S.L. of the original N.S.R.D.C. digital program of Ref. 3, is the main program used. The hydrodynamic coefficient values were

kept the same and all runs were made in forward motion, at a constant speed of 15 knots (25.33 ft/s) and with the submarine approximately "in trim." Appropriate modifications in the equations that lead to the control orders to the planes were made occasionally, as for example in the case of the SOPOC.

In all cases the linear model, derived in Appendix A, was used. The trim-controller part of the program was not operated.

To simulate the effect of an external forcing term, water was removed or added to the trim tanks whose position as defined in Ref. 1 is shown in Fig. IV-4. The depth and pitch error limiters used in the controller of Ref. 1 had to be removed when operation of CMC. was considered. They were used initially in the unbounded controller in order to avoid excessive values of the planes in response to a large step order. Continuation of their use will undesirably alter the values of the K function $\left(K = \left(\frac{0.436}{\delta_{fcom}} \right)^{x} \right)$ and consequently will affect the operation of the CMC.

At first the narrow pulse force shown in Fig. IV-5a was applied on a submarine (forward trim tank) using the BPOC. with bounded controls and with the limiters removed. The controller had weighting factors B=800, C=10, E=1.

From Fig. IV-5b to IV-5e, it is seen that the controller, after the saturation of the bow planes, acted slowly to stabilize the submarine with a steady state depth error of about -65 ft and a steady state pitch error of -0.086 rad. The bow planes never recovered in the time interval of 200 sec. the

simulation was held. In the case of the bounded controller with B=164, C=0.001, E=0.001, application of the same input was destructive as shown in Fig. IV-5f to IV-5i.

The controller with B=800, C-10, E=1 was also tested with the limiters replaced. The controller succeeded now in returning the boat to the ordered zero depth and pitch but with an overshoot of about -30 ft in depth (Fig. IV-6a to 6d). When the weighting factors were changed to C=10, E=1, D=3000, the controller failed completely again (Fig. IV-6e to 6h). In all cases the results were unacceptable and simply show the inadequacy of the designed BPOC with bounds on the controls and with or without error limiters, to handle situations approximated by the application of the pulse force.

All of the subsequent test runs in this chapter simulate motion of the submarine using the CMC Only three kinds of external excitations were considered, the pulse force at the forward trim tank, shown in Fig. IV-5a, a step of the same magnitude at the auxiliary trim tank, and the same step at the forward tank (Fig. IV-7a).

1. Application of Step Force at Auxiliary Trim Tank

Since it was desired to check the operation of the CMC after the saturation of the forward planes, the magnitude of the step was selected big enough to cause saturation.

The CMC was formed by the combination of a SOPOC and a BPOC (bounded) having the same weighting factors.

Under the above conditions, two controllers were tested. The first one with weighting parameters B=800, C=10,

E=1; the second with B=164, C=0.001, E=0.001. There is a distinct difference in the resulting controller orders, which was the reason for the selection of the above two sets of values. The optimal controllers, when the approximation of infinite duration control interval is used, result in time invariant state feedback given in the program in the form (for the order to stern planes):

$$\delta S = Y11 \cdot ZOER + Y21/Uc \cdot ZODOT + Y31 \cdot PERR + Y41/Uc \cdot PIDOT$$

where ZOER = Depth error
ZODOT = Depth rate
PERR = Pitch error
PIDOT = Pitch rate

and Y_{11}, Y_{21}, Y_{31}, Y_{41} are linear combinations of the gains in the $\underline{K}$ matrix.

The values of Y_{11}, etc. in the tested controllers are given in Table IV-1.

Using the closed loop C.E. derived in Appendix C, it is seen that $\delta S1$ of the first controller, if used as the order to the planes in a stern planes only case, is expected to result in a stable submarine (roots with negative real parts). $\delta S1$ of the second controller in a similar use is expected to result in an unstable submarine (roots with positive real parts).

For each of the combined mode controllers, different values of X were tried. The first controller showed improved response in terms of steady state depth error, as X was increased from 0.005 to 500. Results are plotted in Fig. IV-7

to IV-15 and arithmetic values of important response characteristics are collectively presented in Table IV-2. From this table it is seen that for X=0.3 the steady state depth error was approximately -6.4 ft and the steady state pitch error -2.9×10^{-2} rad (1.6°). For X=500, the s.s. depth error was approximately -4.52 ft and the s.s. pitch error again -2.9×10^{-2} rad. Both responses are considered acceptable and so, for this controller the steady state error criterion does not impose any strict limits on X. Control effort differences are not easily distinguishable and simple inspection of the stern plane angle plots is not enough for conclusions.

When the value X=0.5 was used in the C M C , the stern plane angle attained its minimum s.s. value.

The second controller (B=800, C=0.001, E=0.001) was tested for the values of X between 0.5 and 6.0. Numerical results are given in Table IV-3. For values of X greater than 3.75, the system was unstable.

In Fig. IV-16, the response characteristics of the submarine with the C M C controller and X=6 are plotted. The plots are representative of responses resulting from the C M C with values of X greater than 3.75. Responses for X less than 3.75 are plotted in Fig. IV-17, 18. For X=0.5, the stern plane s.s. angle reaches a minimum value. On the other hand, the s.s depth error is minimized for X values near 3.75.

The region of acceptable values of X is considerably reduced in this controller. Again, steady state error and control effort criteria are not enough to distinguish any

"best" value at the exponent. Responses to positive or negative step forces were similar (opposite sign).

2. Application of Step Force at Forward Trim Tank

The step force was applied to the forward trim tank and each C M C tested again. Application of the force to the aft tank was not considered since it will saturate the stern planes first.

Use of the first controller (B=800, C=10, E=1) with values of X greater than 0.9 resulted in unacceptable responses. The submarine followed a sinusoidal path between a depth of -30 to -45 ft.

In the region $0.005 \leq X \leq 0.9$ and among the tested values, X=0.7 gave minimum s.s. depth error of -25.15 ft, steady state pitch error of -0.14 rad (8°) and minimum stern plane s.s. angle (11.16°).

Table IV-4 presents numerical results of the tests, while the corresponding responses are plotted on Fig. IV-19 to IV-23.

In Fig. IV-19, the value of X=1.0 has been used but the curves are characteristic of submarine responses for values of X larger than 0.9.

Use in the second controller (B=164, E=0.001, C=0.001) of X=0.5, gave the first indications of its stabilizing capabilities. After a transient oscillating period the submarine reached a s.s. value of -28 feet in depth and -8° in pitch.

Values of X greater than 0.5 resulted in a sinusoidal response of the boat around an average depth of -40 ft.

The response characteristics for X=0.9 are shown in Fig. IV-24 and are representative of responses when values of X greater than 0.5 are used.

From Table IV-5 and Fig. IV-25 to IV-28, it is seen that values of X around 0.4 result in relative better response characteristics. The value of X=0.005 is destructive for the boat, sending it quickly to the surface at an increasing positive angle. It is noted that when the step was applied at the forward trim tank, permissible values of X became smaller and were restricted also in smaller interval.

Application of the step on the forward trim tank results in higher values for depth and pitch rates than in the case where the step was applied on the auxiliary tank. This can be used as a crude indication that higher rates require smaller values of X, but in any case no indication was found that there is a unique "best" value of X.

Since application of step input forces and moments and use of minimum s.s. errors were not considered satisfactory for the representation of near surface effects and determination of the "best" value of the exponent X, the next step tested application of pulse forces (approximating impulse ones) at the forward trim tank.

3. Narrow Pulse Force at Forward Trim Tank

The magnitude of the pulse force used was the same as for the step, i.e., about 70 tons. Its time duration was 10 s. (Fig. IV-5a). Both controllers with B=800, C=10, E=1 and B=164, C=0.001, E=0.001 were tested. Table IV-6 presents

some of the important characteristics of submarine response when the first controller is in use. Corresponding curves are shown in Fig. IV-29 to IV-36. For values of X larger than 0.6, the submarine is unstable. Responses are similar to those indicated in Fig. IV-29, which correspond to X=0.7. Values of X from 0.005 to 0.6 gave acceptable results. There are small differences in depth and pitch overshoots but these differences are not sufficient to justify conclusions about optimality of a specific value of X. Values of X between 0.45 and 0.6 result in slightly improved response characteristics in comparison with the rest of the permissible values, when depth overshoot is considered.

Tests with the second controller gave, in general, unacceptable results. For values of X larger than 0.2 the boat was unstable. At the value of X=0.005, once again the controller fails and the submarine moves unhelped to surface. The results are shown in Fig. IV- 37 to IV-42 and in Table IV-7.

Judging from the tested values of X, it appears that the best to be expected after the pulse is removed, is a slightly damped sinusoidal response or even a small amplitude undamped one. For this to happen, the value of X is expected to be between 0.1 and 0.2.

4. Simulation with an Alternate Formulation of the CMC

Existing information from previous experimental work by other sources revealed that the value X=0.5 could be "best" for a specific design of combined mode controllers where

(a) the both planes component-controller was designed optimal.

(b) the stern planes component-controller was designed to give the same closed loop C.E. as in (a).

(c) δ_{s1} of the both planes-component-controller if used in a stern planes only submarine, would slightly stabilize it. The optimal both planes controller with B=800, C=10, E=1 fulfills condition (c) and the corresponding C.E. found from program 6. was used in program 4. to give feedback gains in δ_{s2} of the CMC. The controller used in Ref. 1 with D=3000, C=10, E=1 also satisfies the condition (c) and was tested in CMC too.

To complete the picture, the controller with B=164, C=0.001, E=0.001 was used in similar tests, although it was known that its order δ_{s1} would result in an unstable stern only mode submarine.

In Fig. IV-43 to IV-47, the response characteristics of the submarine to a pulse force at FT are presented. The CMC uses a BPOC with B=800, C=10, E=1 and a SOPC resulting from the same closed loop C.E.

Numerical results are collectively presented in Table IV-8. Compared with the results in Table IV-6, they are inferior. The value of X=0.5 doesn't appear to be a "best" one. Rather a value of X=0.1 would be preferable for use in a controller.

The submarine was also simulated with D=3000, C=10, E=1 in BPOC and SOPC with the same closed loop C.E. All the tested values of X (i.e. 1.0, 0.8, 0.6, 0.5. 0.4, 0.15) resulted in unstable operation.

Figures IV-48 to IV-50 and Table IV-9 present the results when in CMC, BOPC and SOPOC with the same weighting C=10, D=3000, E=1 are used.

The last tests are with CMC that makes use of BPOC with B=164, C=0.001, E=0.001 and of SOPC with the same closed loop C.E. For values of X, 0.1, 0.3, 0.5, the submarine is quite unstable.

In Table IV-10, the permissible values of X for use in CMC are collected and presented in a condensed form. It is seen that only in the case of CMC using BPOC and SOPC are there enough cases with "best" value of X around 0.5. Even in these cases, the dependence of the parameter on the input force and on the design of the particular controller (i.e., weighting factors) is apparent.

The strong indication that the "best" value of X lacks generality and that only strict specifications would result in the acceptance of a certain value (for example X=0.5) as optimum in some sense, led in Chapter 5 to a further investigation of the proposed CMC.

Type controller	BPOC	SOPOC	SOPC
Weighting factors	B=800. C=10. E=1.	B=800. C=10. E=1.	—
δ_{s1} order	Y11=-0.020694 Y21=-10.241 Y31= 15.321 Y41=1489.8	—	—
δ_{f1} order	Y12=0.3155 Y22=28.048 Y32=-32.587 Y42=-1871.2	—	—
Closed loop C E	$S^4+1.0044S^3+0.49083S^2+$ $0.1126S+0.0049366$	—	—
Roots	S1=-0.056 S3,4=-0.272±j0319 S2=-0.404	—	—
δ_{s2} order	—	Y11=-0.31623 Y21=-42.188 Y31=53.709 Y41=4020.1	SOA=-0.31653 SOB=1.9959 SOC=64.31 SOD=-5.0764
Closed loop C E	—	—	$S^4+1.0044S^3+0.49083S^2+$ $0.1126S+0.0049366$

TABLE IV-1a. Gains of designed controllers

Type controller	BPOC	SOPOC	SOPC
Weighting factors	B=164. C=0.001 E=0.001	B=164. C=0.001 E=0.001	—
δ_{s1} order	Y11=0.24 Y21=8.7493 Y31=-3.87 Y41=9389.5	—	—
δ_{f1} order	Y12=0.9708 Y22=56.746 Y32=-62.752 Y42=-4394.9	—	—
Closed loop C E	$S^4+4.7425S^3+3.643S^2+$ $1.3716S+0.015405$	—	—
Roots	S1=-0.0116 S3,4=-0.4167±j0.4094 S2=-3.8975	—	—
δ_{s2} order	—	Y11=-1. Y21=-149.77 Y31=193.24 Y41=17739.	SOA=-0.98777 SOB=668.35 SOC=1857.21 SOD=-29929.82
Closed loop C E	—	—	$S^4+4.7425S^3+3.643S^2+$ $1.3716S+0.015405$

TABLE IV-1b. Gains of designed controllers

Type controller	BPOC	SOPOC	SOPC
Weighting factors	C=10. D=3000. E=1.	C=10. D=3000. E=1.	
δ_{s1} order	Y11=-0.030196 Y21=-6.5514 Y31=24.688 Y41=1699.		
δ_{f1} order	Y12=0.31478 Y22=28.844 Y32=-31.983 Y42=-1855.4		
Closed loop C E	$S^4+1.1572S^3+0.65527S^2+$ $0.17872S+0.018267$		
Roots	$S1,2=-0.267\pm j0.0454$ $S3,4=-0.3117\pm j0.3899$		
δ_{s2} order		Y11=-0.31623 Y21=-44.038 Y31=57.591 Y41=4204.	SOA=-1.17128 SOB=-120.9616 SOC=-148.4758 SOD=5606.32
Closed loop C E			$S^4+1.1572S^3+0.65527S^2+$ $0.17872S+0.018267$

TABLE IV-1 C Gains of designed controller

X	S.S.Depth error (ft)	S.S.Pitch error (rad)	Time to ±5% of the S.S. depth (s)	Stern plane angle at S.S (deg)
1000	-4.26	-0.027212	105	-7.
500	-4.522	-0.02875	170	-6.564
16	-4.587	-0.02876	165	-6.563
8	-4.682	-0.02876	162	-6.564
6	-4.737	-0.02876	165	-6.563
4	-4.835	-0.02877	160	-6.554
3.5	-4.688	-0.02751	175	-6.554
2.5	-4.981	-0.02878	160	-6.553
1.5	-4.993	-0.02751	175	-6.555
1.	-5.2	-0.02751	175	-6.563
0.9	-5.263	-0.02751	180	-6.553
0.7	-5.649	-0.02877	167	-6.553
0.5	-5.719	-0.0275	177	-6.125
0.3	-6.393	-0.02875	160	-6.988
0.005	-14.126	-0.029	240	-6.988

TABLE IV-2

Simulation results of the application of a 70-ton step force at the auxiliary trim tank.The submarine was "in trim" and the CMC used optimal design in the combined controllers,with B=800.,C=10.,E=1.

X	S.S.Depth error (ft)	S.S.Pitch error (rad)	Time to ±5% of the S.S. depth (s)	Stern plane angle at S.S (deg)
6	Unstable submarine		-	-
4	Unstable submarine		-	-
3.75	Unstable submarine		-	-
3.5	-5.462	-0.02886	170	-6.825
2.0	-5.48	-0.02886	170	-6.6 to -7.3
0.5	-6.059	-0.02886	170	-6.608

TABLE IV-3

Simulation results of the application of a 70-ton step force at the auxiliary trim tank. The submarine was "in trim" and the CMC used optimal design in the combined controllers with B=164.,C=0.001,E=0.001.

X	S.S.Depth error (ft)	S.S.Pitch error (rad)	Time to ±5% of the S.S. depth (s)	Stern plane angle at S.S. (deg)
500	Sinusoidal variation		-	-
50	Sinusoidal variation		-	-
10	Sinusoidal variation		-	-
2.5	Sinusoidal variation		-	-
1.5	Sinusoidal variation		-	-
1.0	Sinusoidal variation		-	-
0.9	-25.5	-0.14034	130	11.16
0.8	-25.79	-0.14036	170	11.598
0.7	-25.15	-0.14033	110	11.16
0.5	-27.25	-0.14036	140	11.598
0.05	-48.78	-0.14047	150	11.16
0.005	-92.36	-0.14032	210	11.377

TABLE IV-4

Simulation results of the application of a 70 ton step force at the forward trim tank. The submarine was "in trim" and the CMC used optimal design in the combined controllers with B=800.,C=10.,E=1.

X	S.S.Depth error (ft)	S.S.Pitch error (rad)	Time to ±5% of the S.S. depth (s)	Stern plane angle at S.S. (deg)
3.5	Sinusoidal variation		-	-
0.9	Sinusoidal variation		-	-
0.5	-28.	-0.142	475	12.31
0.4	-29.362	-0.141	145	11.86
0.3	-30.694	-0.141	145	12.296
0.05	-113.04	-0.141	160	11.419
0.005	Unstable submarine		-	-

TABLE IV-5

Simulation results of the application of a 70-ton step force at the forward trim tank. The submarine was "in trim" and the CMC used optimal design in the combined controllers with B=164., C=0.001, E=0.001 .

X	Maximum Depth overshoot(ft)		Maximum Pitch overshoot(rad)		Time to positive depth peak(s)
	-	+	-	+	
10	Unstable	submarine	-	-	-
0.9	Unstable	submarine	-	-	-
0.7	Unstable	submarine	-	-	-
0.6	7.56	1.26	0.053	0.012	32
0.55	7.56	0.826	0.0523	0.0094	32
0.5	7.52	0.826	0.0496	0.0096	32
0.45	7.6	1.141	0.0525	0.012	33
0.3	7.75	1.377	0.0531	0.0137	33
0.05	8.26	1.181	0.0367	0.0132	33
0.005	8.42	1.377	0.0291	0.0154	33

TABLE IV-6

Simulation results of the application of a 70-ton x 10 s pulse force at the forward trim tank. The submarine was "in trim" and the CMC used optimal design in the combined controllers with B=800.,C=10.,E=1.

X	Maximum Depth overshoot(ft		Maximum Pitch overshoot(rad)		Time to positive depth peak(s)
	-	+	-	+	
0.5	Unstable submarine				-
0.3	Unstable submarine				-
0.2	Residual oscillations 8.26	2.913	0.0472	0.00425	27.2
0.15	Residual oscillations 8.42	3.94	0.039	0.0164	29.6
0.1	Residual oscillations 8.89	3.86	0.0269	0.0055	32.8
0.05	Residual oscillations 9.37	4.33	0.00157	0.0192	39.

TABLE IV-7

Simulation results of the application of a 70 ton x 10 s pulse force at the forward trim tank.The submarine was " in trim " and the CMC used optimal design in the combined controllers with B=164.,C=0.001,E=0.001

X	Maximum Depth overshoot(ft)		Maximum Pitch overshoot(rad)		Time to positive depth peak(s)
	-	+	-	+	
0.9	Unstable submarine				-
0.7	Unstable submarine				-
0.5	11.14	1.023	0.075	0.0125	27
0.4	10.74	0.157	0.061	0.0094	28
0.3	10.31	2.36	0.049	0.0244	37
0.1	9.13	1.42	0.0031	0.0157	33

TABLE IV-8

Simulation results of the application of a 70 ton x 10 s pulse force at the forward trim tank.The submarine was " in trim " and the CMC used combined controllers with the same CE.(BPOC with B=800.,C=10.,E=1.)

X	Maximum Depth overshoot(ft)		Maximum Pitch overshoot(rad)		Time to positive depth peak(s)
	-	+	-	+	
0.4	7.637	0.945	0.0456	0.14	30
0.5	7.637	0.771	0.0456	0.018	30
0.6	7.48	1.18	0.047	0.0145	30

TABLE IV-9

Simulation results of the application of a 70 ton x 10 s pulse force at the forward trim tank.The submarine was " in trim " and the CMC used optimal design in the combined controllers with D=3000.,C=10.,E=1.

		CMC 1			CMC 2		
		BPOC SOPOC } Same weighting factors			BPOC SOPOC } Same CE		
		B=800,C=10,E=1.	D=3000,C=10,E=1.	B=164,C=0,001,E=0,001	B=800,C=10,E=1	D=3000,C=10,E=1.	B=164,C=0,001,E=0,001
STEP AT AU	"Best" X	1000		3,5			
	"Stable" region of X values	[0,005,1000] Table IV-2	—	[0,5 , 3,5] Table IV-3	—	—	—
STEP AT FT	"Best" X	0,7		0,4 < X < 0,5			
	"Stable" region of X values	[0,005,0,9] Table IV-4	—	[0,005 , 0,5] Table IV-5	—	—	—
PULSE AT FT	"Best" X	0,5	0,4 < X < 0,6	0,05 < X < 0,2	0,1		
		0,69 < X < 1.35	1,29 < X < 1.84	0,85 < X < 0,89	0,29 < X < 1	0,71 < X < 1.14	0,066 < X < 0,133
	"Stable" region of X values	[0,005 ,0,6] Table IV-6	0,4 < X < 0,6 Table IV-9	0,05 < X < 0,2 Table IV-7	[0,1 , 0,5] Table IV-8	None found	None found

TABLE IV-10

Values of 'X' in CMC

V. FURTHER INVESTIGATION OF THE COMBINED MODE CONTROLLER (CMC)

A. ANALYSIS OF CONTROLLER OPERATION

In Chapter IV a controller was devised for the near surface submarine. Its stern plane order was formulated by the combination of the stern plane orders of two separately designed controllers, one for a stern planes only submarine and one for a submarine having both planes available.

The order took the form

$$\delta s_{com} = K\delta s_1 + (1-K)\delta s_2 \quad \text{with} \quad K = \left(\frac{0.436}{\delta f_{com}}\right)^x$$

and was used for values of $K < 1$.

In one case, the two separate controllers were the result of optimal design with the same weighting factors for both. In the second case, while the both planes controller was kept, the stern planes one had its gains adjusted to give the same closed loop C.E. The controllers were used on a number of simulation runs and in order to compare their effectiveness under a sea way, step and pulse forces were applied on the submarine.

It was desired also to determine a value or region of values of the parameter X that would correspond to improved performance of the system. If existence of such a value or region of values was apparent, then a theoretical justification would probably follow.

Examination of the results shows that such a value or region of values is not likely to exist in general. On the

other hand, for a certain controller design and under certain inputs, values of X around 0.5 seem to offer better results.

More specifically, as seen from Table IV-10, this was the case for the C M C using optimal separate controller design with weighting factors B=800, C=10, E=1 or D=3000, C=10, E=1 and when a narrow pulse force was applied at the forward trim tank.

When the controller with B=164, C=0.001, E=0.001 was used, the results, not acceptable in general, showed improvement for values of X around 0.15. It appears that "optimality" of X is directly connected with the design of the separate controllers and the type of applied force. This then implies that generalization of the results is not possible and a specific value of X only accidentally can result as the "best" in a C M C with arbitrary separate controller designs.

To further investigate that view, the following procedure was adopted: A linear relation is assumed between δ_{s1}, δ_{s2} for example $\delta_{s1} = \Lambda \cdot \delta_{s2}$ where Λ is a constant. Then

$$\begin{aligned} \delta_{scom} &= K \cdot \delta_{s1} + (1-K) \cdot \delta_{s2} \\ &= \delta_{s2} + K(\delta_{s1} - \delta_{s2}) \\ &= \delta_{s2} + [1 + K \cdot (\Lambda - 1)] \end{aligned}$$

Setting $M = 1 + K \cdot (\Lambda - 1)$ gives $\delta_{scom} = M \cdot \delta_{s2}$. So under the assumption of a linear relation between the orders δ_{s1}, δ_{s2}, the result is that δ_{scom} can be expressed as a linear combination of δ_{s2} only. δ_{scom} is used in C M C when due to the application of the external forces, a stern planes only submarine configuration has resulted. To each value of M, a stern planes only controller corresponds.

From this family of controllers the one or the ones with the most desirable response characteristics to certain inputs can be found by tests on the stern planes only submarine. Then the "best" value of X in the original expression of will be the one capable of keeping the values of M near the selected one, for all the expected values of $\delta fcom$. In this way, the dependence of X on the controller designs and the applied inputs will have been shown.

The above can be considered as a short description of what now is presented in detail.

From optimal design with the assumption of infinite duration process and in general from the case of state feedback design, $\delta s2$ results in the form

$\delta s2 = SOA \cdot X1 + SOB \cdot X2 + SOC \cdot X3 + SOD \cdot X4$ where

X_1, X_2, X_3, X_4 are the states and SOA, SOB, SOC, SOD are the gains. The closed loop C.E. as found in Appendix C is given by:

$$S^4 + \left[(-A11 - A22) - B1 \cdot SOB - B2 \cdot SOD \right] \cdot S^3$$

$$+ \left[\begin{array}{l} (A11 \cdot A22 - A12 \cdot A21) - B1 \cdot SOA + (A22 \cdot B1 - A21 \cdot B2) \cdot SOB - B2 \cdot SOC \\ \qquad + (A11 \cdot B2 - A12 \cdot B1) \cdot SOD \end{array} \right] \cdot S^2$$

$$+ \left[(B1 \cdot A22 - B2 \cdot A21 + B2 \cdot U_c) \cdot SOA + (B2 \cdot A11 - B1 \cdot A12) \cdot SOC \right] \cdot S$$

$$+ \left[(B1 \cdot U_c \cdot A12 - B2 \cdot U_c \cdot A11) \cdot SOA \right]$$

Then the order $\delta s_{com} = M \cdot \delta s_2$, if substituted in, will result in variation of the position of the eigen values of the above equation, since the gains are multiplied by the parameter M. For each set of values of feedback gains corresponding to a certain design of stern only mode controller, the characteristic equation takes the form:

$$S^4 + (A \cdot M + C_1) \cdot S^3 + (B \cdot M + C_2) \cdot S^2 + (C \cdot M + C_3) \cdot S + (D \cdot M + C_4)$$

For the given submarine at the velocity of 25.33 ft/sec:

$C_1 = 0.1982$

$C_2 = 9.7316 \times 10^{-3}$

$C_3 = 0$

$C_4 = 0$

A, B, C, D calculated in program 8 are functions of the feedback gains. This form is suitable for root loci methods. The loci were plotted for different controller designs. It must be noted here that root loci methods are not in general helpful when impulse responses are examined, since there doesn't appear to be any simple correlation between pole motion and the corresponding transient response of the system. The trouble arises because the pole -zero concepts essentially refer to linear systems. As soon as the poles are in motion, the usual relations between pole and zero positions and overshoot for example, are no longer valid. What was done in that case was to try to use probable existing simple correlation between pole motion and transient response, which would explain accidental "optimality" of the exponent 0.5.

The following cases were examined:

(1) SOPOC and BPOC with weighting B=800, C=10, E=1.

(2) SOPC and BPOC (B=800, C=10, E=1) with the same closed loop C.E.

(3) SOPOC and BPOC with weighting B=164, C=0.001 and E=0.001.

(4) SOPC and BPOC (B=164, C=0.001, E=0.001) with the same closed loop C.E.

(5) SOPOC and BPOC with weighting D=3000, C=10, E=1.

(6) SOPC and BPOC (D=3000, C=10, E=1) with the same closed loop C.E.

To establish an approximating relationship of the form $\delta_{s1} = \eta \cdot \delta_{s2}$, the following simplifying assumptions, valid only for the given submarine, were made:

(1) Average depth rate during application of impulse at FT tank = (-0.7) - (-0.8) ft/s.

(2) Pitch rate = $\dfrac{\text{Depth rate}}{200}$ to $\dfrac{\text{Depth rate}}{150}$ = $\dfrac{\text{D. rate}}{\text{N}}$

The above values are averages derived from tests made on the given submarine by applying forces of about 70 tons in one of the auxiliary tanks. To saturate the forward planes, the forces have to be applied in the forward or the central (auxiliary) trim tanks.

Then:

$$\delta s1 = FSA \cdot x_1 + FSB \cdot x_2 + FSC \cdot x_3 + FSD \cdot x_4$$

$$\delta f1 = \delta fcom = FBA \cdot x_1 + FBB \cdot x_2 + FBC \cdot x_3 + FBD \cdot x_4$$

$$\delta s2 = SOA \cdot x_1 + SOB \cdot x_2 + SOC \cdot x_3 + SOD \cdot x_4$$

Setting $x_1 = x_2 \cdot t$, $x_3 = x_4 \cdot t$ and $x_4 = \frac{x_2}{N}$.

$$\delta s1 = \left[\left(FSA + \frac{FSC}{N}\right) \cdot t + \left(FSB + \frac{FSD}{N}\right)\right] \cdot x_2$$

$$\delta s2 = \left[\left(SOA + \frac{SOC}{N}\right) \cdot t + \left(SOB + \frac{SOD}{N}\right)\right] \cdot x_2$$

$$\delta f com = \left[\left(FBA + \frac{FBC}{N}\right) \cdot t + \left(FBB + \frac{FBD}{N}\right)\right] \cdot x_2$$

1. SOPOC and BPOC with weighting B=800, C=10, E=1

From optimal controllers	From program 8
FSA = -0.20694×10^{-1}	A = 0.77984
FSB = -0.4043	B = 0.45489
FSC = 15.321	C = 0.10600
FSD = 58.51	D = 0.49318×10^{-2}
FBA = 0.3155	
FBB = 1.1073	
FBC = -32.57	
FBD = -73.85	
SOA = -0.31623	
SOB = -1.665	
SOC = 53.71	
SOD = 158.71	

Then, using N = 175

$\delta_{s1} \approx (6.68\text{x}10^{-2} \cdot t - 0.07) \cdot X_2$

$\delta_{s2} \approx (-9.32\text{x}10^{-3} \cdot t - 0.758) \cdot X_2$

$\delta_{f1} \approx (0.129 \cdot t + 0.685) \cdot X_2$ and with $X_2 \approx -0.8$

$\delta_{f1} \approx -0.103 \cdot t - 0.55$

For values of t between 0 and 12 s., (time duration of the pulse force plus 2 s. for which the submarine is considered to continue its path with the same characteristics), δ_{f1} varies between 0.55 and 1.79. The ratio $r = \frac{0.436}{|\delta_{f1}|}$ will vary from 0.79 to 0.244. The average value of r will be 0.433.

Also, $\frac{\delta_{s1}}{\delta_{s2}} = \frac{6.68 \times 10^{-2} \cdot t - 0.07}{-9.32 \times 10^{-3} \cdot t - 0.758}$ and

t	0	2	4	6	8	10	12
δ_{s1}/δ_{s2}	0.092	-0.082	-0.192	-0.285	-0.36	-0.418	-0.469

The average value of $\left(\frac{\delta_{s1}}{\delta_{s2}}\right)$ is approximately -0.245. From $\delta_{scom} = \delta_{s2} + K(\delta_{s1} - \delta_{s2})$ and for $\delta_{s1} \approx -0.245\,\delta_{s2}$

$$\delta_{scom} = \delta_{s2} \cdot (1 - 1.245\,K)$$
$$= \delta_{s2} \cdot m$$

The corresponding root locus is plotted in Fig. V-1. For a system with a dominant pair of complex roots, values of in the region of ζ = 0.4-0.6 would be suggested. In this case, the order $m \cdot \delta_{s2}$ was used in simulation runs of a stern only mode submarine with m taking the values 0.3, 0.6, 0.8, 1, 1.2.

To compensate for the effect of the forward planes, the magnitude and the time duration of the applied force were reduced to about 35 tons and 6 sec. correspondingly. The results are shown in Fig. V-2 to V-4 and suggest values of m between 0.3-0.6 (ζ = 0.22- 0.4). Then,

$$0.3 < 1 - 1.245K < 0.6 \quad \text{leads to}$$

$$0.562 > K > 0.321$$

Using $K = \left(\frac{0.436}{\delta_{fcom}}\right)^X = (0.433)^X$

$$0.562 > (0.433)^X > 0.321 \quad \text{and}$$

$$0.69 < X < 1.35.$$

If in the the above calculations the extreme values of $\frac{\partial s_1}{\partial s_2} = 0.092$ and $\partial f = -0.55$ are used, they result in

$1.12 < X < 3.52$

If the values $\frac{\partial s_1}{\partial s_2} = -0.469$ and $\partial f_1 = -1.79$ were used, then

$0.524 < X < 0.92$

Finally, if the value N=200 had been used, it would result, using average values for $\frac{\partial s_1}{\partial s_2}$ and r, in:

$\partial s_1 = -0.165 \cdot \partial s_2$

$r = 0.390$

$0.54 < X < 1.13.$

2. SOPC and BPOC (B=800, C=10, E=1) with the same closed loop C.E.

From BPOC

$FSA = -0.20694 \times 10^{-1}$

$FSB = -0.4043$

$FSC = 15.321$

$FSD = 58.51$

$FBA = 0.3155$

$FBB = 1.1073$

$FBC = -32.57$

$FBD = -73.85$

From program 4

$SOA = -0.31653$

$SOB = 1.996$

$SOC = 64.31$

$SOD = -5.076$

From program 6

$$C.E. = S^4 + 1.0044S^3 + 0.49083S^2 + 0.1126S + 0.49366 \times 10^{-2}$$

From program 8

$A = 0.80618$

$B = 0.48109$

$C = 0.1126$

$D = 0.49365 \times 10^{-2}$

Again, using N=175

$$\delta s_1 \approx (6.68\times10^{-2}\cdot t - 0.07)\cdot x_2$$

$$\delta s_1 \approx (-0.103\times t - 0.55)\cdot x_2$$

$$\delta s_2 = 5.095\times10^{-2}\cdot t + 1.967$$

$$|r|_{av} = 0.43$$

$$(\delta s_1/\delta s_2) = 0.159$$

Then

$$\delta s_{com} = \delta s_2\cdot(1-0.841\,K)$$
$$= \delta s_2\cdot m$$

From the corresponding root locus in Fig. V-5a, b and for $\zeta = 0.2 - 0.4$, m is found between 0.34 - 0.64. So,

$$0.34 < 1 - 0.841\cdot K < 0.64$$

$$0.784 > K > 0.428$$

$$0.288 < X < 1.0$$

3. SOPOC and BPOC with weighting B=164, C=0.001, E=0.001

From optimal controllers

FSA = 0.24
FSB = 0.3459
FSC = -4.2
FSD = 370.9
FBA = 0.9705
FBB = 2.240
FBC = -62.6
FBD = -173.37
SOA = -1.0
SOB = -5.9127
SOC = 193.24
SOD = 700.3158

From program 8

A = 4.0547
B = 1.7757
C = 0.34961
D = 0.015596

with N=175

$$\delta s_1 = (0.216 \cdot t + 2.465) \cdot x2$$

$$\delta s_2 \approx (0.104 \cdot t - 1.91) \cdot x2$$

$$\delta f_1 \approx (0.612 \cdot t + 1.249) \cdot x2 = -0.49 \cdot t - 1 \quad (x_2 = -0.8)$$

$$r_{av} \approx \left(\frac{0.436}{|\delta f_1|} \right)_{av} \approx 0.162$$

$$\left(\frac{\delta s_1}{\delta s_2} \right)_{av} \approx -3.56$$

For $\zeta = 0.2-0.4$, the root locus (Fig. V-6a, b) gives

$\eta = 0.035-0.092$

Then

$$\delta s_{com} = \delta s_2 + K(-3.56 \delta s_2 - \delta s_2)$$
$$= \delta s_2 (1 - 4.56 \cdot K)$$

$$0.035 < 1-K \cdot 4.56 < 0.092$$
$$0.965 < -K \cdot 4.56 < 0.908$$
$$0.211 > K > 0.200$$
$$0.85 < X < 0.89$$

4. SOPC and BPOC (B=164, C=0.001, E=0.001) with the same closed loop C.E.

From BPOC

FSA = 0.24

FSB = 0.34592

FSC = -4.200

FSD = 370.9

FBA = 0.97051

FBB = 2.240

FBC = -62.6

FBD = -173.37

C.E. from program 6

$S^4 + 4.7425S^3 + 3.6443S^2 + 1.3713S + 0.015405$

From program 4

SOA =-0.98777

SOB = 668.35

SOC = 1857.2

SOD = -22229.8

From program 8

A = 4.543

B = 3.6345

C = 1.3713

D = 0.015405

Again, $\delta_{s1} = 0.216 \cdot t + 2.465$

$\delta_{f1} \simeq 0.612 \cdot t + 1.249$

$r_{av} \simeq 0.162$

Also, $\delta_{s2} = 9.62 \cdot t + 497.3$

and $\left(\frac{\delta_{s1}}{\delta_{s2}}\right)_{av} \simeq 0.007$

From the root locus (Fig. V-7a, b) and for $\zeta = 0.2-0.4$,

$\eta = 0.12-0.22$

$$0.12 < 1 - K \cdot 0.993 < 0.22$$

$$0.886 > K > 0.785$$

$$0.066 < X < 0.133$$

5. SOPOC and BPOC with weighting D=3000, C=10, E=1.

From optimal controllers

FSA = -0.030196

FSB = -0.25864

FSC = 24.688

FSD = 67.0746

FBA = 0.31478

FBB = 1.13873

FBC = -31.983

FBD = -73.249

SOA = -0.31623

SOB = -1.7386

SOC = 57.59

SOD = 165.97

From program 8

A = 0.81664

B = 0.49528

C = 0.10839

D = 0.49318E-02

Using the above,

$$\delta_{s1} = (0.111 \cdot t + 0.124) \cdot x_2$$

$$\delta_{s2} \simeq (0.01285 \cdot t - 0.79) \cdot x_2$$

$$\delta_{f1} \simeq (0.132 \cdot t + 0.720) \cdot x_2$$

$$= -0.1056 \cdot t - 0.576$$

Then

$$\gamma_{av} = 0.417$$

$$\left(\frac{\delta_{s1}}{\delta_{s2}}\right)_{av} = -1.16$$

$$\delta_{scom} = \delta_{s2} + K(-1.16\,\delta_{s2} - \delta_{s2})$$

$$= \delta_{s2} \cdot (1 - 2.16\,K) = M \cdot \delta_{s2}$$

From the root locus (Fig. V-8a, b) and for ζ = 0.2-0.4,

M = 0.3-0.6

Then

$0.3 < 1-2.16\ K < 0.6$

$0.324 > K > 0.185$

$1.29 < X < 1.84$

6. SOPC and BPOC (D=3000, C=10, E=1) with the same closed loop C.E.

From BPOC

FSA = -0.030196

FSB = -0.25864

FSC = 24.688

FSD = 67.0746

FBA = 0.31478

FBB = 1.13873

FBC = -31.983

FBD = -73.249

From program 6

$C.E. = S^4 + 1.1572S^3 + 0.65527S^2 + 0.17872S + 0.018267$

From program 4

SOA = -1.1713

SOB = -120.9616

SOC = -148.4758

SOD = 5606.32

From program 8

A = 0.95901

B = 0.64553

C = 0.17872

D = 0.018267

Again, $\delta_{s1} \approx (0.111 \cdot t + 0.124) \cdot x_2$

$$\delta_{f1} \approx (0.132 \cdot t + 0.720) \cdot x_2$$

$$\approx -0.1056 \cdot t - 0.576$$

$$r_{av} \approx 0.417$$

Also, $\delta_{s2} = (-2.02 \cdot t - 88.9) \cdot x_2$

and $\left(\dfrac{\delta_{s1}}{\delta_{s2}}\right)_{av} = -7.41 \times 10^{-3} \rightarrow \delta_{scom} = \delta_{s2}(1 - 1.00741K) = \delta_{s2} \cdot \eta.$

From the root locus (Fig. V-9b) and for ζ =0.2-0.4, η = 0.46-0.63

$$0.536 > K > 0.367$$

$$0.71 < X < 1.14$$

It was not expected that use of the previously, almost heuristically found values of exponents would result in each case in "best" response to impulse inputs. Only for the controller using weighting B=800, C=10, E=1, simulations were done for the stern only mode submarine to find the value of η in the order $\eta \cdot \delta_{s2}$ that would result in "best" response. From that point, the condition ζ = 0.2-0.4 was arbitrarily used in the rest of the controllers, although from the corresponding root loci distinct differences in the positions and movement of roots are apparent.

If the results would approximate that of the simulation in Chapter 4, it could be said that the best value of the exponent is the one that corresponds to a stern only mode system with order $\eta \cdot \delta_{s2}$ and η taking the value resulting to the better response of a stern only planes submarine to an impulse input. The form of the order allows smooth switching, but the

only thing that determines the value of the exponent and can result in a different value each time, is the design of the separate controllers in CMC.

For easy comparison, the ranges of X variation for each case of controller, derived in that section under the assumption of linear relation of the form $\delta_{s1} = \Lambda \cdot \delta_{s2}$, are incorporated and shown encircled in Table IV-10. Furthermore, the above analysis indicates that using the relation $\delta_{s1}/\delta_{s2} = \Lambda$ in an inverse way, then:

$$\delta_{scom} = K\delta_{s1} + (1-K)\frac{\delta_{s1}}{\Lambda} = \delta_{s1}\cdot\left[K + \frac{1-K}{\Lambda}\right] =$$

$$= \delta_{s1}\left[K\frac{(\Lambda-1)}{\Lambda} + \frac{1}{\Lambda}\right] = \delta_{s1}\cdot M_1$$

The above suggests that a controller having $\delta_{scom} = \delta_{s1}\cdot[\alpha K + \beta]$ for $K < 1$ could be equivalent to the proposed CMC. This controller will not make use of a separate stern only mode design but there will be some difficulty in defining the values of α, β . The parameter M_1 will depend on the specific design of the BPOC and the specific submarine.

To demonstrate and check the above, root loci for $\delta_{s1}\cdot M$ were plotted for the three BPOC (Fig. V-10).

For the controller B=800, C=10, E=1, M_1 is selected from values that it is expected to give roots satisfying the criterion of dominant complex pair.

Then $M_{1\,LOWER} = 0.4 = M_L$

$M_{1\,UPPER} = 2.1 = M_U.$

or $$M_L < \frac{K(\Lambda-1)}{\Lambda} + \frac{1}{\Lambda} < M_u$$

For $\Lambda = -4$ (Indicated from the ratio $\frac{\delta s1}{\delta s2} = -0.245$ found before)

$$(M_L+0.25)\cdot\frac{4}{5} < K < (M_u+0.25)\cdot\frac{4}{5}$$

Using for K as before the average $(0.433)^X$, it is found:

$$\frac{\log\left[(M_L+0.25)\cdot\frac{4}{5}\right]}{\log 0.433} > X > \frac{\log\left[(M_u+0.25)\cdot\frac{4}{5}\right]}{\log 0.433}$$

and

$$0.781 > X > -0.755$$

Thus, the control order to be tested will be

$$\delta scom = \delta s1\cdot\left[1.25\cdot\left(\frac{0.436}{\delta fcom}\right)^X - 0.25\right]$$

Values of X between -1.0 and 0.75 were tested and the results are shown on Fig. V-11 to V-14. The submarine responses are acceptable. Numerical results are collectively presented in Table V-1. It is seen that the maximum depth overshoot with X = -1 is lower than any one achieved with the previously tested CMC schemes. The controller has the same degree of generality as the proposed one when the exponent X is fixed to a specific value.

It is functioning effectively only with a specific system of submarine and controller and already failed when tested with BPOC having B=164, C=0.001, E=0.001.

B. EVALUATION OF CMC

The conclusions concerning the proposed CMC are:

a. It functions satisfactorily for certain designs of the separate BPOC and SOPC or SOPOC that compose it.

b. It is desirable that δ_{s1} in BPOC, if used in a SOPC as the order to the planes, will result in a stable submarine.

c. In the cases of satisfactory operation there appears to exist a relation between the orders δ_{s1} , δ_{s2}, which also can result in a specific "optimal" value of X, not necessarily 0.5.

d. In any case, the CMC using BPOC/SOPOC or BPOC/SOPC (same C.E.) schemes may not be the near surface optimum one. As for example, when in one of the simulation runs (BPOC with B=800, C=10, E=1 and SOPC with the same C.E.) the value of γ_{21} in δ_{s2} was mistakenly written 5.056 instead of the correct 50.56, the resultant submarine response (Fig. V-15) was much improved compared with that corresponding to the correct value (Fig. IV-44)!

The last thing that was examined was the possibility that for a given submarine, use of optimal theory for a BPOC and SOPOC would always result in the same relation between δ_{s1}, δ_{s2}, possibly a linear one, independently of the values of the weighting parameters.

A combining function of the proposed form could then have elements of generality, at least for a family of controllers. This general relationship doesn't appear to exist when the optimal design is closely examined.

The steady state solution of the matrix Riccati equation can be found in terms of the eigenvectors of the 2n x 2n partitioned matrix $\underline{M} = \left[\begin{array}{c|c} \underline{A}^T & \underline{Q} \\ \hline \underline{BR}^{-1}\,\underline{B}^T & -\underline{A} \end{array}\right]$

The eigenvalues and corresponding eigenvectors of $\underline{M}$ are found:

$$\lambda = \left[\begin{array}{c|c} \underline{\lambda}_{+} & \underline{0} \\ \hline \underline{0} & \underline{\lambda}_{-} \end{array}\right]$$

where the subscripts denote the signs of the real parts of the eigenvalues, and the corresponding eigenvectors are

$$\underline{V}^{1} = \left[\begin{array}{c|c} \underline{V}_{T+} & \underline{V}_{T-} \\ \hline \underline{V}_{B+} & \underline{V}_{B-} \end{array}\right]$$

$\underline{V}^{1}$ is partitioned and the solution to the problem is

$$\underline{K} = \underline{V}_{T+} \; \underline{V}_{B+}^{-1}$$

The relationship between the values of $\underline{K}$'s in the two combined controllers is a function of $\underline{B}\;\underline{R}^{-1}\;\underline{B}^{T}$ of the controllers, if $\underline{A}$, $\underline{Q}$ are kept the same. (S O P O C has $\underline{B} = \begin{bmatrix} 0 \\ B_{11} \\ 0 \\ B_{12} \end{bmatrix}$, $\underline{R} = C$)

(B P O C has $\underline{B} = \begin{bmatrix} 0 & 0 \\ B_{11} & B_{21} \\ 0 & 0 \\ B_{12} & B_{22} \end{bmatrix}$, $\underline{R} = \begin{bmatrix} C & 0 \\ 0 & C \end{bmatrix}$)

since this is the only term that changes in $\underline{M}$.

Now if $\underline{Q}$ and $\underline{R}$ are changed, resulting in another set of optimal controllers with new weighting factors, it becomes evident that only by chance could the same relationship between S O P O C. and B P O C be found.

X	Maximum Depth overshoot(ft)		Maximum Pitch overshoot(rad)		Time to positive peak(s)
	-	+	-	+	
0.5	10.	1.496	$2.74\text{x}10^{-2}$	$1.83\text{x}10^{-2}$	32
-0.5	8.2	1.338	$3.34\text{x}10^{-2}$	$1.54\text{x}10^{-2}$	32
-0.75	8.	1.42	$3.31\text{x}10^{-2}$	$1.62\text{x}10^{-2}$	32
-1.	7.32	1.57	$2.4\text{x}10^{-2}$	$1.66\text{x}10^{-2}$	32

TABLE V-1

Simulation results of the application of a 70 ton x 10 s pulse force at the forward trim tank. The submarine was 'in trim' and the controller had $\delta_{scom} = \delta_{s1} \cdot \left\{ 1.25 \cdot \left(\frac{0.436}{\delta_{fcom}} \right)^{x} - 0.25 \right\}$ and B=800.,C=10.,E=1.

VI. DISCUSSION, CONCLUSIONS AND RECOMMENDATIONS

The problem of optimum near surface control of a submarine is relatively new and not an easy one to solve. Much of the experimental data that would help in gaining a better understanding of this subject has been classified. Optimal theory has made the setting up of a large class of optimal control problems almost routine. Again, this is not the case here.

Inequality constraints on the control variables suggested approximate methods and exact approaches for the solution, with which the author was not familiar.

It becomes necessary in a number of important fields of engineering to devise controllers for plants which exhibit very substantial changes in their characteristics, due in most instances to a changing environment. Such a devised controller was tested and analyzed since design of the optimum near surface control system would require both the classified experimental data and advanced knowledge of optimal control theory.

The proposed scheme of control order for use after the saturation of the forward planes introduces, in fact, an intentional nonlinearity, modifying the system characteristics. The theory of intentional introduction of nonlinearities into the system (linear model) is still decisively incomplete. A suitable approach is not evident for a straightforward analysis. A more extended examination could perhaps include describing function methods, phase plane description using a second order model or even consideration of the minimum time problem.

As K appears in δ_{scom} , it is a variable weighting factor that in a way selects corresponding parts of the δ_{s1} and δ_{s2} orders, after the forward planes are saturated. Because of the way K is defined, i.e. $K = \left(\frac{\delta_{fmax}}{\delta_{fcom}}\right)^{X}$ for $\delta_{fcom} > \delta_{fmax}$, its values of interest range between 1 and 0.

The value of corresponds to either a value of δ_{fmax} infinite for small values of X, or to a large value of X (10^2 - 10^5), depending on the value of the ratio $\frac{0.436}{\delta_{fcom}}$.

For a stabilizing controller, δ_{fcom} after saturation, is expected to take values in a limited range. The exponent X directs or scales the value of K somewhere in the region (0,1).

There is no indication nor even possibility that an exponent X=0.5 could automatically result in a value of K selecting the right proportion of δ_{s1} and δ_{s2} orders, to obtain the best response for every given controlled submarine and to any inputs.

The simulation runs show that even if the value of X=0.5 results in an order δ_{scom} which satisfies criteria of "optimality" in response to a certain input, changing the input will reveal disadvantages in other respects, which could be softened by another value of X.

The submarine is already a nonlinear system. It may behave quite differently with different input functions. The proposed controller was tested for a limited number of step and impulse inputs at a constant velocity. The logical design of a nonlinear system requires a complete description of the input signals.

As demonstrated in Ref. 6, the design of a linear system to minimize the mean square error with random input signals requires a knowledge of the second probability distribution functions of the signal and noise components of input. In contrast, the design to yield the nonlinear system which is optimum on the basis of the mean square error criterion theoretically requires knowledge of all probability distributions.

To the most elegant design problem, involving the determination of nonlinear elements appropriate for insertion in either a linear or nonlinear system to improve overall dynamic performance, perhaps the major difficulty lies in the problem of phrasing the specifications in a form amenable to analytical design techniques. The marked dependence of the performance characteristics of nonlinear systems on the particular input signals means that in general the specifications must include precise description of input and desired output. For the specifications developed during this work, the most promising scheme of Combined Mode Controller appears to be the one that combines two separate optimal designs. It is then preferable that δ_{s1} also corresponds to the order of a stabilizing Stern Only Plane Controller.

The exact tracking controller can also be used in the Combined Mode Controller.

The Combined Mode Controller can be tested under a fictitious seaway simulated by the parallel application of step and sinusoidal forces.

Finally as an alternate solution, a model reference method can be worked. In that case, an improved linear model is required. The model can be used in an stochastic estimator which will provide the necessary decision logic to modify the controller parameters.

APPENDIX A

ON LINEARIZATION

In Appendix F the general equations of motion, given in terms of a set of axes fixed in the boat, are repeated from Ref. 1. These are six nonlinear scalar equations, representing the components of dynamic equilibrium in each of the six degrees of freedom. Also, six additional equations, relating the motions of the boat in body axes to the orientation and motion of the boat with respect to fixed axes are given.

The above mentioned set of twelve nonlinear equations, characterize the submarine and are sufficient to determine its response to an arbitrary set of time dependent forces and moments. Where only vertical motions are of interest, considerable simplification may be achieved. Although this constrains the applicability of the results, somehow the constraint will not be too severe, since submarines spend the vast majority of their underway time traveling on a straight course between the proverbial points A and B.

Further, for the class of problems in which the submarine perturbations from equilibrium are small, the equations of motion may be linearized and thus lead to even greater simplification with acceptable accuracy.

Basically a Taylor's series expansion of all variables is made about an equilibrium condition. After substituting these expansions into the equation of motion, terms of higher than first order, are omitted. For small perturbations about the equilibrium condition, the actual equations are very closely approximated by the linearized equations. As the magnitude of the perturbations increases the accuracy of the linearized equations is degraded.

The equilibrium condition used, will be for the submarine traveling horizontally at a steady speed and with a steady pitch angle (assumed zero for simplicity). As part of the equilibrium condition the "in trim" condition of weight and longitudinal center of gravity is included.

The initial reference values of all the variables are denoted by the subscript i and the small perturbations are indicated as follows:

- changes in U, V, W etc. are indicated by small letters, i.e.

  $U = u_i + u$, $P = p_i + p$, etc.

- changes in control surfaces are denoted by

  δs, δb, δr

In this paper the submarine is considered as a rigid body and in Appendix B, derivation of the required equations from first principles is presented. (Ref. 9)

The kinematic and dynamic equations of motion derived in Appendix are collected below for ready reference. The XZ plane of the coordinate system is taken at the plane of symmetry of the boat so that

$$I_{yz} = I_{xy} = 0.$$

Then

$$\begin{aligned}
X &= M(\dot{U}+QW-RV) \\
Y &= M(\dot{V}+RU-PW) \\
Z &= M(\dot{W}+PV-QU) \\
K &= I_x\dot{P}+QR(I_z-I_y) \\
M &= I_y\dot{Q}+RP(I_x-I_z) \\
N &= I_z\dot{R}+PQ(I_y-I_x)
\end{aligned} \tag{1}$$

The details of linearization of equations for Z,M,X are carried through.

The hydrodynamic forces and moments depend on the orientation, configuration and motion variables. Thus:

$$\begin{aligned}
&Z=Z(U,V,W,\dot{U},\dot{V},\dot{W},P,Q,R,\dot{P},\dot{Q},\dot{R},\theta,\phi,\psi,Ds,Dr,Db). \\
&X=X(U,V,W,\dot{U},\dot{V},\dot{W},P,Q,R,\dot{P},\dot{Q},\dot{R},\theta,\phi,\psi,Ds,Dr,Db). \\
&M=M(U,V,W,\dot{U},\dot{V},\dot{W},P,Q,R,\dot{P},\dot{Q},\dot{R},\Theta,\phi,\psi,Ds,Dr,Db).
\end{aligned} \tag{2}$$

For steady equilibrium the reference values of the variables are taken as constants. Making the appropriate substitutions:

$$Z = m\left[\dot{\omega} + (p_i+p)(v_i+v) - (q_i+q)(u_i+u)\right]$$

$$M = I_Y\dot{q} + (r_i+r)(q_i+q)(I_Y - I_X) \qquad (3)$$

$$X = m\left[\dot{u} + (q_i+q)(\omega_i+\omega) - (r_i+r)(v_i+v)\right]$$

where Z,M,X are given by:

$$Z = Z(u_i+u,\ v_i+v,\ \dots\dots\ D_b+\delta_b)$$

$$M = M(u_i+u,\ v_i+v,\ \dots\dots\ D_b+\delta_b) \qquad (4)$$

$$X = X(u_i+u,\ v_i+v,\ \dots\ D_b+\delta_b)$$

The Taylor expansion of Eqn. (4) is

$$Z = \left[1 + (uD_u + vD_v + \dots + \delta_b D_{\delta_b}) + \frac{(uD_u + vD_v + \dots \delta_b D_{\delta_b})^2}{2!} + H.O.T\right] Z(u_i, v_i \dots \delta_b)$$

$$M = \left[1 + (uD_u + vD_v + \dots + \delta_b D_{\delta_b}) + \frac{(uD_u + vD_v + \dots \delta_b D_{\delta_b})^2}{2!} + H.O.T\right] M(u_i, v_i \dots \delta_b)$$

$$X = \left[1 + (uD_u + vD_v + \dots + \delta_b D_{\delta_b}) + \frac{(uD_u + vD_v + \dots \delta_b D_{\delta_b}}{2!} + H.O.T\right] X(u_i, v_i \dots \delta_b)$$

(5)

Since we are dealing with small perturbations, from the equilibrium condition, the second and higher order terms may be omitted, leaving only the first two terms of each of the above series.

Writing the D operators in their partial derivative notation

$$Z = Z(u_i, v_i, \ldots \delta b) + u\frac{\partial Z}{\partial u}\Big|_i + v\frac{\partial Z}{\partial v}\Big|_i + \cdots + \delta b\frac{\partial Z}{\partial \delta b}\Big|_i$$

$$M = M(u_i, v_i, \ldots \delta b) + u\frac{\partial M}{\partial u}\Big|_i + v\frac{\partial m}{\partial v}\Big|_i + \cdots + \delta b\frac{\partial m}{\partial \delta b}\Big|_i \qquad (6)$$

$$X = X(u_i, v_i, \ldots \delta b) + u\frac{\partial x}{\partial u}\Big|_i + v\frac{\partial x}{\partial v}\Big|_i + \cdots + \delta b\frac{\partial x}{\partial \delta b}\Big|_i$$

where the subscript i denotes that the partial derivative is taken at the initial equilibrium condition.

The use of only first order terms, implies that the forces and moments vary linearly with the disturbance variables for small enough disturbances.

On substituting Eqns (6) in Eqns (3) one obtains:

$$Z_i + u\frac{\partial z}{\partial u}\Big|_i + v\frac{\partial z}{\partial v}\Big|_i + \cdots + \delta b\frac{\partial z}{\partial \delta b}\Big|_i = m(\dot{w} + (p_i+p)(v_i+v) - (q_i+q)(u_i+u))$$

$$M_i + u\frac{\partial m}{\partial u}\Big|_i + v\frac{\partial M}{\partial v}\Big|_i + \cdots + \delta b\frac{\partial m}{\partial \delta b}\Big|_i = I_y\dot{q} + (r_i+r)(p_i+p)(I_y - I_x)$$

$$X_i + u\frac{\partial x}{\partial u}\Big|_i + v\frac{\partial x}{\partial v}\Big|_i + \cdots + \delta b\frac{\partial x}{\partial \delta b}\Big|_i = m\left[\dot{u} + (q_i+q)(w_i+w) - (r_i+r)(v_i+v)\right]$$

(7)

Since Z_i, M_i, X_i are equal to the unperturbed terms on the right hand side of Eqns. (7) respectively, they may be cancelled out. Retaining only first order terms and writing Z_u for etc. gives:

$$Z_u u + Z_v v + Z_w w + \dot{u} Z_{\dot{u}} + \dot{v} Z_{\dot{v}} + \dot{w} Z_{\dot{w}} + Z_p p + Z_q q + Z_r r + \dot{q} Z_{\dot{q}} +$$
$$\dot{p} Z_{\dot{p}} + \dot{r} Z_{\dot{r}} + \theta Z_\theta + \phi Z_\phi + \psi Z_\psi + \delta_s Z_{\delta_s} + \delta_b Z_{\delta b} + \delta_r Z_{\delta r} =$$
$$m(\dot{w} - q_i u - q u_i + p_i v + p v_i)$$

$$(X_{\dot{u}} - m)\dot{u} + X_{\dot{v}} \dot{v} + X_{\dot{w}} \dot{w} + X_u u + (X_v + m r_i) v + (X_w - m q_i) w + X_{\dot{p}} \dot{p} +$$
$$X_{\dot{q}} \dot{q} + X_{\dot{r}} \dot{r} + X_p p + (X_q - m w_i) q + (X_r + m v_i) r + X_\theta \theta + X_\phi \phi +$$
$$X_\psi \psi + X_{\delta b} \delta b + X_{\delta_s} \delta s + X_{\delta r} \delta r = 0$$

(8)

$$M_u u + M_v v + M_w w + M_{\dot{u}} \dot{u} + M_{\dot{v}} \dot{v} + M_{\dot{w}} \dot{w} + M_p p + M_q q + M_r r +$$
$$M_{\dot{q}} \dot{q} + M_{\dot{p}} \dot{p} + M_{\dot{r}} \dot{r} + M_\theta \theta + M \phi \phi + M_\psi \psi + M_{\delta_s} \delta s + M_{\delta_b} \delta b + M_{\delta_r} \delta r =$$
$$I_y \dot{q} + (r_i p + r p_i)(I_y - I_x)$$

For the straight, level symmetrical travel, these equations may be considerably simplified.

For a truly symmetrical boat it is clear that the side force Y, rolling moment K, yawing moment N, and rudder moment are zero. Thus the derivatives of the assymmetric moments and forces with respect to the symmetric variables U, W, Q, δr, Θ are all zero.

Furthermore, v_i, p_i, q_i, r_i, ϕ_i are equal to zero. Since we use stability axes $\omega_i = 0$ and u_i is the reference boat speed u_o. Since we are considering level flight $\theta_i = 0$. In addition the following approximations are made:

1. We may neglect the derivatives of the symmetric forces and moments X, Z, M with respect to the asymmetric variables v, p, r, ϕ, δ_r

2. We may neglect all the acceleration derivatives except $Z\dot{w}$, $Z\dot{q}$, $M\dot{w}$, $M\dot{q}$. When the above simplifications are made, the linearization of Eqns. (1), for calm sea operation, results in

$$(mD - X_u)u - X_w w - [X_q D + X_\Theta]\Theta = 0$$

$$-Z_u u + [(m - Z_{\dot{\omega}})D - Z_\omega]\omega - [Z_{\dot{q}}D^2 + (Z_q + mU_o)D + Z_\Theta]\Theta - Z_{\delta_s}\delta_s - Z_{\delta_b}\delta_b$$

$$-M_u u - (M_{\dot{\omega}}D + M_\omega)\omega + [(I_Y - M_{\dot{q}})D^2 - M_q D - M_\Theta]\Theta - M_{\delta_s}\delta_s - M_{\delta_b}\delta_b \quad (9)$$

Also

$q = \dot{\Theta}$

$(\text{Depth})' = \omega - \upsilon_0 \Theta$

The above equations are now compared with the corresponding Equations of Ref. 4 and those of Ref. 1, where linearization was accomplished by dropping all nonlinear terms. To make the comparison easier the equations are written in similar form.

I. Derived

$$(m - Z_{\dot{\omega}})\dot{\omega} - Z_{\dot{q}}\dot{q} = Z_\omega \omega + (Z_q + m u_0) q + [Z_u u + Z_\Theta \Theta] + \text{control forces}$$

$$-M_{\dot{\omega}}\dot{\omega} + (I_y - M_{\dot{q}})\dot{q} = M_\omega \omega + M_q q + [M_u u + M_\Theta \Theta] + \text{control forces}$$

II. Reference 1

$$(m - Z_{\dot{\omega}})\dot{\omega} - e Z_{\dot{q}}\dot{q} = \frac{u}{e} Z_\omega \omega + u Z_q q + \text{control forces}$$

$$-\frac{M_{\dot{\omega}}}{e}\dot{\omega} + (I_y - M_{\dot{q}})\dot{q} = \frac{u}{e^2} M_\omega \omega + \frac{u}{e} M_q q + \text{control forces}$$

III. Reference 4

$$(m - Z_{\dot{\omega}})\dot{\omega} - Z_{\dot{q}}\dot{q} = Z_\omega \omega + (Z_q + m) q + \text{control forces}$$

$$-M_{\dot{\omega}}\dot{\omega} + (I_y - M_{\dot{q}})\dot{q} = M_\omega \omega + M_q q + M_\Theta \Theta + \text{control forces}$$

Except for the fact that some equations make use of dimensionalized and others of nondimensionalized coefficients, the terms in the equations are almost identical.

The hydrostatic restoring term $\mathcal{M}_\theta\Theta$ appearing in the set of equations in I and II above, exists because the introduction of a pitch angle Θ on a submerged submarine results in the creation of an hydrostatic moment $\mathcal{M}$, equal to $B\bar{z} \cdot \Delta\Theta$ in ft-tons, opposing that angle. This gives rise to an $\mathcal{M}_\theta \cdot \Theta$ term in the pitch equation having a nonzero value. The ways that the existence of $\mathcal{M}_\theta \cdot \Theta$ influences stability in the vertical plane are defined in Ref. 4.

In the nonlinear equations of motion about the Body Axis System y-Axis in references 1 and 3, the term appears as $\bar{B}z \sin\Theta$ and is neglected during the linearization process of reference 1, although it could have been included as $\bar{B}z \cdot \Theta$, since small angles are considered.

The terms $Z_\theta \theta$, $Z_u u$, $M_u u$ do not appear in reference 1 or in 3, from where the nonlinear equations were originally taken. Finally, the term muq (or $mu_0 q$) is neglected in the linearized model of reference 1.

This thorough, yet not complete, investigation of the already derived and used linear model, was soon justified, when the realization in the form of simulation of the design of the stern only planes control system was attempted. Initially the model of reference 1 was used unchanged. After a few tests it became apparent that the model was insufficient for the present purpose.

The term "muq" was included in the equations without any improvement in the simulation results.

Use of the linearized equation

$$(\text{Depth})' = w - u \cdot \Theta \qquad \text{instead of}$$

$$(\text{Depth})' = w \qquad \text{which in fact was}$$

used when the original vector $\begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix}$ was rewritten in the state form $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix}$, gave acceptable results in the respect of the linearized model mathematical representation and simulation results.

Insertion of the term $\bar{B}z \cdot \Theta$ was not considered in order to keep the linear model as simple as possible and because a constant speed was used in all test runs.

Details on changes due to the application of the described steps are given below for future reference, and for a constant forward speed of 15 knots (25.33 ft/sec).

1. As already mentioned, in reference 1, the problem was initially treated as a linear tracking problem. The state equations were:

$\underline{x} = \underline{A}\ \underline{x} + \underline{B}\ \underline{u}$, and the performance measure to be minimized was:

$$J = 1/2\left[\underline{x}(t_f)-\underline{r}(t_f)\right]^T \underline{H}\left[\underline{x}(t_f)-\underline{r}(tf)\right] + 1/2\int_{t_0}^{t_f}\left\{\left[\underline{x}(t)-r(t)\right]^T \underline{Q}\left[\underline{x}(t)-\underline{r}(t)\right]+\underline{u}^T(t)\,\underline{R}\,\underline{u}(t)\right\}dt$$

where $\underline{r}$ represents the ordered depth and pitch.

The following approximations were also made for small perturbations:

state variable form of equations

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & A_{11} & 0 & A_{21}\\ 0 & 0 & 0 & 1\\ 0 & A_{12} & 0 & A_{22}\end{bmatrix}\cdot\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0 & 0\\ B_{11} & B_{21}\\ 0 & 0\\ B_{12} & B_{22}\end{bmatrix}\cdot\begin{bmatrix}D_s\\ D_b\end{bmatrix}$$

where

x_1 = Depth

$x_2 = \dot{x}_1$ = Rate of depth change

x_3 = Pitch

$x_4 = \dot{x}_3$ = Rate of pitch change.

Using $\underline{r} = \begin{bmatrix}Z_{ord}\\ 0\\ \theta_{ord}\\ 0\end{bmatrix}$ for the command inputs then

$x_1 - Z_{ord}$ = Depth error = E_1

x_2 = Depth rate error = E_2

$x_3 - \Theta_{ord}$ = Pitch error = E_3

x_4 = Pitch rate error = E_4

and the performance measure became

$$J = 1/2 \int_{t_o}^{t_f} \left(\underline{E}^T \underline{Q}\, \underline{E} + \underline{u}^T \underline{R}\, \underline{u} \right) dt \qquad (\text{for } \underline{H} = \underline{0})$$

Also:
$$\begin{bmatrix} \dot{E}_1 \\ \dot{E}_2 \\ \dot{E}_3 \\ \dot{E}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & A_{11} & 0 & A_{21} \\ 0 & 0 & 0 & 1 \\ 0 & A_{12} & 0 & A_{22} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} + \underline{B} \cdot \begin{bmatrix} D_s \\ D_b \end{bmatrix}$$

The optimal solution $\underline{U}^* = -\underline{R}^{-1}\, \underline{B}^T\, \underline{K}\, \underline{E}$ in the simulation program of reference 1 gave the equations:

$$\text{DSAD} = Y_{11} \cdot \text{ZOER} + Y_{21} \cdot \text{ZODOT}/u + Y_{31} \cdot \text{PERR} + Y_{41} \cdot \text{PIDOT}/u$$

$$\text{DBAD} = Y_{12} \cdot \text{ZOER} + Y_{22} \cdot \text{ZODOT}/u + Y_{32} \cdot \text{PERR} + Y_{41} \cdot \text{PIDOT}/u$$

In the described model:

$$A_{11} = \left[\underbrace{(I_y - M_{\dot q})}_{AEE} Z_w + \underbrace{(e Z_{\dot q})}_{AEC} \frac{M_\omega}{e} \right] \cdot u/e \cdot Inv \qquad = -4.3769 \times 10^{-2}$$

$$A_{21} = \left[AEE \cdot Z_q + AEC \cdot M_q/e \right] \cdot u \cdot Inv \qquad = -6.2278$$

$$A_{12} = \left[\underbrace{\left(\frac{M_{\dot\omega}}{e} \right)}_{ACE} \cdot Z_\omega + \underbrace{(m - Z_{\dot\omega})}_{ACC} \cdot \frac{M_\omega}{e} \right] \cdot \frac{u}{e} \cdot Inv \qquad = 4.7727 \times 10^{-4}$$

$$A_{22} = \left[ACE \cdot Z_q + ACC \cdot M_q/\ell\right] \cdot u \cdot Inv \qquad = -0.15443$$

$$B_{11} = \left[AEE \cdot Z_{\delta s} + AEC \cdot \frac{M_{\delta s}}{\ell}\right] \cdot u^2/\ell \cdot Inv \qquad = -0.42783$$

$$B_{12} = \left[ACE \cdot Z_{\delta s} + ACC \cdot \frac{M_{\delta s}}{\ell}\right] \cdot u^2/\ell \cdot Inv \qquad = -9.4019 \times 10^{-2}$$

$$B_{21} = \left[AEE \cdot Z_{\delta b} + AEC \cdot \frac{M_{\delta b}}{\ell}\right] \cdot u^2/\ell \cdot Inv \qquad = -0.24839$$

$$B_{22} = \left[ACE \cdot Z_{\delta b} + ACC \cdot \frac{M_{\delta b}}{\ell}\right] \cdot u^2/\ell \cdot Inv \qquad = 2.0465 \times 10^{-3}$$

and $Inv = ACC \cdot AEE - ACE \cdot AEC = 6.3713 \times 10^4$

2. If the term $u \cdot m \cdot q$ is included in the equations of the linear model then

$$A_{21} = \left(AEE \cdot (Z_q + M) + AEC \cdot \frac{M_q}{\ell}\right) \cdot U \cdot INV \qquad = 7.4422$$

$$A_{22} = \left(ACE \cdot (Z_q + M) + ACC \cdot \frac{M_q}{\ell}\right) \cdot U \cdot INV \qquad = -0.1623$$

3. If the term $\overline{BZ} \cdot \theta$ is included then setting $BG = \frac{\overline{BZ}}{\ell^5}$, ($\overline{BZ} = B \cdot Z_B$), then

$$\begin{bmatrix} \dot{w} \\ \dot{\theta} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} M - Z_{\dot{w}} & 0 & \ell \cdot Z_{\dot{q}} \\ 0 & 1 & 0 \\ \frac{-M_{\dot{w}}}{\ell} & 0 & I_y - M_{\dot{q}} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \frac{U \cdot Z_w}{\ell} & 0 & U(Z_q + m) \\ 0 & 0 & 1 \\ \frac{UM_w}{\ell} & BG & \frac{UM_q}{\ell} \end{bmatrix} \cdot \begin{bmatrix} w \\ \theta \\ q \end{bmatrix} +$$

$$+\begin{bmatrix} M-Z\dot{w} & 0 & -\ell\cdot Z\dot{q} \\ 0 & 1 & 0 \\ \frac{-M\dot{w}}{\ell} & 0 & Iy-M\dot{q} \end{bmatrix}^{-1} \begin{bmatrix} \frac{U^2\cdot Zds}{\ell} & \frac{U^2\cdot Zdb}{\ell} \\ 0 & 0 \\ \frac{U^2 Mds}{\ell^2} & \frac{U^2 Mdb}{\ell^2} \end{bmatrix} \begin{bmatrix} ds \\ db \end{bmatrix}$$

and finally

$$\begin{bmatrix} \dot{w} \\ \dot{\theta} \\ \dot{q} \end{bmatrix} \begin{bmatrix} A_{11} & \frac{BG\cdot\ell\cdot Z\dot{q}}{Determ} & A_{21} \\ 0 & 0 & 1 \\ A_{12} & \frac{BG\cdot(m-Z\dot{\omega})}{Determ} & A_{22} \end{bmatrix} \begin{bmatrix} w \\ \theta \\ q \end{bmatrix} + \begin{bmatrix} B_{11} & B_{21} \\ 0 & 0 \\ B_{12} & B_{22} \end{bmatrix} \begin{bmatrix} ds \\ db \end{bmatrix}$$

where $Determ = (M-Z\dot{w})(Iy-M\dot{q})-M\dot{w}\cdot Z\dot{q} = \frac{1}{INV}$

$$= \frac{1}{6.3713 \times 10^4}$$

and A_{11}, A_{21}, A_{12}, A_{22}, B_{11}, B_{21}, B_{12}, B_{22} have the same values as before. With $BG \approx 2.44 \times 10^{-6}$ then

$$\alpha = \frac{BG\cdot\ell\cdot Z\dot{q}}{Determ} \simeq 0.0013$$

$$\beta = \frac{BG\cdot(M-Z\dot{w})}{Determ} \simeq 0.0025$$

To study the effect of the changes in the values of the elements of matrix $\underline{A}$, the eigenvalues of the plant matrix were found and are given at the end of this Appendix.

To check for controllability the third order system (w, Θ, q) was decoupled using linear transformation. Kalman criterion was also used. In all cases the plant was shown to be controllable.

a. Original A matrix

$$\begin{bmatrix} 0 & 1.0 & 0 & 0 \\ 0 & -0.43769\ \mathrm{E}{-01} & 0 & -6.2278 \\ 0 & 0.0 & 0 & 1.0 \\ 0 & 0.47727\ \mathrm{E}{-03} & 0 & -0.15443 \end{bmatrix}$$

Eigenvalues

0.0, 0.0, -0.89659 E-01, -0.108539

b. Matrix A after the addition of "$-u \cdot \Theta$"

$$\begin{bmatrix} 0 & 1.0 & -25.33 & 0 \\ 0 & -0.43769\ \mathrm{E}{-01} & 0.0 & -6.2278 \\ 0 & 0.0 & 0.0 & 1.0 \\ 0 & 0.47727\ \mathrm{E}{-03} & 0.0 & -0.15443 \end{bmatrix}$$

Eigenvalues

Same as above.

In the state model used in that paper the above matrix was substituted.

c. Original matrix A with terms "m·u·q" added:

$$\begin{bmatrix} 0 & 1.0 & 0 & 0 \\ 0 & -0.43769\ \mathrm{E}{-01} & 0 & 7.4422 \\ 0 & 0.0 & 0 & 1.0 \\ 0 & 0.47727\ \mathrm{E}{-03} & 0 & -0.16123 \end{bmatrix}$$

Eigenvalues

0.0, 0.0, -0.18826 E-01, -0.18617

d. Above matrix A with term "$-u \cdot \Theta$" included

$$\begin{bmatrix} 0 & 1.0 & .25.33 & 0 \\ 0 & -0.43769\ \mathrm{E}{-01} & 0 & 7.4422 \\ 0 & 0.0 & 0 & 1.0 \\ 0 & 0.47727\ \mathrm{E}{-03} & 0 & -0.16123 \end{bmatrix}$$

Eigenvalues

0.0, 0.0, -0.18826 E-01, -0.18617

APPENDIX B

DERIVATION OF EQUATIONS OF MOTION OF RIGID BOAT

1. Rigid Body Equation

In this Section the submarine is considered as a rigid body and a derivation of its equations of motion from first principles is presented.

From Newton's second law, the force F acting through the center of gravity c of the boat, in vector notation is

$$\overline{F} = M \frac{d\overline{U}_o}{dt} \tag{1}$$

where m is the mass of the boat and $\overline{U}_o$ its velocity. The moment of the boat $\overline{G}$ is equal to the time rate of change of its ngular momentum and is given by

$$\overline{G} = \frac{d\overline{h}}{dt} \tag{2}$$

where the angular momentum $\overline{h}$ is, in terms of a particle of mass dm located a vector distance $\overline{r}$ from the center of gravity.

$$\overline{h} = \int_V \overline{r} \, x(\vec{U}_o + \overline{w} \; x \; \overline{r}) \qquad dm$$

where the integration is taken over the whole boat. Since $\bar{U}$ is the same for all particles and $\int \bar{r}\, d\, m = 0$ the above equation becomes, after expanding the vector triple product,

$$\bar{h} = \int \left[\bar{\omega}\ (\bar{r} \cdot \bar{r}) - \bar{r}\ (\bar{\omega} \cdot \bar{r}) \right] \quad dm.$$

Let $\bar{r}$ and $\bar{\omega}$ be expressed by

$$\begin{aligned} \bar{r} &= x\ \bar{i} + y\ \bar{j} + z\ \bar{k} \\ \bar{\omega} &= P\ \bar{i} + Q\ \bar{j} + R\ \bar{k} \end{aligned} \qquad (3)$$

where x, y and z are the scalar components of $\bar{r}$, P, Q and R are the scalar components of $\bar{\omega}$ and $\bar{i}$, $\bar{j}$ and $\bar{k}$ are unit vectors in the direction of x, y and z.[1] Substituting these expressions in Equation 3 gives for the scalar components of $\bar{h}$, in terms of the moments and products of inertia A, B, ... F,

$$\begin{aligned} h_x &= I_x\ P - I_{xy}\ Q - I_{zx}\ R \\ h_y &= -I_{xy}\ P + I_y\ Q - I_{yz}\ R \\ h_z &= -I_{zx}\ P - I_{yz}\ Q + I_z\ R \end{aligned} \qquad (4)$$

---

[1] All rotations (control surfaces, angular velocity components, etc.) and moment components are positive according to the usual right handed convention; i.e. from the x-axis to the y-axis to the z-axis to the x-axis.

where

$$I_x = \int (y^2 + z^2)\ dm, \qquad I_{yz} = \int yz\ dm,$$

$$I_y = \int (z^2 + x^2)\ dm, \qquad I_{zx} = \int zx\ dm,$$

$$I_z = \int (x^2 + y^2)\ dm, \quad \text{and} \quad I_{xy} = \int xy\ dm.$$

If , y and z are taken as the coordinates of dm in a non-rotating frame of reference, with origin at the center of gravity of the boat, C, then it is clear that in general the moments and products of inertia as well as the angular velocity components P, Q and R will vary with time as the boat rotates. This is an unnecessary complication which can be avoided if the coordinate system 0 x y z is fixed in the boat and allowed to rotate with it. Though this introduces additional terms the resulting equations are much simpler since now the inertia terms will remain constant. Thus Equation (1) becomes in terms of the velocity components of U_o (U,V,W) and angular velocity $\bar{\omega}$

$$\bar{F} = m \frac{d}{dt} (U\ \bar{i} + V\ \bar{j} + W\ \bar{k})$$

$$= m \left[(\dot{U}\ \bar{i} + \dot{V}\ \bar{j} + \dot{W}\ \bar{k}) + (U \frac{d\bar{i}}{dt} + V \frac{d\bar{j}}{dt} + W \frac{d\bar{k}}{dt}) \right]$$

$$= m \left[\frac{\delta \bar{U}_o}{\delta t} + \bar{\omega} \times \bar{U}_o \right] \qquad (5)$$

since $\frac{d\bar{i}}{dt} = \bar{\omega} \times \bar{i}$, $\frac{\partial \bar{j}}{\partial t} = \bar{\omega} \times \bar{j}$ and $\frac{d\bar{k}}{dt} = \bar{\omega} \times \bar{k}$. The

operator $\frac{\delta}{\delta t}$ has the definition implied by Equation (5). Similarly, Equation (2) becomes

$$\overline{G} = \frac{\delta \overline{h}}{\delta t} + \overline{\omega} \times \overline{h}. \tag{6}$$

Combining Equations (5) and (6) with Equation (3) gives for the scalar components of $\overline{F}$ in the $\overline{i}$, $\overline{j}$ and $\overline{k}$ directions

$$\begin{aligned} F_x &= m\,(\dot{U} + QW - RV) \\ F_y &= m\,(\dot{V} + RU - PW) \\ F_z &= m\,(\dot{W} + PV - QU) \end{aligned} \tag{7}$$

and for the scalar components of $\overline{G}$ in the $\overline{i}$, $\overline{j}$ and $\overline{k}$ directions

$$\begin{aligned} K &= \dot{h}_x + Q\,h_z - R\,h_y \\ M &= \dot{h}_y + R\,h_x - P\,h_z \\ N &= \dot{h}_z + P\,h_y - Q\,h_x \end{aligned} \tag{8}$$

Equations (7) and (8) are the Euler equations of motion of the submarine.

3. Motion of Boat Relative to Fixed Coordinate

Solution of Equations (7) and (8) gives the linear velocity components U, V, W and angular velocity components P, Q, R relative to the O x y z axes fixed in the boat. To

obtain the motion of the boat center of gravity it is necessary to express the linear velocities relative to a fixed coordinate system.

We therefore define a fixed orthogonal coordinate system 0 x_o y_o z_o in which the x_o y_o plane is fixed parallel to the equilibrium plane of the free water surface and the z_o direction is positive downward. The orientation of the boat axes (x,y,z) relative to the fixed axes (x_o, y_o, z_o) is shown in Figure B-1. It is assumed that at first the two reference frames are parallel. Then the orientation of the boat is determined by considering the following three rotations, in the order indicated, where all rotations are in the positive direction.

i. Rotate the x_o,y_o axes about z_o through the angle of yaw Ψ to the position (x_1,y_1). Then the direction cosines between (x_o,y_o,z_o) and (x_1,y_1,z_1) are given by

	x_1	y_1	z_1
x_o	$\cos \Psi$	$-\sin \Psi$	0
y_o	$\sin \Psi$	$\cos \Psi$	0
z_o	0	0	1

Thus a vector $x_1 \bar{i}_1 + y_1 \bar{j}_1 + z_1 \bar{k}_1$ in the x_1, y_1, z_1 system has the scalar components

$$x_o = x_1 \cos\Psi - y_1 \sin\Psi$$

$$y_o = x_1 \sin\Psi + y_1 \cos\Psi$$

$$z_o = z_1$$

in the (x_o, y_o, z_o) system. Expressed in matrix notation these equations may be written as:

$$\begin{vmatrix} x_o \\ y_o \\ z_o \end{vmatrix} = \begin{vmatrix} \cos\Psi & -\sin\Psi & 0 \\ \sin\Psi & \cos\Psi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x_1 \\ y_1 \\ z_1 \end{vmatrix} \qquad (9a)$$

Thus Equation (9a) represents the first rotation.

ii. Rotate the (x_1, z_1) axes about the y_1 axis through an angle of pitch θ to (x_2, y_2, z_2). This may be expressed in matrix notation by

$$\begin{vmatrix} x_1 \\ y_1 \\ z_1 \end{vmatrix} = \begin{vmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{vmatrix} \begin{vmatrix} x_2 \\ y_2 \\ z_2 \end{vmatrix} \qquad (9b)$$

iii. Rotate the (y_2, z_2) axes about the x_2 axis through an angle of roll ϕ to (x,y,z) the actual orientation of the boat. This may be expressed by

$$\begin{vmatrix} x_2 \\ y_2 \\ z_2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} \qquad (9c)$$

Combining Equations (9a), (9b), (9c) gives for the resultant of all three rotations

$$\begin{vmatrix} x_o \\ y_o \\ z_o \end{vmatrix} = \begin{vmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} \qquad (10)$$

By carrying out the operations indicated by Equation (10), the direction cosines between the χ,y,z axes and the X, y_o, z_o axis are obtained. These are tabulated below.

	x	y	z
x_o	$\cos\theta \cos\psi$	$-\cos\phi \sin\psi$ $+ \sin\theta \sin\phi \cos\psi$	$\sin\phi \sin\psi$ $+ \sin\theta \cos\phi \cos\psi$
y_o	$\cos\theta \sin\psi$	$\cos\phi \cos\psi$ $+ \sin\theta \sin\phi \sin\psi$	$-\sin\phi \cos\psi$ $+ \sin\theta \cos\phi \sin\psi$
z_o	$-\sin\theta$	$\cos\theta \sin\phi$	$\cos\theta \cos\phi$

Resolving the U, V, W velocity components in the x_o,y_o,z_o directions gives for the velocity components of the center of gravity of the boat in fixed coordinates

$$\frac{dx_o}{dt} = U \cos\theta \cos\psi + V (\sin\theta \sin\phi \cos\psi - \cos\phi \sin\psi)$$
$$+ W (\sin\phi \sin\psi + \sin\theta \cos\phi \cos\psi)$$

$$\frac{dy_o}{dt} = U \cos\theta \sin\psi + V (\cos\phi \cos\psi + \sin\theta \sin\phi \sin\psi)$$
$$+ W (\sin\theta \cos\phi \sin\psi - \sin\phi \cos\psi)$$

$$\frac{dz_o}{dt} = -U \sin\theta + V \cos\theta \sin\phi + W \cos\theta \cos\phi \qquad (11)$$

To obtain x_o y_o z_o from integration of these equations, as a function of time, in the most general case would clearly require a very considerable effort. However, for most problems involving the motions of submarines these equations may be considerably simplified.

4. Angular Orientation of the Submarine

The angular orientation of the submarine (θ, ϕ, ψ) may be expressed in terms of the angular velocity components (P,Q,R) by expressing the angular velocity of the boat in terms of the angles (θ, Φ, ψ). Thus the resultant rotation of the boat from orientation (θ, ϕ, ψ) to orientation $(\theta+d\theta, \phi+d\phi, \psi+d\psi)$ in time dt, using the same order and axes of rotation as in the previous section, may be represented by

$$d\bar{\Omega} = (\psi + d\psi)\, \bar{k}_o + (\theta + d\theta)\, \bar{j}_1' + (\phi + d\phi)\, \bar{i}'$$

$$- \psi\, \bar{k}_o - \bar{\theta}\, j_1 - \Phi\, \bar{i}$$

where the subscripts indicate the axes along which the unit vectors $\bar{i}$, $\bar{j}$ and $\bar{k}$ are taken and $\bar{j}_1' \rightarrow \bar{j}_1$ and $\bar{i}' \rightarrow \bar{i}$ as $dt \rightarrow 0$

$$d\bar{\Omega} \doteq d\psi\,\bar{k}_o + d\theta\,\bar{j}_1 + d\phi\,\bar{i}$$

and

$$\bar{\omega} = \frac{d\bar{\Omega}}{dt} = \dot{\psi}\,\bar{k}_o + \dot{\theta}\,\bar{j}_1 + \dot{\phi}\,\bar{i} = P\,\bar{i} + Q\,\bar{j} + R\,\bar{k} \tag{12}$$

Resolving $\dot{\psi}\,\bar{k}_o$ and $\dot{\theta}\,\bar{j}_1$ along the body axes x,y,z by the use of Equations (9b), (9c) and (10) gives

$$\begin{aligned} P &= \dot{\phi} - \dot{\psi}\sin\theta \\ Q &= \dot{\theta}\cos\phi + \dot{\psi}\cos\theta\sin\phi \\ R &= \dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi \end{aligned} \tag{13}$$

Solving Equation 13 for $\dot{\theta}$, $\dot{\phi}$ and $\dot{\psi}$ gives

$$\begin{aligned} \dot{\theta} &= Q\cos\phi - R\sin\phi \\ \dot{\phi} &= P + Q\sin\phi\tan\theta + R\cos\phi\tan\theta \\ \dot{\psi} &= (Q\sin\phi + R\cos\phi)\sec\theta. \end{aligned} \tag{14}$$

APPENDIX C

INVESTIGATION FOR VALUES OF WEIGHTING FACTORS IN THE COST FUNCTION

A. STERN PLANES SUBMARINE

In the design of the optimal controller for the stern planes only submarine, one of the problems faced with was the proper selection of weighting parameters that would result in the desired submarine response. At that point the idea of setting the submarine "in trim" had not been considered yet. So, when the values C=10, D=3000, E=1, taken from Ref. 1, were tested, the resulting gains and finally the controller orders failed to keep the submarine at the ordered depth.

A laborious trial and error procedure gave unacceptable results. The need for a more systematic search for the proper values of the weighting parameters led to the method described below.

In short, this method uses linear feedback of states and the closed loop CE for root placement at selected positions. Then a linear system of equations is formed from corresponding relations in the optimal design and the simple state feedback design. Solution of the system gives values for the unknown factors.

The problem is not solved exactly but the method helps to indicate magnitude relations between the factors and range of values from which they can be selected.

The solution to the linear regulator problem gave the optimal control law

$$U^* = -R^{-1} \cdot \underline{B}^T \cdot \underline{K} \cdot \underline{E}$$

where $R^{-1} = CI = 1/C$, or

$$U^* = -CI \cdot \begin{bmatrix} 0 & B_1 & 0 & B_2 \end{bmatrix} \cdot \begin{vmatrix} K_{11} & K_{21} & K_{31} & K_{41} \\ K_{12} & K_{22} & K_{32} & K_{42} \\ K_{13} & K_{23} & K_{33} & K_{43} \\ K_{14} & K_{24} & K_{34} & K_{44} \end{vmatrix} \cdot \underline{E} \quad .$$

where for simplicity B_{11}, B_{12} were replaced by B_1, B_2.

Then

$$U^* = -CI \cdot \left[B_1 \cdot K_{12} + B_2 \cdot K_{14} \,\vdots\, B_1 \cdot K_{22} + B_2 \cdot K_{24} \,\vdots\, B_1 \cdot K_{32} + B_2 \cdot K_{34} \,\vdots\, B_1 \cdot K_{42} + B_2 \cdot K_{44} \right] \cdot \underline{E}$$

$$= -CI \cdot \begin{bmatrix} F_{11} & F_{21} & F_{31} & F_{41} \end{bmatrix} \cdot \underline{E}$$

Deviating from optimal control methods, the problem of controlling the linearized model by use of linear feedback of states is examined. Ds is replaced by

$$SOA \cdot E_1 + SOB \cdot E_2 + SOC \cdot E_3 + SOD \cdot E_4$$

where SOA, SOB, SOC, SOD are the feedback gains.

Then

$$\underline{\dot{E}} = \begin{vmatrix} 0 & 1 & -U & 0 \\ 0 & A_{11} & 0 & A_{21} \\ 0 & 0 & 0 & 1 \\ 0 & A_{12} & 0 & A_{22} \end{vmatrix} \cdot \underline{E} + \begin{vmatrix} 0 \\ B_1 \\ 0 \\ B_2 \end{vmatrix} \left[SOA \cdot E_1 + SOB \cdot E_2 + SOC \cdot E_3 + SOD \cdot E_4 \right]$$

or

$$\underline{\dot{E}} = \begin{vmatrix} 0 & 1 & -U & 0 \\ SOA \cdot B_1 & A_{11} + SOB \cdot B_1 & SOC \cdot B_1 & A_{21} + SOD \cdot B_1 \\ 0 & 0 & 0 & 1 \\ SOA \cdot B_2 & A_{12} + SOB \cdot B_2 & SOC \cdot B_2 & A_{22} + SOD \cdot B_2 \end{vmatrix} \cdot \underline{E}$$

To find the CE the following matrix is formed

$$\begin{vmatrix} S & -1 & U & 0 \\ -SOA \cdot B_1 & S \cdot A_{11} - SOB \cdot B_1 & -SOC \cdot B_1 & -A_{21} - SOD \cdot B_1 \\ 0 & 0 & S & -1 \\ -SOA \cdot B_2 & -A_{12} - SOB \cdot B_2 & -SOC \cdot B_2 & S - A_{22} - SOD \cdot B_2 \end{vmatrix}$$

Setting the determinant of the above equal to zero gives

$$s^4 + (-A_{11}-A_{22}-SOB\cdot B_1-SOD\cdot B_2)\cdot s^3$$

$$+ \begin{pmatrix} A_{11}\cdot A_{22}+A_{11}\cdot SOD\cdot B_2+A_{22}\cdot SOB\cdot B_1 \\ -A_{12}\cdot A_{21}-A_{12}\cdot SOD\cdot B_1-A_{21}\cdot SOB\cdot B_2 \\ -SOC\cdot B_2-SOA\cdot B_1 \end{pmatrix} \cdot s^2$$

$$+ \begin{pmatrix} -SOC\cdot B_1\cdot A_{12}+A_{11}\cdot SOC\cdot B_2 \\ +SOA\cdot B_1\cdot A_{22}-A_{21}\cdot SOA\cdot B_2 \\ +U\cdot SOA\cdot B_2 \end{pmatrix} \cdot s$$

$$+ (U\cdot SOA\cdot B_1\cdot A_{12}-U\cdot SOA\cdot B_2\cdot A_{11})$$

$$= 0$$

where

A_{11} = -0.43769 E-01

A_{12} = 0.47727 E-03

A_{21} = -6.2278

A_{22} = -0.15443

B_1 = -0.42783

B_2 = -0.94019 E-02

The original positions of the plant eigenvalues are considered and new positions selected according to some specified criterion (for example selection of a pair of relative dominant complex roots with negative real parts). The new CE is formed and comparison with (1) gives the gains SOA, SOB, SOC, SOD (program 4).

for roots:

$s_1 = -0.065$

$s_2 = -0.51$

$s_{3,4} = -0.00102 \pm j0.486$

the CE is $S^4 + 0.57524\ S^3 + 0.2702\ S^2 + 0.13533\ S + 0.77979\ E\text{-}02$

Then SOA = -0.5

SOB = -7.03

SOC = 32.5

SOD = 360.0

The order DSAD = -0.5·ZOER-7.03·ZODOT+32.5·PERR+360·PIDOT gave the submarine the satisfactory behavior shown in Figure C-1, for zero depth and pitch ordered.

Another test was made,

for roots

$s_1 = -0.082$

$s_2 = -1.998$

$s_{3,4} = -0.00447 \pm j0.0642$

and CE $S^4 + 2.08976\ S^3 + 0.18699\ S^2 + 0.0010087\ S + 0.679973\ E\text{-}03$

the feedback gains were:

SOA = -0.0436

SOB = -3.49

SOC = 0.0523

SOD = 360.0

It has to be noticed here, that even if the values for s_1, s_2, s_3, s_4 in the examined cases appear to contain some magic relative to their selection, in fact they resulted from a limited trial and error circular procedure. The path was from roots to the CE, to the feedback gains, to simulation, to modification of the gains to improve controller performance, to new corresponding CE and finally to the given values of roots.

Figure C-2 corresponds to the use of a feedback controller with the gains found in the second test. The results are acceptable.

Finally for roots s_1=0.1, s_2=-1.0, $s_{3,4}$=-0.1 ± j0.015 and CE S^4+1.3 S^3 + 0.33 S^2 +0.031 S^1 + 0.0010225, the gains FBA= -0.0656, FBB= -1.47, FBC= 26.2, FBD= 184.1 were found and tested. (Fig. C-3)

The main reason for these repeated tests was to check how well assumptions concerning stability of the linear model were valid on the simulated nonlinear submarine..

During runs at the speed of 15 knots with both planes available, it was found that for a submarine "in trim" the following quantities representing ballast, had to be added in the three trim tanks.[1] AT= -6.4×10^{-5}, AU= 8.8×10^{-5}, FT= 6.4×10^{-5}.

---

[1] To give weight in pounds the above numbers are multiplied by (Length of Submarine)^3.

Figures C-4 to C-6 show the behavior of the submarine for each of the examined control laws as before but with the required ballast added. Judging from the S.S. error values the performance of all controllers has now been improved.

The next step was an effort to correlate the results from classic feedback control with that of the optimal theory.

For the feedback gains

SOA = -0.5

SOB = -7.03

SOC = 32.5

SOD = 360.5

the control order was:

$$\text{DSAD}= -0.5 \cdot E_1 - 7.03 \cdot E_2 + 32.5 \cdot E_3 + 360 \cdot E_4$$

From the optimal law

$$\text{DSAD}^* = -\text{CI} \cdot \left[F_{11} \cdot E_1 + F_{21} \cdot E_2 + F_{31} \cdot E_3 + F_{41} \cdot E_4 \right]$$

where use of the definitions in main program of Ref. 1 is made.

Comparison of the two orders show the following correspondance:

$$-\text{CI} \cdot F_{11} = -0.5 = \text{SOA}$$

$$-\text{CI} \cdot F_{21} = -7.03 = \text{SOB}$$

$-CI \cdot F_{31} = 32.5 = SOC$

$-CI \cdot F_{41} = 360.0 = SOD$

This gives four equations, linear, algebraic, in K's, i.e.

$F_{11} = B_1 \cdot K_{12} + B_2 \cdot K_{14} = -C \cdot SOA = C \cdot 0.5$

$F_{21} = B_1 \cdot K_{22} + B_2 \cdot K_{24} = -C \cdot SOB = C \cdot 7.03$

$F_{31} = B_1 \cdot K_{23} + B_2 \cdot K_{34} = -C \cdot SOC = -C \cdot 32.5$

$F_{41} = B_1 \cdot K_{24} + B_2 \cdot K_{44} = -C \cdot SOD = -C \cdot 360.0$

Also from the nonlinear algebraic equations in matrix form

$$\dot{\underline{K}} = \underline{0} = -\underline{K} \cdot \underline{A} - \underline{A}^T \cdot \underline{K} - \underline{Q} + \underline{K} \cdot \underline{B} \cdot \underline{R}^{-1} \cdot \underline{B}^T \cdot \underline{K}$$

the following ten equations written explicitly are used:

$(F_{11})^2 \cdot CI - E = 0$

$(F_{11} \cdot F_{21}) \cdot CI - (K_{11} + K_{12} \cdot A_{11} + K_{14} \cdot A_{12}) = 0$

$(F_{11} \cdot F_{31}) \cdot CI + U \cdot K_{11} = 0$

$(F_{11} \cdot F_{41}) \cdot CI - (K_{12} \cdot A_{21} + K_{13} + K_{14} A_{22}) = 0$

$(F_{21})^2 \cdot CI - 2 \cdot (K_{12} + A_{11} \cdot K_{22} + A_{12} \cdot K_{24}) - A = 0$

$(F_{21} \cdot F_{31}) \cdot CI - (K_{13} + A_{11} \cdot K_{23} + K_{34} \cdot A_{12}) + U \cdot K_{21} = 0$

$(F_{21} \cdot F_{41}) \cdot CI - (K_{14} + A_{11} \cdot K_{24} + K_{44} \cdot A_{12} + A_{21} \cdot K_{22} + K_{23} + A_{22} \cdot K_{24}) = 0$

$(F_{31})^2 \cdot CI + 2 \cdot U \cdot K_{13} - D = 0$

$(F_{31} \cdot F_{41}) \cdot CI - (A_{21} \cdot K_{32} + K_{33} + A_{22} \cdot K_{34}) + U \cdot K_{14} = 0$

$(F_{41})^2 \cdot CI - 2 \cdot (K_{24} \cdot A_{21} + K_{34} + K_{44} \cdot A_{22}) - B = 0$

Substituting in the above:

$F_{11} = -C \cdot SOA$

$F_{21} = -C \cdot SOB$

$F_{31} = -C \cdot SOC$

$F_{41} = -C \cdot SOD$

and defining also:

$x(1) = K_{11}$

$x(2) = K_{12}$

$x(3) = K_{13}$

$x(4) = K_{14}$

$x(5) = K_{22}$

$x(6) = K_{23}$

$x(7) = K_{24}$

$x(8) = K_{33}$

$x(9) = K_{34}$

$x(10) = K_{44}$

$x(11) = E$ (Depth error weighting factor)

$x(12) = D$ (Pitch error weighting factor)

x(13) = A (Depth error rate weighting factor)

x(14) = B (Pitch error rate weighting factor)

A system of 14 linear algebraic equations is formed, from which given the specific feedback control gains SOA, SOB, SOC, SOD, the steady state constant K values and the corresponding values for the weighting factors in $\underline{Q}$ of an equivalent optimal controller can be found. The system written in the matrix form $\underline{A}\ \underline{X} = \underline{B}$ is:

$$\left|\begin{array}{cccccccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & AA & 0 & AB & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-U & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & AC & 1 & AD & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 2AA & 0 & 2AB & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & -U & 1 & 0 & 0 & AA & 0 & 0 & AB & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & AC & 1 & (AA+AD) & 0 & 0 & AB & 0 & 0 & 0 & 0 \\
0 & 0 & -2U & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & -U & 0 & AC & 0 & 1 & AD & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2AC & 0 & 2 & 2AD & 0 & 0 & 0 & 1 \\
0 & BA & 0 & BB & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & BA & 0 & BB & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & BA & 0 & 0 & BB & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & BA & 0 & 0 & BB & 0 & 0 & 0 & 0
\end{array}\right| \cdot X = \left|\begin{array}{c}
(FBA)^{2} * C \\
FBA*FBB*C \\
FBA*FBC*C \\
FBA*FBD*C \\
(FBB)^{2} * C \\
FBB*FBC*C \\
FBB*FBD*C \\
(FBC)^{2} * C \\
FBC*FBD*C \\
(FBD)^{2} * C \\
-FBA*C \\
-FBB*C \\
-FBC*C \\
-FBD*C
\end{array}\right|$$

C, the control effort weighting parameter, is considered known in the above. Its value is determined by the designer.

For the gains in (1) and C=1, solution of the system gave E=0.25, D=-0.6686 x 10^4, A=-0.1137 x 10^3, B=0.233 x 10^6. The negative values of A and D are not acceptable in the optimal analysis. So C was made equal to 0.001 and as a result of the form of equations (right hand side multiplied by C), the above values were changed to E=0.00025, D=-6.686, A=-0.1137, B=233. Having reduced their values, A and D were then neglected and the optimal method tested with values of E, B, C around the found ones.

The values of C=-.001, E=0.001, B=164 were selected for subsequent use and results of their use are shown in Fig. C-7 and C-8 with the submarine "in trim" and "out of trim."

The S.S. errors are comparable with that of the feedback controllers and acceptable.

As for a first indication of the behaviour of the controllers and submarine under a simple harmonic seaway, the sinusoidal forcing function:

AU = 2·E-05*sin(t)

was introduced. Results are presented in Figures C-9 to C-12.

The described method has the disadvantage of the possible negative weighting factors that can be found. Variation of the value of C can be used to lessen the magnitude of these negative values and indicate a region of the positive ones.

A good linear model will improve the results and allow use of the CE (1) for a systematic search for the "best" placement of roots either by parameter plane or root locus methods. Using the described method the controller C=10, B=800, E=1 was also found, corresponding to a closed loop CE $S^4+0.978037S^3+0.464618S^2+0.106002S+0.493183E-02$ and roots

$$s_1 = -0.06077$$

$$s_2 = 0.40235$$

$$s_{3,4} = -0.25745 \pm j0.36798.$$

Results of its use are shown in Figure C-10 with the sinusoidal input at AU. This controller, having acceptable performance, was also used in the formulation of the CMC.

B. BOTH PLANES SUBMARINE

Extension of the method developed in the case of the both planes submarine was proved very tedious and of doubtful usefulness.

For this, only a few results will be presented that would be useful in similar studies and the intermediate steps will be omitted.

Use of

$$DSAD = Y_{11} \cdot E_1 + Y_{21}/U \cdot E_2 + Y_{31} \cdot E_3 + Y_{41}/U \cdot E_4$$

$$DBAD = Y_{12} \cdot E_1 + Y_{22}/U \cdot E_2 + Y_{32} \cdot E_3 + Y_{42}/U \cdot E_4$$

in $\dot{\underline{E}} = \underline{A}\ \underline{E} + \underline{B}\ \underline{U}$, results in the closed loop CE given below.

Y_{11}, Y_{21}, Y_{31}, Y_{41}, Y_{12}, Y_{22}, Y_{32}, Y_{42} were defined in Ref. (1) and the following substitutions were also made for the general case of feedback controller.

$FSA = Y_{11}$ $\qquad FBA = Y_{12}$

$FSB = Y_{21}/U$ $\qquad FBB = Y_{22}/U$

$FSC = Y_{31}$ $\qquad FBC = Y_{32}$

$FSD = Y_{41}/U$ $\qquad FBD = Y_{42}/U$

Then

$$S^4 + S^3 \cdot \left(-A_{11} - B_{11} \cdot FSB - B_{21} \cdot FBB - A_{22} - B_{12} \cdot FSD - B_{22} \cdot FBD\right)$$

$$+ S^2 \cdot \left(\begin{array}{l} A_{11} \cdot A_{22} + B_{11} \cdot A_{22} \cdot FSB + B_{21} \cdot A_{22} \cdot FBB + A_{11} \cdot B_{12} \cdot FSD \\ + B_{21} \cdot B_{12} \cdot FBB \cdot FSD + A_{11} \cdot B_{22} \cdot FBD + B_{11} \cdot B_{22} \cdot FSB \cdot FBD \\ - A_{21} \cdot A_{12} - A_{12} \cdot B_{11} \cdot FSD - A_{12} \cdot B_{21} \cdot FBD - A_{21} \cdot B_{12} \cdot FSB \\ - B_{21} \cdot B_{12} \cdot FBD \cdot FSB - A_{21} \cdot B_{22} \cdot FBB - B_{11} \cdot B_{22} \cdot FSD \cdot FBB \\ - B_{11} \cdot FSA - B_{21} \cdot FBA - B_{12} \cdot FSC - B_{22} \cdot FBC \end{array}\right)$$

$$+ S \cdot \left(\begin{array}{l} A_{22} \cdot B_{11} \cdot FSA + B_{22} \cdot FBD \cdot B_{11} \cdot FSA + A_{22} \cdot B_{21} \cdot FBA \\ + B_{12} \cdot FSD \cdot B_{21} \cdot FBA - A_{21} \cdot B_{12} \cdot FSA - B_{21} \cdot FBD \cdot B_{12} \cdot FSA \\ - A_{21} \cdot B_{22} \cdot FBA - B_{11} \cdot FSD \cdot B_{22} \cdot FBA + A_{11} \cdot B_{12} \cdot FSC \\ + B_{21} \cdot FBB \cdot B_{12} \cdot FSC + A_{11} \cdot B_{22} \cdot FBC + B_{11} \cdot FSB \cdot B_{22} \cdot FBC \\ - A_{12} \cdot B_{11} \cdot FSC - B_{22} \cdot FBB \cdot B_{11} \cdot FSC - A_{12} \cdot B_{21} \cdot FBC \\ - B_{12} \cdot FSB \cdot B_{21} \cdot FBC + U \cdot B_{12} \cdot FSA + U \cdot B_{22} \cdot FBA \end{array}\right)$$

$$+\left(\begin{array}{l} B_{22}\cdot FBC\cdot B_{11}\cdot FSA+B_{12}\cdot FSC\cdot B_{21}\cdot FBA-B_{21}\cdot FBC\cdot B_{12}\cdot FSA \\ -B_{11}\cdot FSC\cdot B_{22}\cdot FBA+U\cdot A_{12}\cdot B_{11}\cdot FSA-U\cdot A_{11}\cdot B_{12}\cdot FSA \\ +U\cdot B_{22}\cdot FBB\cdot B_{11}\cdot FSA-U\cdot B_{21}\cdot FBB\cdot B_{12}\cdot FSA \\ +U\cdot A_{12}\cdot B_{21}\cdot FBA-U\cdot A_{11}\cdot B_{22}\cdot FBA \\ +U\cdot B_{12}\cdot FSB\cdot B_{21}\cdot FBA-U\cdot B_{11}\cdot FSB\cdot B_{22}\cdot FBA \end{array}\right)$$

$$= 0$$

The simplest case will be to accept the relations:

$$FBA = FSA$$

$$FBB = FSB$$

$$FBC = FSC$$

$$FBD = FSD$$

Then the resultant CE will be of the form

$$S^4 + (b_1\cdot FSB + d_1\cdot FSD + e_1)\cdot S^3$$

$$+ (a_2\cdot FSA + b_2\cdot FSB + C_2\cdot FSC + d_2\cdot FSD + e_2)\cdot S^2$$

$$+ (a_3\cdot FSA + b_3\cdot FSB + C_3\cdot FSC + e_3)$$

$$+ (a_4\cdot FSA)$$

Parameter plane methods will help now in selecting positions for root placement and give values for gains. The next in difficulty but more interesting case will be to accept relations of the form

FBA = -K·FSA

FBB = -K·FSB

FBC = -K·FSC

FBD = -K·FSD

The characteristic equation can result, by the application of some stability test, into conditions on FSA, FSB, FSC, FSD for stable roots in the form of a system of inequalities or again parameter plane methods can be used to give values of the feedback gains.

Finally desired values can be given to four of the feedback gains and the other four used as parameters.

To correlate the results with that of optimal control analysis on altered form of the DSAD, DBAD equations is used, i.e.

$$\begin{bmatrix} DSAD \\ DBAD \end{bmatrix} = - \begin{bmatrix} 1/C_1 & 0 \\ 0 & 1/C_2 \end{bmatrix} \begin{bmatrix} F_{11} & F_{21} & F_{31} & F_{41} \\ F_{12} & F_{22} & F_{32} & F_{42} \end{bmatrix} \underline{E}$$

$$= \begin{bmatrix} -1/C_1(F_{11}\cdot E_1+F_{21}\cdot E_2+F_{31}\cdot E_3+F_{41}\cdot E_4) \\ -1/C_2(F_{12}\cdot E_1+F_{22}\cdot E_2+F_{32}\cdot E_3+F_{42}\cdot E_4) \end{bmatrix}$$

Then

$FSA = -1/C_1\cdot F_{11}$ $\qquad$ $FBA = -1/C_2\cdot F_{12}$

$FSB = -1/C_1\cdot F_{21}$ $\qquad$ $FBB = -1/C_2\cdot F_{22}$

$FSC = -1/C_1 \cdot F_{31}$ $\quad$ $FBC = -1/C_2 \cdot F_{32}$

$FSD = -1/C_1 \cdot F_{41}$ $\quad$ $FBD = -1/C_2 \cdot F_{42}$

With linear feedback gains selected for a desired CE and using the defining equations of $F_{11}=B_{11} \cdot K_{12}+B_{12} \cdot K_{14}$, $F_{12}=B_{21} \cdot K_{12}+B_{22} \cdot K_{14}$, etc. together with the Riccati algebraic equations for the steady state solution, a system of eighteen linear equations in sixteen unknowns (C_1, C_2, A, B, E, D, K_{11}, K_{22}, K_{33}, K_{44}, K_{12}, K_{13}, K_{23}, K_{14}, K_{24}, K_{34}) results.

This requires the introduction of two additional non-diagonal weighting factors as unknowns in the $\underline{Q}$ matrix and consequently in the algebraic equations. Solution of this system of eighteen linear equations will give the corresponding optimal control weighting factors, where once again the possibility of negative values is not excluded.

APPENDIX D

SIMPLIFIED TRIM ANALYSIS

As the submarine changes speed, hydrodynamic forces and moments change in proportion to velocity squared. When the submarine is at a neutral bubble, these changes are in equilibrium and is neither necessary to change the weight of the submarine nor to shift the center of gravity

Necessary changes of variable ballast are accomplished with the trim system. In this paper three trim tanks are used and shown in Fig. IV-4.

To determine the required ballast at the speed of 15 knots, (the constant speed of all simulation runs), the following approximate method was used.

The equation of motion along the body axis system z-axis when w, v, p, q, r and their derivatives are set equal to zero gives:

$$W_b = - \frac{\rho/2 \cdot \ell^2 \cdot Zuu \cdot U^2}{\cos\theta \cdot \cos\phi}$$

where $W_b > 0 \Rightarrow$ ballast to be added

$W_b < 0 \Rightarrow$ ballast to be blown

For $\theta = \phi = 0$, $Zuu = -0.1\text{x}10^{-3}$ (Ref. 3), $\ell = 415$, $\rho/2 \approx 1$,

$W_b = 17.22 \cdot U^2$

This parabola is plotted in Fig. D-1.

For $U = 25.33$ ft/s $W_b = 11050$ lbf.

From the equation of motion about the body axis system y-axis

$$X_G = \frac{B \cdot Z_B \cdot \sin\theta + \rho/2 \cdot \ell^2 \cdot Muu \cdot U^2}{W \cdot \cos\theta}$$

where

X_G = X coordinate of the CG

W = $m \cdot g$ = weight of submarine = $0.625\text{x}10^6\text{x}32 = 2\text{x}10^7$ lbf

B = buoyant force

Z_B = separation of submarines c.b and c.m

For $\theta = 0$, $Muu = 0.4 \times 10^{-4}$ (Ref. 3)

$$X_G = \frac{\ell^3 \cdot Muu \cdot U^2}{W} = 1.43 \times 10^{-4} \cdot U^2$$

This curve is plotted in Fig. D-2

For $U = 25.33$ $XG \approx 0.1$

The above calculations suggest addition of 11050 lbs in the auxilliary trim tank to compensate for W_b. Additional ballast has to be added since $X_G = 0.1 \neq 0$.

For FT = -AT

$$W \cdot X_G = FT \cdot (175.5+215.5)$$

$$FT = \frac{W \cdot X_G}{391}$$

Setting $W = 2x10^7 + 11.05x10^3$

FT $\approx$ 5118 lbf and AT $\approx$ -5118 lbf.

If these values were used in the program, they would appear as $AT=-7.16x10^{-5}$, $FT=7.16x10^{-5}$, $AU=1.546x10^{-4}$

The values used were

$AT=-6.4x10^{-5}$, $FT=6.4x10^{-5}$, $AU=8.8x10^{-5}$.

APPENDIX E

In an effort to find limits on the exponent x in

$$K = \left(\frac{0.436}{\delta fcom}\right)^{x},$$

different computer programs were developed.

The programs use almost the same approximations but different criteria each time lead to different regions of variation of x.

A short description of each one is given next and intermediate steps are omitted where the analysis is straightforward. Because of the approximations used it was not expected that the results would be much restrictive, even sometimes acceptable. Further refinements are possible. In all cases the root loci analysis already described in Chapter V gave better results.

1. The first two programs (programs 9 and 10) result from an analysis of the response to a step input of the submarine with the CMC and optimal design in the combined controllers. The criterion used was minimization of a function of steady state depth and pitch errors.

From the designed optimal controllers

$$\delta s_1 = FSA*X_1+FSB*X_2+FSC*X_3+FSD*X_4 \quad \Big\} \text{ stern planes order in BPOC}$$

$$\delta s_2 = SOA*X_1+SOB*X_2+SOC*X_3+SOD*X_4 \quad \Big\} \text{ stern planes order in SOPOC}$$

$$\delta f_1 = FBA*X_1+FBB*X_2+FBC*X_3+FBD*X_4 \quad \Big\} \text{ Fairwater planes order in BPOC}$$

First approximation

$$X_1 = X_2 * t$$

$$X_3 = X_4 * t$$

Substituting in δf_1 and solving for t results in:

$$t = \frac{\delta f1-FBB*X_2-FBD*X_4}{FBA*X_2+FBC*X_4}$$

Second approximation

$X_2 = P * X_4$ (P experimentally found between 150-200 for the given submarine and acceptable response)

Then

$$t = \frac{\delta f_1-(FBB+FBD/P)*X_2}{(FBA+FBC/P)*X_2}$$

$$\delta s_1=\delta f_1*\left[\frac{FSA+FSC/P}{FBA+FBC/P}\right]+X_2*\left[\frac{(-FSA-FSC/P)*(FBB+FBD/P)}{FBA+FBC/P}+FSB+FSD/P\right]$$

$$\delta s_2 = \delta f_1 * \left[\frac{SOA+SOC/P}{FBA+FBC/P}\right] + X_2 * \left[\frac{(-SOA-SOC/P)*(FBB+FBD/\rho)}{FBA+FBC/\rho} + SOB+SOD/P\right]$$

Since

$\delta scom = \delta s_2 + K\,(\delta s_1 - \delta s_2)$ then at the steady state

with $X_2 = 0$

$$\delta scom = \delta f1 * \left[\left(\frac{SOA+SOC/P}{FBA+FBC/P}\right) + K*\left(\frac{FSA+FSC/P-SOA-SOC/P}{FBA+FBC/P}\right)\right]$$

Using the limiting condition

$$-0.436 < \delta som < 0.436$$

and setting

$$H = \delta f1 * \left[\frac{FSA+FSC/P-SOA-SOC/P}{FBA+FBC/P}\right.$$

Then for $H > 0$ and $\dfrac{0.436}{|\delta fcom|} < 1$

$$\frac{\log\left[\dfrac{-0.436-\delta f1*\left(\frac{SOA+SOC/P}{FBA+FBC/P}\right)}{H}\right]}{\log\left(\dfrac{0.436}{|\delta fcom|}\right)} > x > \frac{\log\left[\dfrac{0.436-\delta f_1*\left(\frac{SOA+SOC/P}{FBA+FBC/P}\right)}{H}\right]}{\log\left(\dfrac{0.436}{|\delta fcom|}\right)}$$

where $\delta fcom = \delta f1$

For $H < 0$ the signs of the inequality are inversed.

In order for the above unequalities to be of some use, the value of $_{f1}$ to be substituted in, must be specified. From the linear model

$$\begin{vmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{vmatrix} = \begin{vmatrix} 0 & 1 & -U & 0 \\ 0 & A_{11} & 0 & A_{21} \\ 0 & 0 & 0 & 1 \\ 0 & A_{12} & 0 & A_{22} \end{vmatrix} \circ \begin{vmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ B_{11} & B_{21} \\ 0 & 0 \\ B_{12} & B_{22} \end{vmatrix} U + \begin{vmatrix} 0 \\ F \\ 0 \\ M \end{vmatrix}$$

where F, M are terms due to external step forces and moments. and with

$$\begin{aligned} U &= K*\delta_{s1} + (1-K)*\delta_{s2} \\ &= \left[K*FSA+(1-K)*SOA\right]*X_1 + \\ &\quad \left[K*FSB+(1-K)*SOB\right]*X_2 + \\ &\quad \left[K*FSC+(1-K)*SOC\right]*X_3 + \\ &\quad \left[K*FSD+(1-K)*SOD\right]*X_4 \end{aligned}$$

it is finally found

$$\text{s.s depth error} = \frac{K*R_1+R_2}{K*R_3+R_4}$$

$$\text{s.s pitch error} = \frac{K*R_5+R_6}{K*R_3+R_4}$$

R_1, R_2, R_3, R_4, R_5, R_6 are given explicitly in programs 9 and 10.

The function to be minimized is

$$(\text{s.s depth error})^2 + (p * \text{s.s pitch error})^2$$

Taking derivatives with respect to K gives K as a solution of the quadratic equation

$$K^2 + K * \frac{\left[R_1^2R_4 + R_1R_2R_3 - 2P^2R_3R_5R_6\right]}{\left[R_1^2R_3 - p^2R_3R_5\right]} + \frac{\left[R_1R_2R_4 - p^2R_3R_6^2\right]}{\left[R_1^2R_3 - P^2R_3R_5^2\right]} = 0 \qquad (1)$$

At steady state

$$\delta_{fcom} = FBA * X_{1ss} + FBC * X_{3ss} \qquad (2)$$

It was also found that

$$X_{1ss} = \frac{R_1K + R_2}{R_3K + R_4} \quad , \quad X_{2ss} = \frac{R_5K + R_6}{R_3K + R_4} \qquad (3)$$

and that the value of K minimizing $X_{1ss}{}^2 + (P \cdot X_{3ss})^2$ is the solution of (1). Substituting this value in (3) and using (3) in (2) results in the desired value in the x inequalities to give the region of x variation. This region is expected to be reduced if the values ±0.436 in the limiting condition are lowered depending on the magnitude of the applied moments and forces. Also, it is possible instead of using in the above the value of K minimizing the function of depth and pitch errors, to use

a second order approximating system and express the steady state errors as functions of specified ζ (for example $\zeta=$ 0.4-0.6), settling time (for example 120 sec) and input forces (for example 70 tons). These values of X_{1ss}, X_{3ss} will subsequently give the value of δ_{f1ss} to be used in the x inequalities.

2. The application of step inputs to the system leads to the saturation of the forward planes and a different from zero angle of the stern planes, before the CMC is energized.

For a more elaborate analysis the effect of the above must be considered. This is done in programs 11 to 13. In program 11 the effect of saturated forward planes is translated to initial depth and pitch errors.

Programs 12 and 13 are almost the same as 9 and 10 but properly modified to accept results of program 11.

3. The next approach was to put limits on the exponent of K considering the response of the system to impulse inputs.

The starting point was to consider a second order approximating system, find the closed loop CE with $U=K\delta_{s1}+(1-K)\delta_{s2}$, accept a value of ζ and use the form

$S^2+2\zeta W_n S+W_n^2$ and the relations between the power coefficients to get a corresponding value of K suitable for use with the inequalities from the step response. (Programs 9-13)

In program 14 the found CE of the form $S^2+A(K)S+B(K)$ is set equal to $S^2+2\zeta W_n+W_n^2$ and a relation derived which gives values of K with ζ as a parameter.

Finally in program 15 further approximations are made to result in an expression giving K with parameters P(150-250), ζ, and X_2 (average value ±0.8 depending on side of response) and δ_{fcom} the independent variable . Using this program, the CMC with separate optimal controllers having weighting factors B=800, C=10, E=1, also $\zeta = 0.6-0.8$, P=220-250 and $X_2 = -0.2$, values of K varying as function of δ_{fcom} were found. The results are shown in Fig. E-1 to 6 where also $\left(\frac{0.436}{\delta fcom}\right)^x$ is plotted for different values of x.

In these figures the region of comparison is for δ_{fcom} between 0.74 and 0.9. The curve with X=1.2 is the one that better approximates the K-curve found by the program.

4. Additional informations for the use of the programs.

a. When forces or moments are desired to be introduced in the right side of the linear model state equations, the quantities already divided by either L^3 or L^5 have then to be multiplied by the matrix

$$\begin{bmatrix} XMA & XMC \\ XMB & XMD \end{bmatrix} = \begin{bmatrix} 61.72 & -5288.51 \\ -0.0031 & 1035.02 \end{bmatrix}$$

For example application of the step $-2.208\text{x}10^{-3}$ at the FT results in the terms

$$\begin{bmatrix} 61.72 & -5288.51 \\ -0.0031 & 1035.02 \end{bmatrix} \begin{bmatrix} -2.208\text{x}10^{-3} \\ \dfrac{-2.208\text{x}10^{-3} \cdot X_{FT}}{415^2} \end{bmatrix} = \begin{bmatrix} FRC \\ TRQ \end{bmatrix}$$

b.

SCA, SCB, SCC, SCD are the same as FSA, FSB, FSC, FSD

BCA, BCB, BCC, BCD are the same as FBA, FBB, FBC, FBD

A_{11}, A_{21}, A_{12}, A_{22} are the same as AA, AC, AB, AD

B_{11}, B_{21}, B_{12}, B_{22} are the same as BA, BC, BB, BD

APPENDIX F

EQUATIONS OF MOTION

The following set of equations are referred to a body fixed system of axes which are coincident with the principal axes of inertia of the body. The origin of this axis-system is located at the assumed center of mass of the body

Equation of Motion Along the Body Axis System x-Axis

$$
\begin{aligned}
m(\dot{u} - vr + wq) = {} & \frac{\rho}{2} \ell^4 \left[X_{qq}' q^2 + X_{rr}' r^2 + X_{rp}' rp \right] \\
& + \frac{\rho}{2} \ell^3 \left[X_{\dot{u}}' \dot{u} + X_{vr}' vr + X_{wq}' wq \right] \\
& + \frac{\rho}{2} \ell^2 \left[X_{uu}' u^2 + X_{vv}' v^2 + X_{ww}' w^2 \right] \\
& + \frac{\rho}{2} \ell^2 u^2 \left[X_{\delta r \delta r}' \delta_r^{\,2} + X_{\delta s \delta s}' \delta_s^{\,2} + X_{\delta b \delta b}' \delta_b^{\,2} \right] \\
& + \frac{\rho}{2} \ell^2 X_{vvn'}' (n' - 1) v^2 \\
& + \frac{\rho}{2} \ell^2 X_{wwn'}' (n' - 1) w^2 \\
& + \frac{\rho}{2} \ell^2 u^2 X_{\delta s \delta s n'}' (n' - 1) \delta_s^{\,2} \\
& + \frac{\rho}{2} \ell^2 u^2 X_{\delta r \delta r n'}' (n' - 1) \delta_r^{\,2} \\
& - \Sigma W_i \sin \theta \\
& + (F_x)_P
\end{aligned}
$$

Equation of Motion Along the Body Axis System y-Axis

$$m(\dot{v} - wp + ur) = \frac{\rho}{2}\ell^4 \left[Y_{\dot{r}}{}' \dot{r} + Y_{\dot{p}}{}' \dot{p} \right]$$

$$+ \frac{\rho}{2}\ell^4 \left[Y_{pq}{}' pq + Y_{p|p|}{}' p|p| \right]$$

$$+ \frac{\rho}{2}\ell^3 \left[Y_{\dot{v}}{}' \dot{v} + Y_{wp}{}' wp + Y_{v|r|}{}' \frac{v}{|v|} \left|(v^2 + w^2)^{\frac{1}{2}}\right| \, |r| \right]$$

$$+ \frac{\rho}{2}\ell^3 \left[Y_{r}{}' ur + Y_{|r|\delta r}{}' u|r|\delta r + Y_{p}{}' up \right]$$

$$+ \frac{\rho}{2}\ell^3 \; Y_{rn'}{}' (n' - 1)\, ur$$

$$+ \frac{\rho}{2}\ell^2 \left[Y_{*}{}' u^2 + Y_{v}{}' uv + Y_{v|v|}{}' v\left|(v^2 + w^2)^{\frac{1}{2}}\right| \right]$$

$$+ \frac{\rho}{2}\ell^2 u^2 \, Y_{\delta r}{}' \delta_r$$

$$+ \frac{\rho}{2}\ell^2 u^2 \, Y_{\delta r n'}{}' (n' - 1)\, \delta_r$$

$$+ \frac{\rho}{2}\ell^2 \, Y_{vn'}{}' (n' - 1)\, uv$$

$$+ \frac{\rho}{2}\ell^2 \, Y_{v|v|n'}{}' (n' - 1)\, v\left|(v^2 + w^2)^{\frac{1}{2}}\right|$$

$$+ \frac{\rho}{2}\ell^2 \; Y_{wv}{}' wv\#$$

$$+ \frac{\rho}{2}\ell^2 (F_y)_{vs} \; \frac{v^2 + w^2}{U} \; (-w) \; \operatorname{Sin} \omega t$$

$$+ \Sigma W_i \sin\phi \cos\theta$$

# Multiplied by $\frac{u}{U}$ for large angles of attack near -90°

Equation of Motion Along the Body Axis System z-Axis

$$m(\dot{w} - uq + vp) = \frac{\rho}{2}\ell^4 Z_{\dot{q}}{}' \dot{q}$$

$$+ \frac{\rho}{2}\ell^4 \left[Z_{rr}{}' r^2\# + Z_{rp}{}' rp\#\right]$$

Note 1

$$+ \frac{\rho}{2}\ell^3 \left[Z_{\dot{w}}{}' \dot{w} + Z_{vr}{}' vr\# + Z_{vp}{}' vp + \Delta Z_{vp}{}' vp\#\right]$$

$$+ \frac{\rho}{2}\ell^3 \left[Z_{q}{}' uq + Z_{|q|\delta s}{}' u|q|\delta s + Z_{w|q|}{}' \frac{w}{|w|}\left|(v^2 + w^2)^{\frac{1}{2}}\right| |q|\right]$$

$$+ \frac{\rho}{2}\ell^3 Z_{qn'}{}' (n' - 1) uq$$

$$+ \frac{\rho}{2}\ell^2 \left[Z_{*}{}' u^2 + Z_{w}{}' uw + Z_{w|w|}{}' w\left|(v^2 + w^2)^{\frac{1}{2}}\right|\right]$$

$$+ \frac{\rho}{2}\ell^2 \left[Z_{|w|}{}' u|w| + Z_{ww}{}' \left|w(v^2 + w^2)^{\frac{1}{2}}\right| + Z_{vv}{}' v^2\#\right]$$

$$+ \frac{\rho}{2}\ell^2 u^2 \left[Z_{\delta s}{}' \delta s + Z_{\delta b}{}' \delta b\right]$$

$$+ \frac{\rho}{2}\ell^2 \left[Z_{wn'}{}' (n' - 1) uw + Z_{w|w|n'}{}' (n' - 1) w\left|(v^2 + w^2)^{\frac{1}{2}}\right|\right]$$

$$+ \frac{\rho}{2}\ell^2 u^2 Z_{\delta sn'}{}' (n' - 1) \delta_s$$

$$+ \frac{\rho}{2}\ell^2 (F_z)_{vs} \frac{v^2 + w^2}{U} v \sin \omega t$$

$$+ \Sigma W_i \cos\theta \cos\phi .$$

# Multiplied by $\frac{u}{U}$ for large angles of attack near -90°

Note 1
when not multiplied by $\frac{u}{U}$ add to $Z_{vp}{}'$

Equation of Motion About the Body Axis System x-Axis

$$
\begin{aligned}
I_x\dot{p} + (I_z - I_y)\,qr = {} & \frac{\rho}{2}\ell^5 \left[K_{\dot{p}}'\,\dot{p} + K_{qr}'\,qr + K_{\dot{r}}'\,\dot{r} + K_{p|p|}'\,p|p|\right] \\
& + \frac{\rho}{2}\ell^4 \left[K_p'\,up + K_r'\,ur + K_{\dot{v}}'\,\dot{v} + K_{wp}'\,wp\right] \\
& + \frac{\rho}{2}\ell^3 \left[K_*'\,u^2 + K_v'\,uv + K_{v|v|}'\,v\left|(v^2 + w^2)^{\frac{1}{2}}\right|\right] \\
& + \frac{\rho}{2}\ell^3\; K_{vw}'\,vw \\
& + \frac{\rho}{2}\ell^3\, u^2 K_{\delta r}'\,\delta_r \\
& + Bz_B \sin\phi\,\cos\theta
\end{aligned}
$$

Equation of Motion About the Body Axis System y-Axis

$$
\begin{aligned}
I_y\dot{q} + (I_x - I_z)\,rp = {} & \frac{\rho}{2}\ell^5 \left[M_{\dot{q}}'\,\dot{q} + M_{rr}'\,r^2\# + M_{rp}'\,rp + \overset{\text{Note 1}}{\Delta M_{rp}'\,rp\#}\right] \\
& + \frac{\rho}{2}\ell^4 \left[M_q'\,uq + M_{|q|\delta s}'\,u|q|\delta s + M_{|w|q}'\,\left|(v^2 + w^2)^{\frac{1}{2}}\right|q\right] \\
& + \frac{\rho}{2}\ell^4 \left[M_{\dot{w}}'\,\dot{w} + M_{vr}'\,vr\# + M_{vp}'\,vp\#\right] \\
& + \frac{\rho}{2}\ell^4 M_{qn'}'\,(n' - 1)\,uq \\
& + \frac{\rho}{2}\ell^3 \left[M_{*}'\,u^2 + M_w'\,uw + M_{w|w|}'\,w\left|(v^2 + w^2)^{\frac{1}{2}}\right|\right] \\
& + \frac{\rho}{2}\ell^3 \left[M_{|w|}'\,u|w| + M_{ww}'\,\left|w\,(v^2 + w^2)^{\frac{1}{2}}\right| + M_{vv}'\,v^2\#\right] \\
& + \frac{\rho}{2}\ell^3\,u^2 \left[M_{\delta s}'\,\delta_s + M_{\delta b}'\,\delta_b\right] \\
& + \frac{\rho}{2}\ell^3 M_{wn'}'\,(n' - 1)\,uw \\
& + \frac{\rho}{2}\ell^3 M_{w|w|n'}'\,(n' - 1)\,w\left|(v^2 + w^2)^{\frac{1}{2}}\right| \\
& + \frac{\rho}{2}\ell^3\,u^2 M_{\delta sn'}'\,(n' - 1)\,\delta_s \\
& + Bz_B \sin\theta \\
& - \Sigma W_i\, x_{ti} \cos\theta \cos\phi
\end{aligned}
$$

# Multiply by $\frac{u}{U}$ for large angles of attack near -90°

Note 1
when not multiplied by $\frac{u}{U}$ add to M_{rp}'

Equation of Motion About the Body Axis System z-Axis

$$I_z \dot{r} + (I_y - I_x)\,pq = \frac{\rho}{2} \ell^5 \left[N_{\dot{r}}{}' \dot{r} + N_{pq}{}' pq + N_{\dot{p}}{}' \dot{p} \right]$$

$$+ \frac{\rho}{2} \ell^4 \left[N_r{}' ur + N_{|r|\delta r}{}' u|r|\delta r + N_{|v|r}{}' \left|(v^2 + w^2)^{\frac{1}{2}}\right| r \right]$$

$$+ \frac{\rho}{2} \ell^4 \left[N_p{}' up + N_{\dot{v}}{}' \dot{v} + N_{wp}{}' wp \right]$$

$$+ \frac{\rho}{2} \ell^4 \; N_{rn'}{}' (n' - 1)\, ur$$

$$+ \frac{\rho}{2} \ell^3 \left[N_*{}' u^2 + N_v{}' uv + N_{v|v|}{}' v\left|(v^2 + w^2)^{\frac{1}{2}}\right| \right]$$

$$+ \frac{\rho}{2} \ell^3 \; u^2 N_{\delta r}{}' \delta_r$$

$$+ \frac{\rho}{2} \ell^3 \; u^2 N_{\delta rn'}{}' (n' - 1)\, \delta_r$$

$$+ \frac{\rho}{2} \ell^3 \; N_{vn'}{}' (n' - 1)\, uv$$

$$+ \frac{\rho}{2} \ell^3 \; N_{v|v|n'}{}' (n' - 1)\, v\left|(v^2 + w^2)^{\frac{1}{2}}\right|$$

$$+ \frac{\rho}{2} \ell^3 \; N_{wv}{}' \; wv\#$$

$$+ \Sigma W_i \, x_{ti} \cos\theta \sin\phi$$

# Multiply by $\frac{u}{U}$ for large angles of attack near -90°

AUXILIARY EQUATIONS

$$\dot{\phi} = p + \dot{\psi} \sin\theta$$

$$\dot{\theta} = (q - \dot{\psi} \cos\theta \sin\phi) / \cos\phi$$

$$\dot{\psi} = (r + \dot{\theta} \sin\phi) / \cos\theta \cos\phi$$

$$\dot{x}_o = u \cos\theta \cos\psi + v (\sin\phi \sin\theta \cos\psi - \cos\phi \sin\psi) + w (\sin\phi \sin\psi + \cos\phi \sin\theta \cos\psi)$$

$$\dot{y}_o = u \cos\theta \sin\psi + v (\cos\phi \cos\psi + \sin\phi \sin\theta \sin\psi) + w (\cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi)$$

$$\dot{z}_o = - u \sin\theta + v \cos\theta \sin\phi + w \cos\theta \cos\phi$$

$$U = (u^2 + v^2 + w^2)^{\frac{1}{2}}$$

$$(F_x)_P = \frac{\rho}{2} \ell^2 u^2 \left[a_1' + a_2' n' + a_3' n'^2 \right] \qquad \text{when} \quad k_1 < n'$$

$$= \frac{\rho}{2} \ell^2 u^2 \left[b_1' + b_2' n' + b_3' n'^2 \right] \qquad \text{when} \quad k_2 < n' < k_1$$

$$= \frac{\rho}{2} \ell^2 u^2 \left[c_1' + c_2' n' + c_3' n'^2 \right] \qquad \text{when} \quad k_3 < n' < k_2$$

$$= \frac{\rho}{2} \ell^2 u^2 \left[d_1' + d_2' n' + d_3' n'^2 \right] \qquad \text{when} \quad n' < k_3$$

a_1', a_2', a_3'
b_1', b_2', b_3'
c_1', c_2', c_3'
d_1', d_2', d_3'

Sets of non-dimensional coefficients used in the propulsion equation above. The set which will be in effect at any time during a simulated maneuver will depend on the value of n' and the numbers k_1, k_2, k_3.

NOMENCLATURE

All symbols used in the equations of motion and in the auxiliary equations and relationships which appear in this report are defined below. Any dimensions involved will be consistent with the foot-pound-second system of units. All angles are in degrees. The Fortran variables corresponding to these symbols are shown in Appendix B .

SYMBOL	DEFINITION
•	A dot over any symbol signifies differentiation with respect to time.
B	Buoyancy force which is positive upwards.
m	Mass of the submarine including the water in the free flooding spaces.
ℓ	Overall length of the submarine
U	Linear velocity of origin of body axes relative to an earth-fixed axis system.
u	Component of U along the body x-axis.
v	Component of U along the body y-axis.
w	Component of U along the body z-axis.

u_c	Command speed: A steady value of u for a given propeller rpm when α, β and control surface angles are zero. Sign changes with propeller reversal.
x	Longitudinal axis of the body fixed coordinate axis system.
y	Transverse axis of the body fixed coordinate axis system.
z	Vertical axis of the body fixed coordinate axis system.
x_o	Distance along the x_o axis of an earth-fixed axis system.
y_o	Distance along the y_o axis of an earth-fixed axis system.
z_o	Distance along the z_o axis of an earth-fixed axis system.
p	Component of angular velocity about the body fixed x-axis.
q	Component of angular velocity about the body fixed y-axis.
r	Component of angular velocity about the body fixed z-axis.
z_B	The z coordinate of the center of buoyance (CB) of the submarine.

α	Angle of attack.
β	Angle of drift.
δb , D_b	Deflection of bowplane (or sailplane)
δr , D_r	Deflection of rudder.
δs , D_s	Deflection of sternplane.
n'	The ratio u_c/u.
θ	Pitch angle.
Ψ	Yaw angle.
ϕ	Roll angle.
ρ	Mass density of sea water.
W_i	Weight of water blown from a particular ballast tank identified by the integer assigned to the index i.
ω	Angular velocity.
t	Time.
x_{ti}	Location along the body x-axis of the center of mass of the i^{th} ballast tank when this tank is filled with sea water.

$(F_x)_p$	Propulsion force (see auxiliary equations and relationships).
I_x	Moment of inertia of a submarine about the x-axis.
I_y	Moment of inertia of a submarine about the y-axis.
I_z	Moment of inertia of a submarine about the z-axis.
$K_{\dot{p}}'$, K_p', $K_{p\lvert p\rvert}'$, K_{qr}', K_r', $K_{\dot{r}}'$, $K_{\dot{v}}'$, K_{wp}', K_*', K_v', $K_{v\lvert v\rvert}'$, K_{vw}', $K_{\delta r}'$	Non-dimensional constants each of which is assigned to a particular force term in the equation of motion about the body x-axis.
$M_{\dot{q}}'$, M_{rr}', M_{rp}', $\Delta M_{rp}'$, M_q', $M_{\lvert q\rvert \delta s}'$, $M_{\lvert w\rvert q}'$, $M_{\dot{w}}'$, M_{vr}', M_{vp}', M_{qn}', M_*', M_w', $M_{w\lvert w\rvert}'$, $M_{\lvert w\rvert}'$, M_{ww}', M_{vv}', $M_{\delta s}'$, $M_{\delta b}'$, M_{wn}'', $M_{w\lvert w\rvert n}''$, $M_{\delta sn}''$	Non-dimensional constants each of which is assigned to a particular force term in the equation of motion about the body y-axis.

$N_{\dot{r}}'$, N_{pq}', $N_{\dot{p}}'$, N_{r}', $N_{\|r\|\delta r}'$, $N_{\|v\|r}'$, N_{p}', $N_{\dot{v}}'$, N_{wp}', N_{rn}', N_{*}', N_{v}', $N_{v\|v\|}'$, $N_{\delta r}'$, $N_{\delta rn}'$, N_{vn}', $N_{v\|v\|n}'$, N_{wv}'	Non-dimensional constants each of which is assigned to a particular force term in the equation of motion about the body z-axis.
X_{qq}', X_{rr}', X_{rp}', $X_{\dot{u}}'$, X_{vr}', X_{wq}', X_{uu}', X_{vv}', X_{ww}', $X_{\delta r\delta r}'$, $X_{\delta s\delta s}'$, $X_{\delta b\delta b}'$, X_{vvn}', X_{wwn}', $X_{\delta s\delta sn}'$, $X_{\delta r\delta rn}'$	Non-dimensional constants each of which is assigned to a particular force term in the equation of motion along the body x-axis.
$Y_{\dot{r}}'$, $Y_{\dot{p}}'$, Y_{pq}', $Y_{p\|p\|}$, $Y_{\dot{v}}'$, Y_{wp}', $Y_{v\|r\|}'$, Y_{r}', $Y_{\|r\|\delta r}'$, Y_{p}', Y_{rn}', Y_{*}', Y_{v}', $Y_{v\|v\|}'$, $Y_{\delta r}'$, $Y_{\delta rn}'$, Y_{vn}', $Y_{v\|v\|n}'$, Y_{wv}', $(F_{y})_{vs}$	Non-dimensional constants each of which is assigned to a particular force term in the equation of motion along the body y-axis
$Z_{\dot{q}}'$, Z_{rr}', Z_{rp}', $Z_{\dot{w}}'$, Z_{vr}, Z_{vp}', $\Delta Z_{vp}'$, Z_{q}', $Z_{\|q\|\delta s}$, $Z_{w\|q\|}'$, Z_{qn}', Z_{*}', Z_{w}', $Z_{w\|w\|}'$, $Z_{\|w\|}'$, Z_{ww}', Z_{vv}', $Z_{\delta s}'$, $Z_{\delta b}'$, Z_{wn}', $Z_{w\|w\|n}'$, $Z_{\delta sn}'$, $(F_{z})_{vs}$	Non-dimensional constants each of which is assigned to a particular force term in the equation of motion along the body z-axis

A	Coefficient matrix of the state vector
B	Coefficient matrix of the input vector
X	State vector
E	Error vector
u	Input vector
R	Control effort weighting matrix
C,C1,C2	Control effort weighting parameters
CI,CI1,CI2	Control effort inverse weighting parameters
Q	Weighting matrix in quadratic performance index
A	Depth rate weighting factor
B	Pitch rate weighting factor
D	Pitch error weighting factor
E	Depth error weighting factor
BG	BZ/1 =B Z /1
X	Longitudinal CG of the submarine
CE	Characteristic equation
SOPOC	Stern only planes optimal controller
SOPC	Stern only planes controller
BPOC	Both planes optimal controller
CMC	Combined mode controller
FSA,FSB,FSC,FSD SCA,SCB,SCC,SCD	Feedback gains in
FBA,FBB,FBC,FBD BCA,BCB,BCC,BCD	Feedback gains in
SOA,SOB,SOC,SOD FA,FB,FC,FD	Feedback gains in
EOM	Eqs of motion

Stern plane command in both planes submarine

Fairwater plane command in both planes submarine

Stern plane command in stern plane only submarine

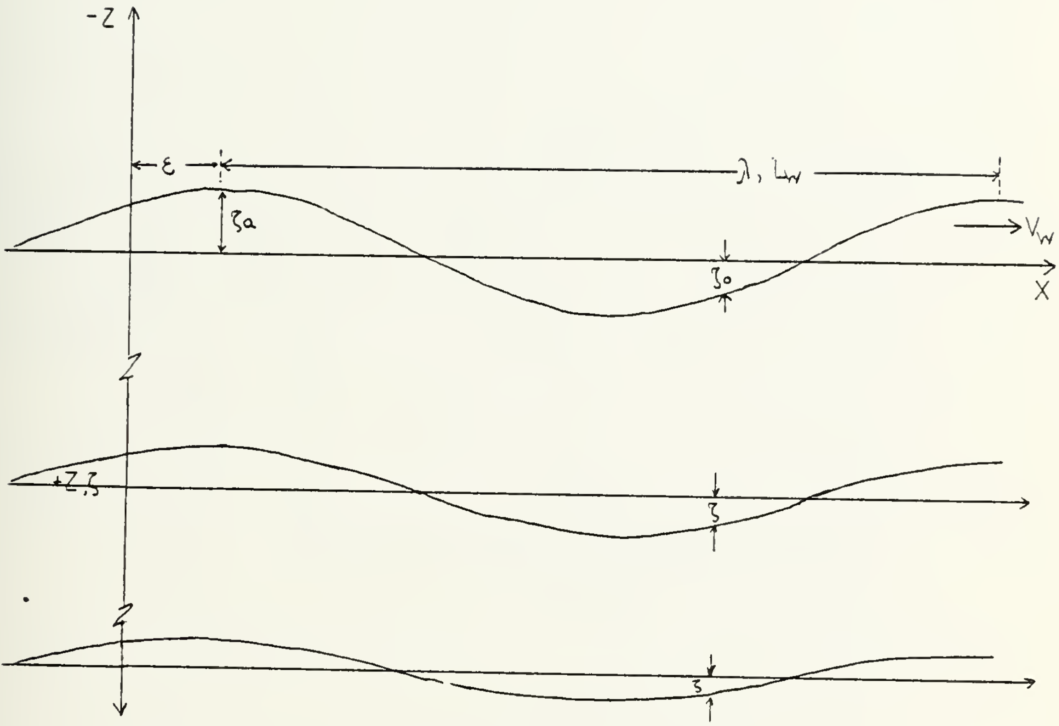


Fig. I-1. Coordinates of waves

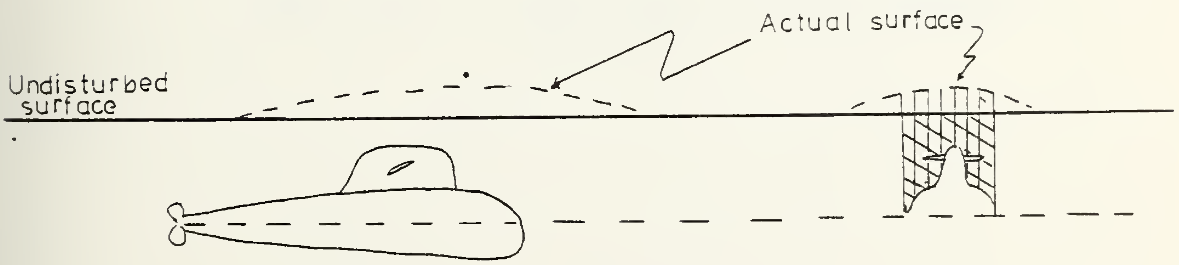


Fig. I-2. Near surface "venturi" effect

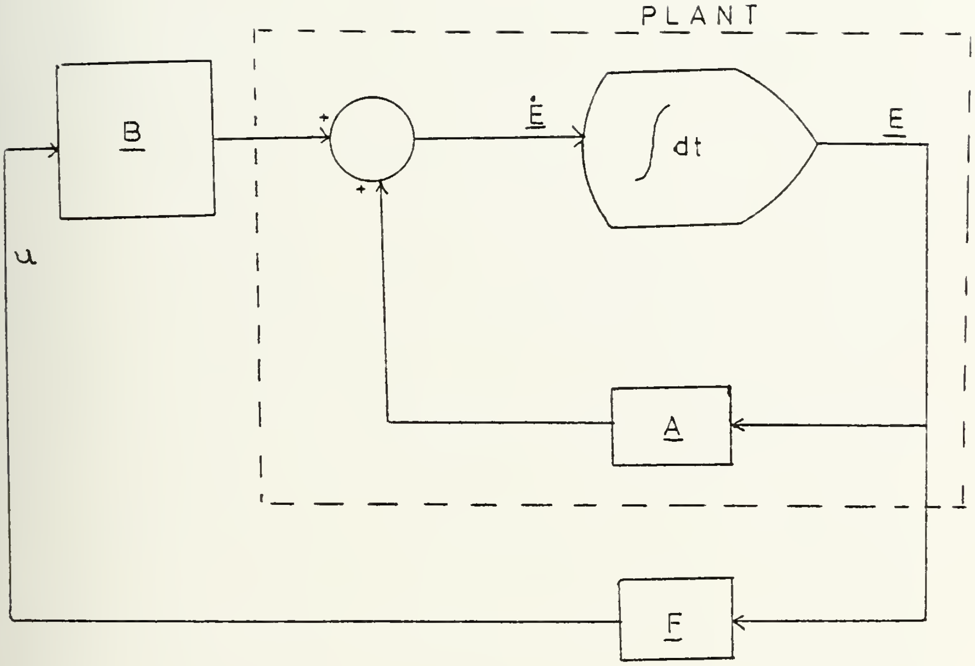


Fig. II-1. Plant and optimal feedback controller.

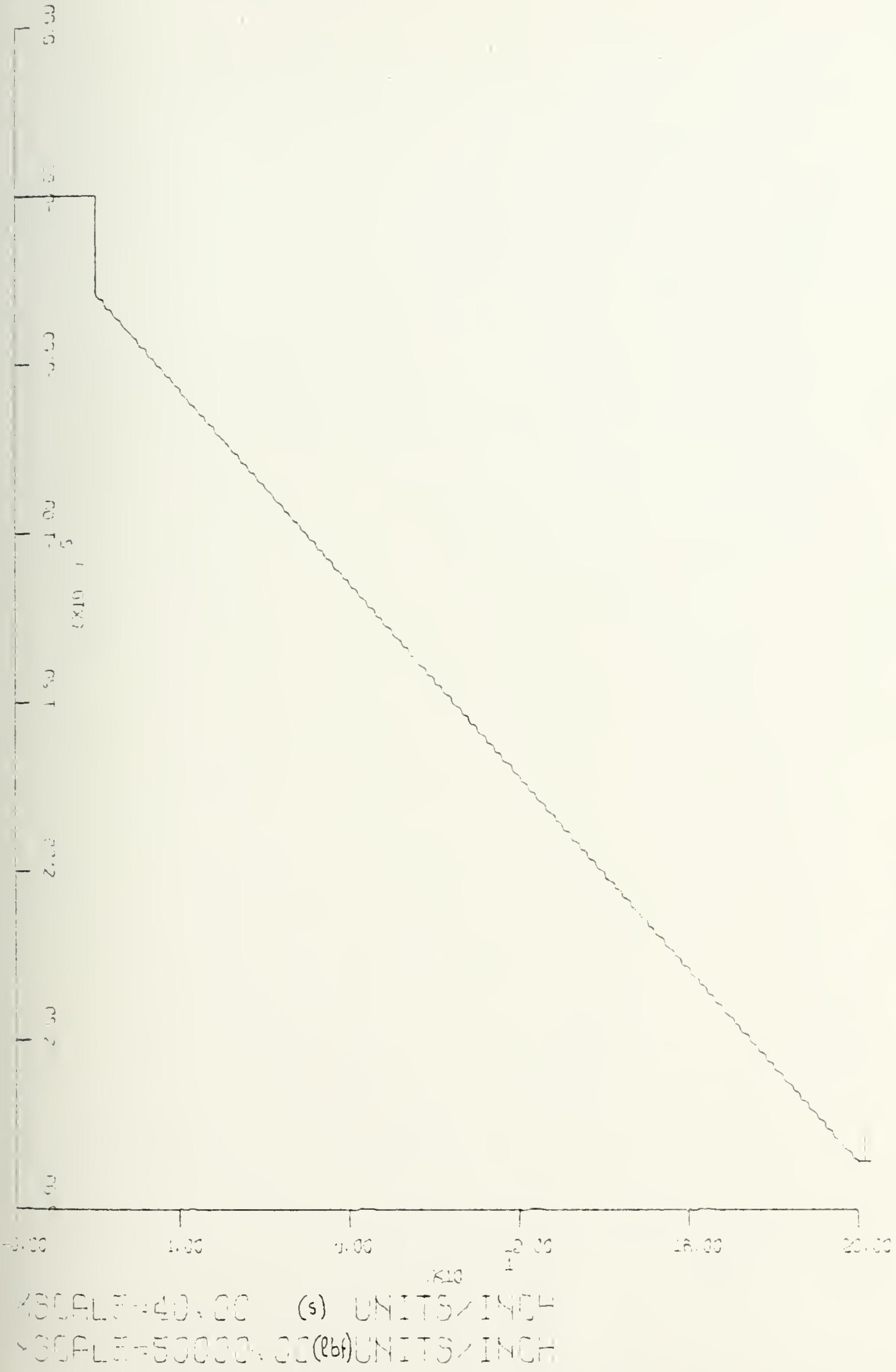


Fig. III-1a. Ramp force at AU

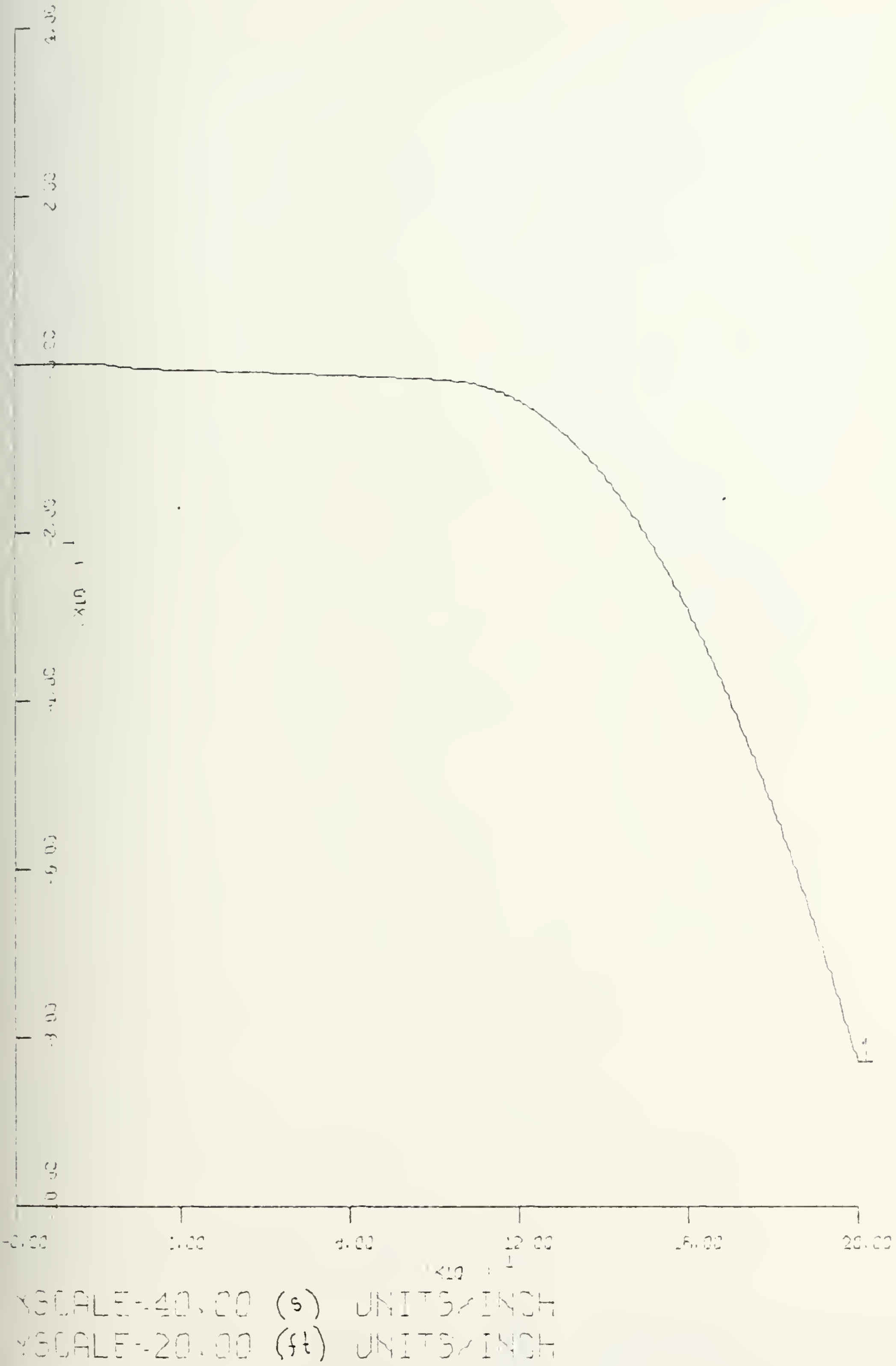


Fig. III-1b. Depth vs. Time. Response to a ramp force at AU. Bounded controller with error limiters. Submarine not "in trim."

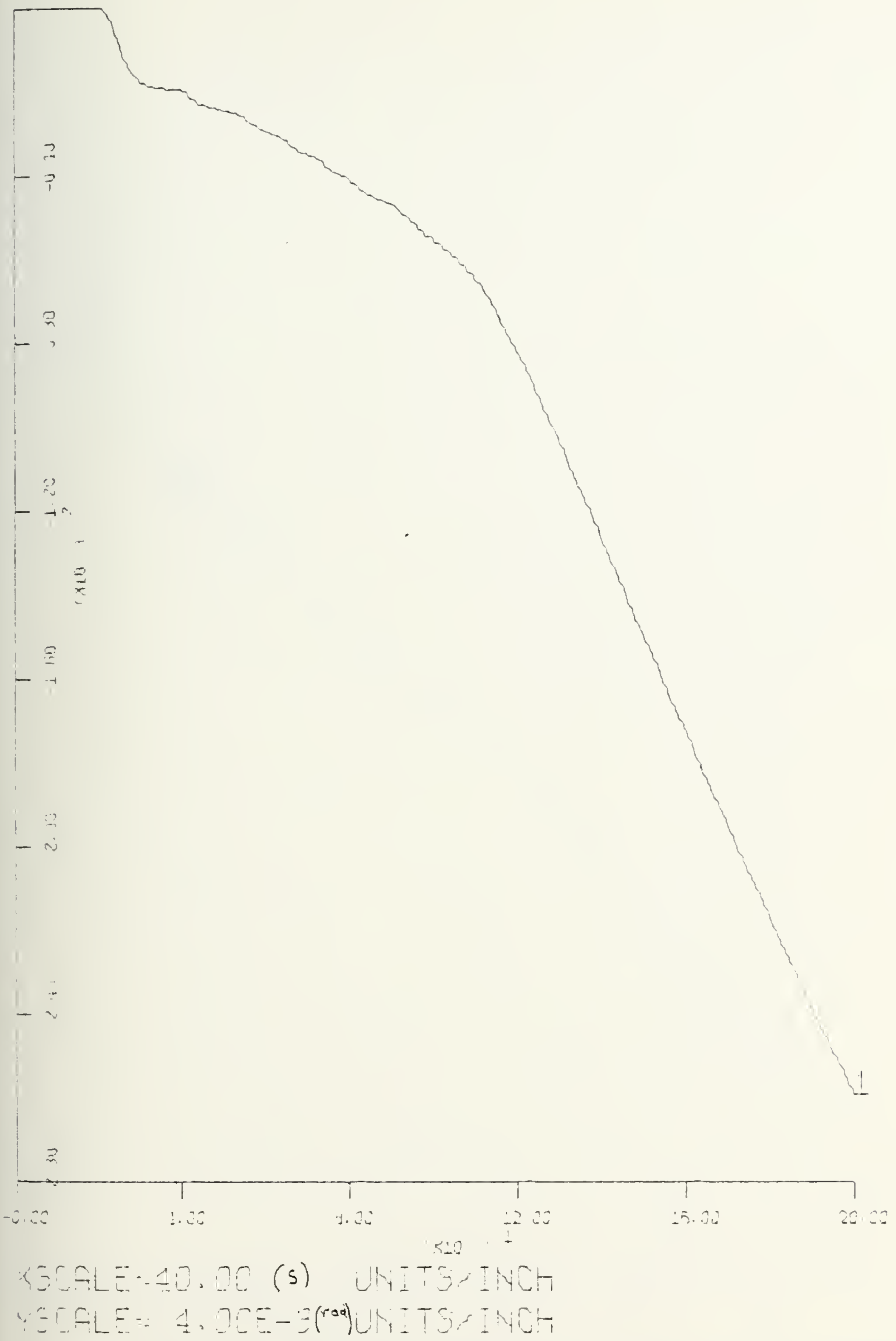


Fig. III-1c. Pitch vs. Time. Response to a ramp force at AU. Bounded controller with error limiters. Submarine not "in trim."

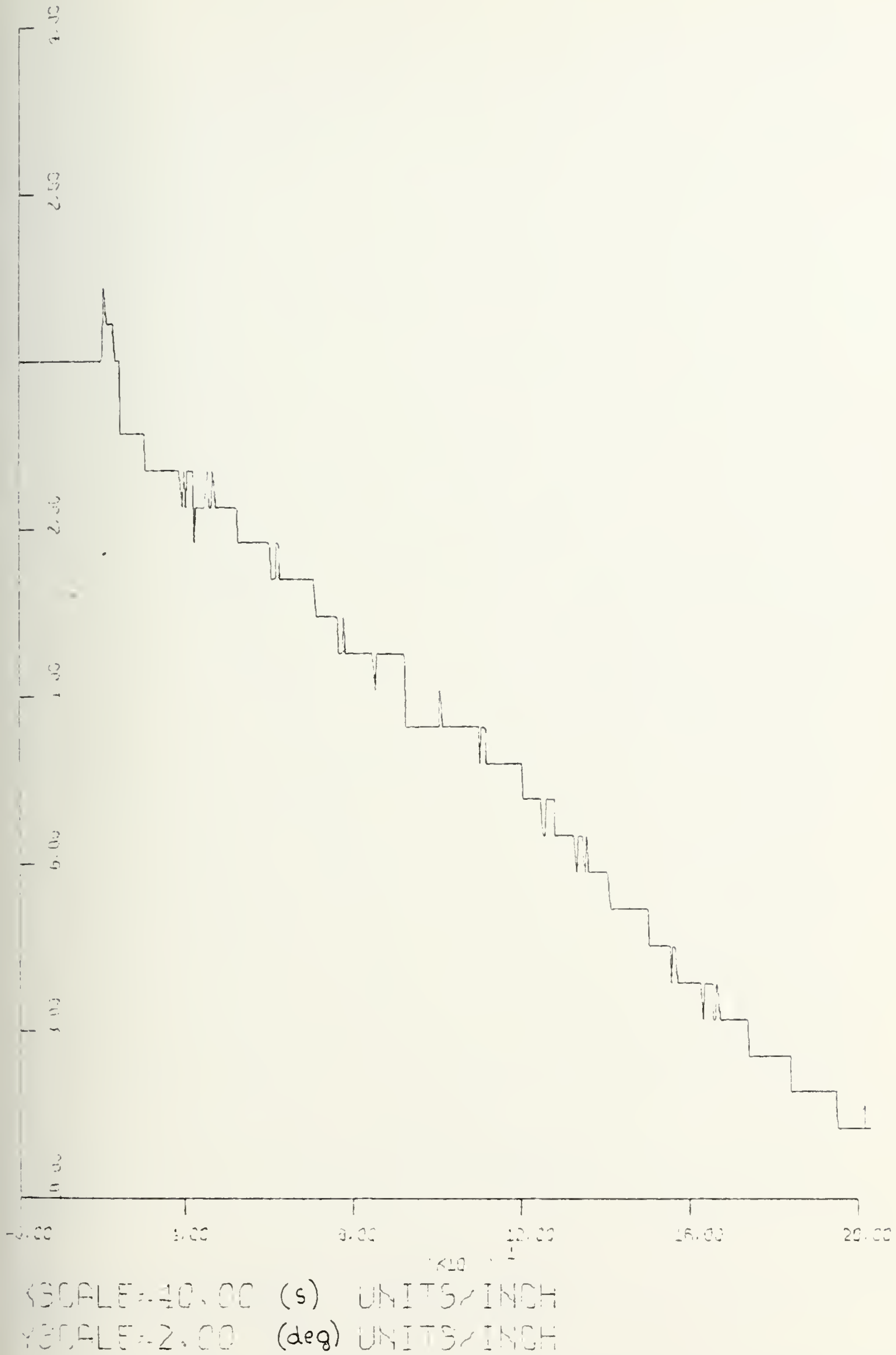


Fig. III-1d. Stern Plane Angle vs. Time. Response to a ramp force at AU. Bounded controller with error limiters. Submarine not "in trim."

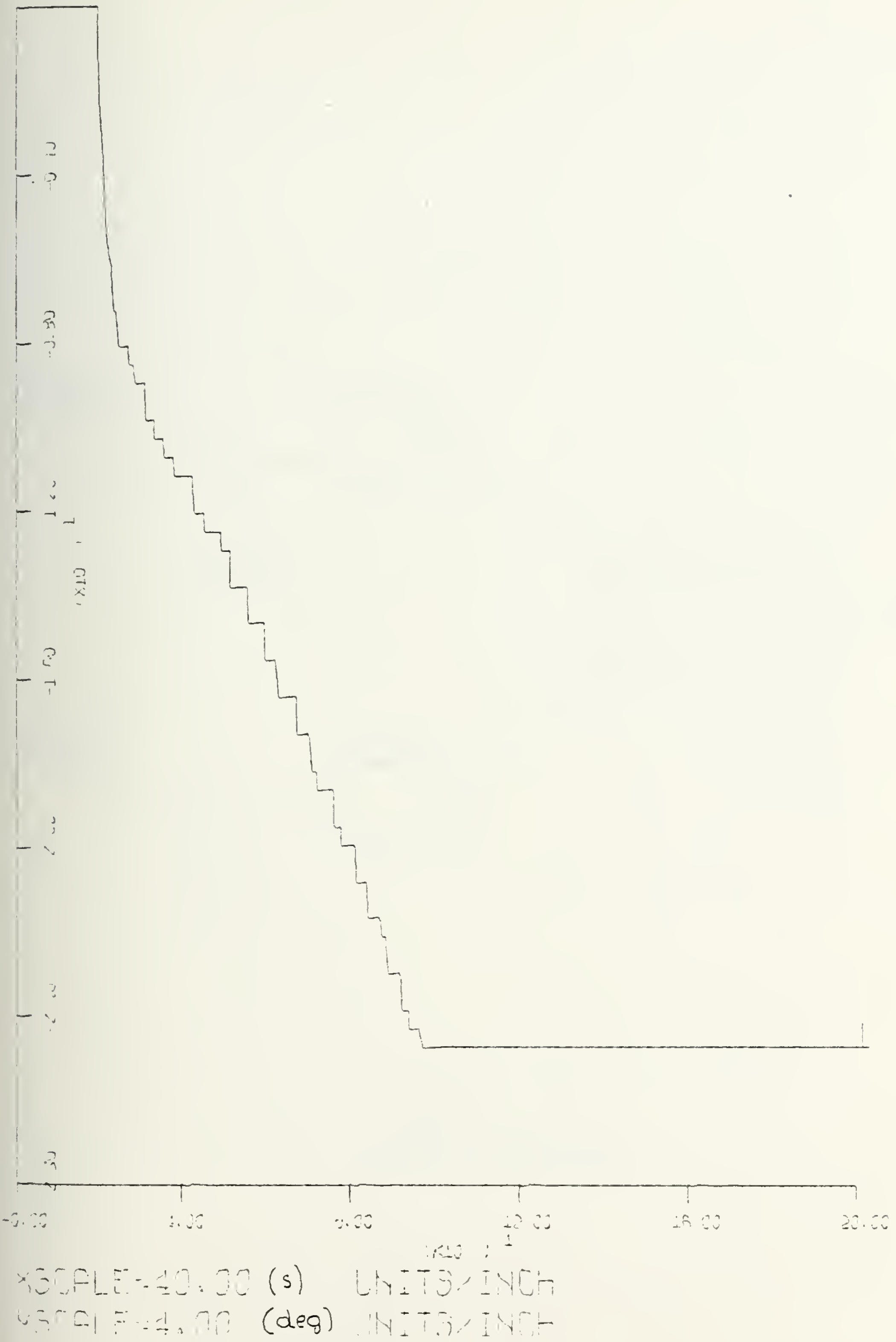


Fig. III-1e. Fairwater Plane Angle vs. Time. Response to a ramp force at AU. Bounded controller with error limiters. Submarine not "in trim."

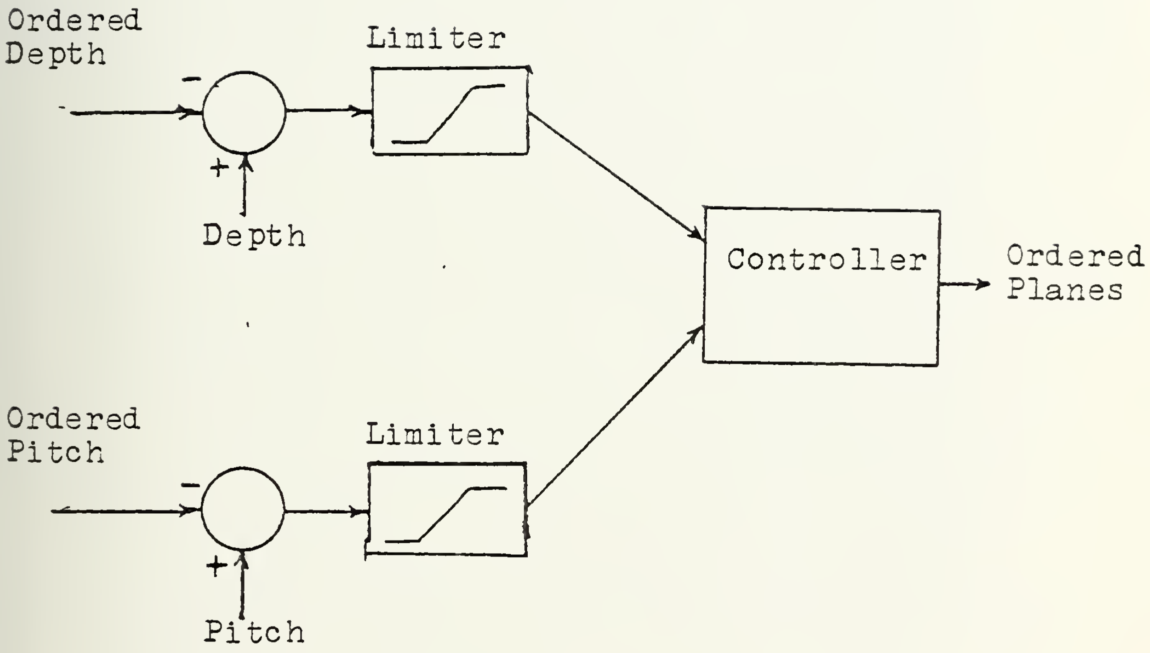
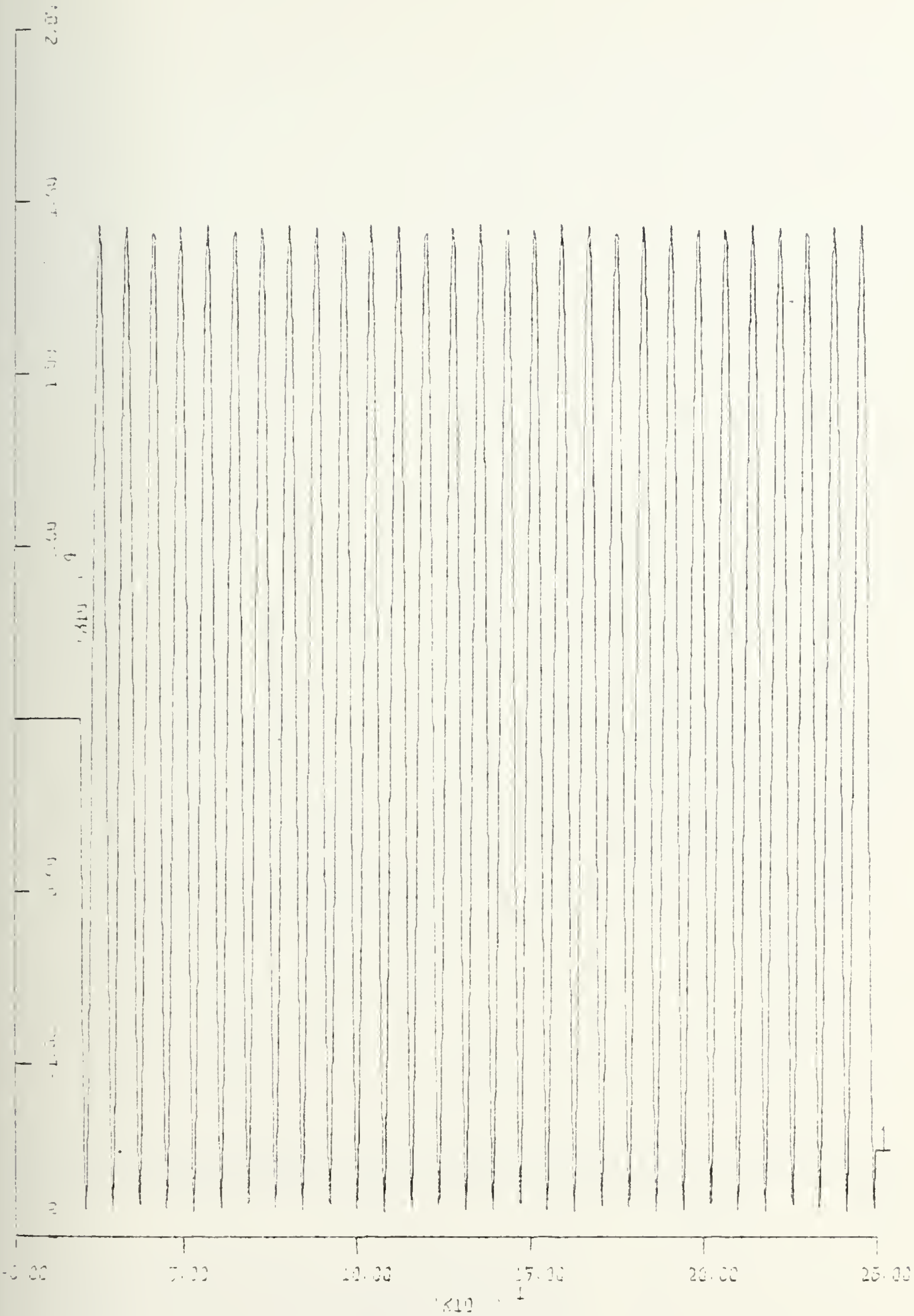


Figure III-2

Error Limiters



XSCALE=50.00 (s) UNITS/INCH
YSCALE=5000.00 (lbs) UNITS/INCH

Fig. III-3a. Sinusoidal force at AU

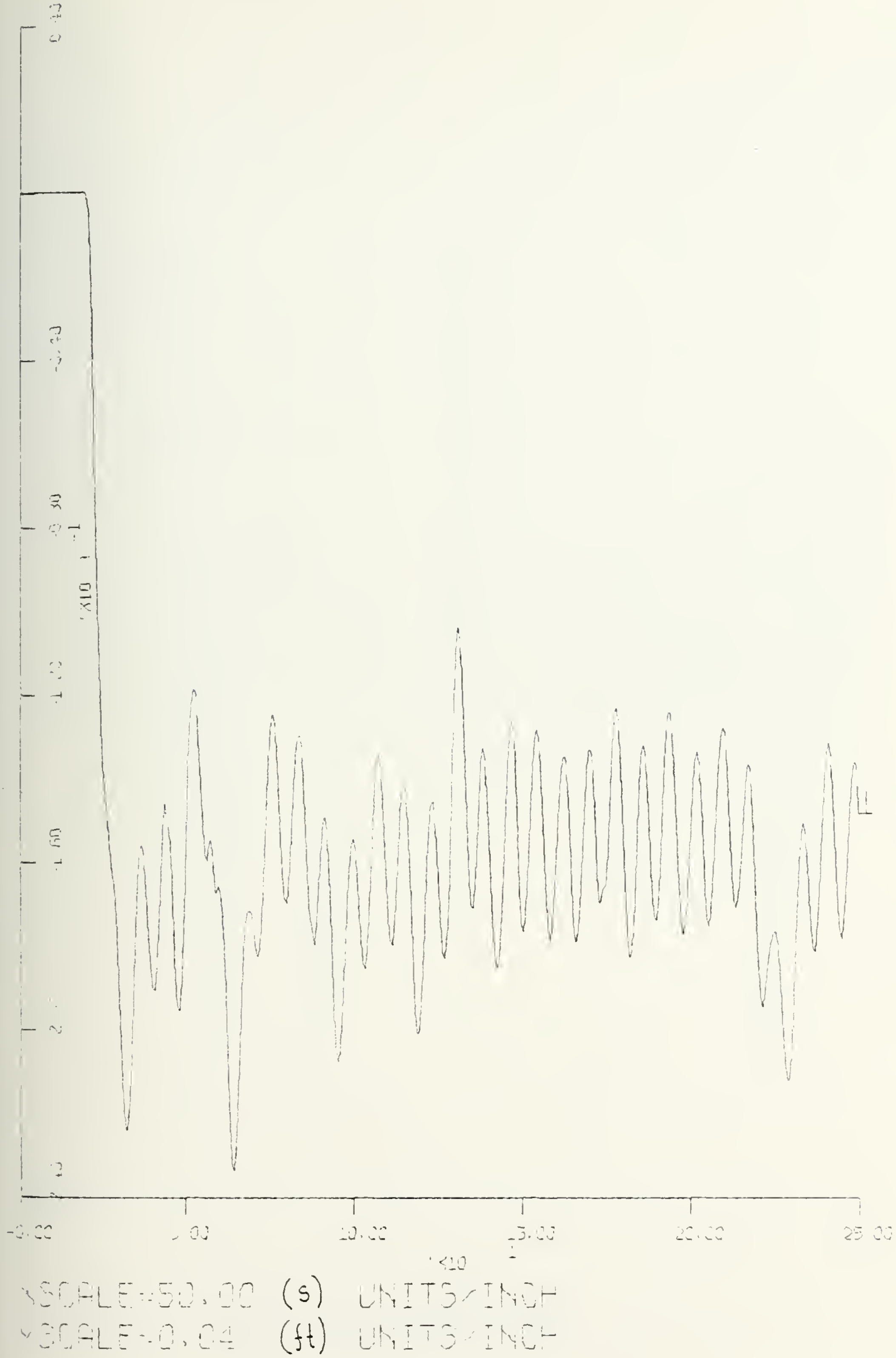


Fig. III-3b. Depth vs. Time. Response to a sinusoidal force at AU. Bounded controller without noise filters. Submarine not "in trim."

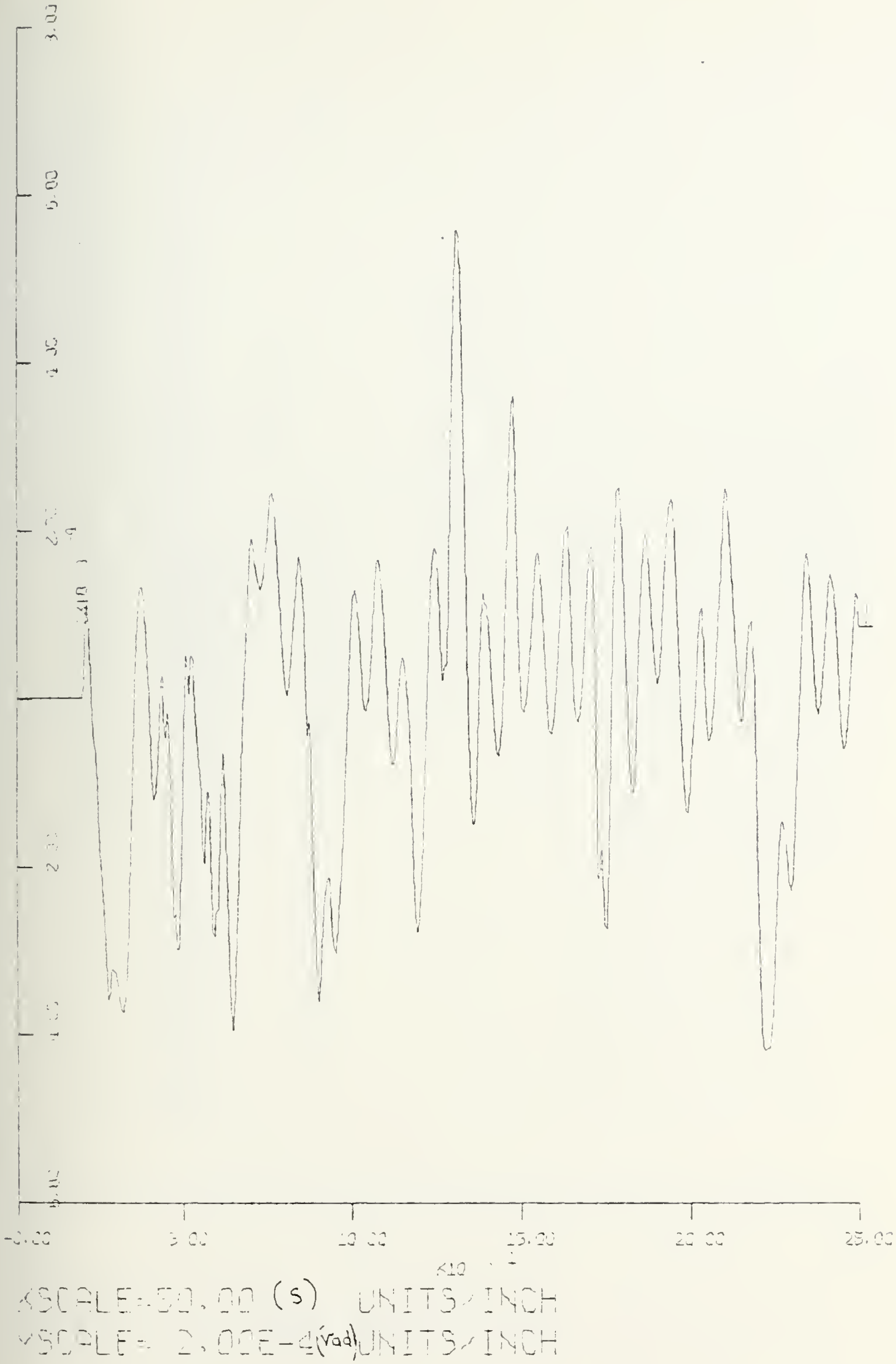


Fig. III-3c. Pitch vs. Time. Response to a sinusoidal force at AU. Bounded controller without noise filters. Submarine not "in trim."

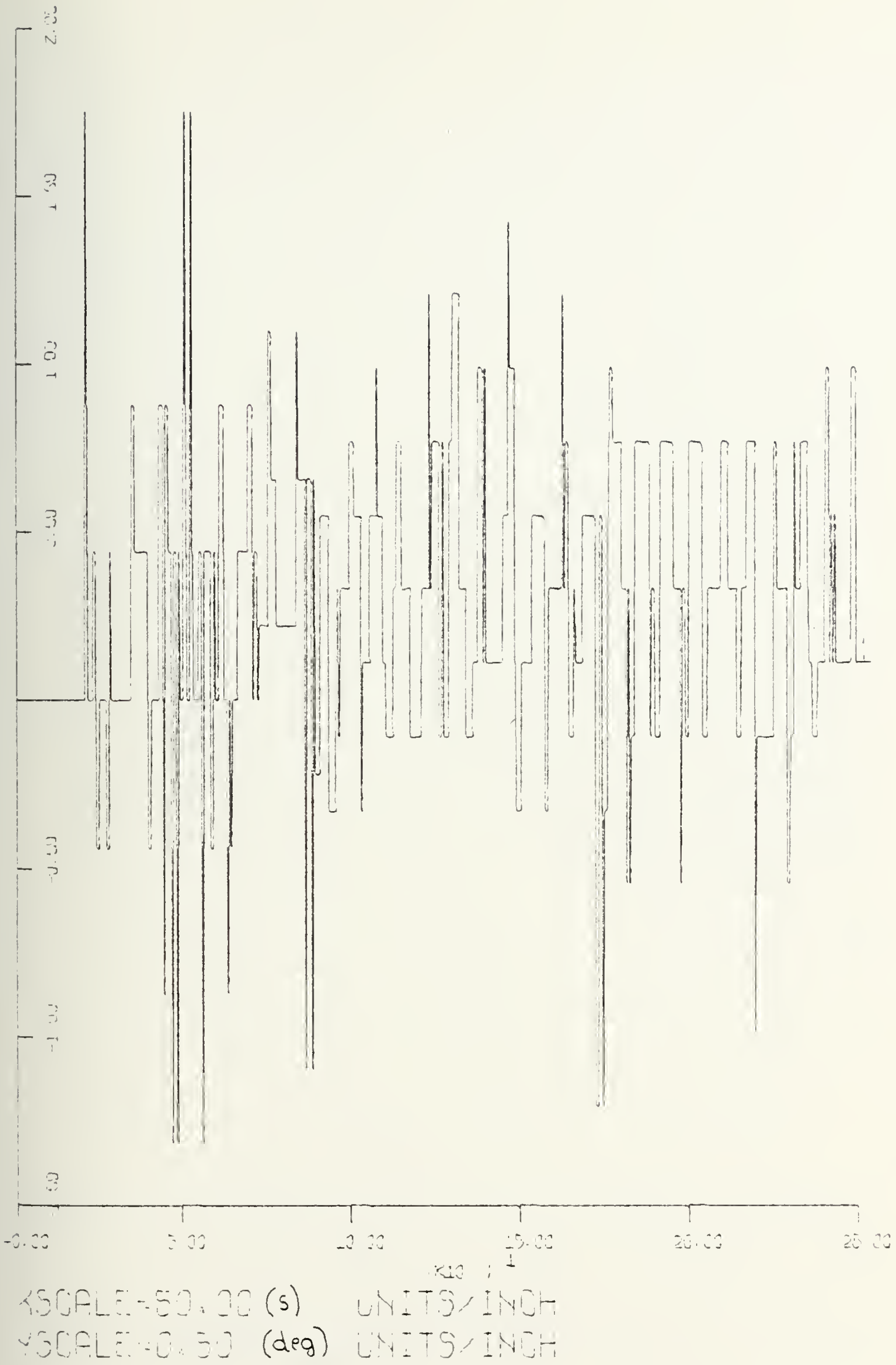


Fig. III-3d. Stern Plane Angle vs. Time. Response to a sinusoidal force at AU. Bounded controller without noise filters. Submarine not "in trim."

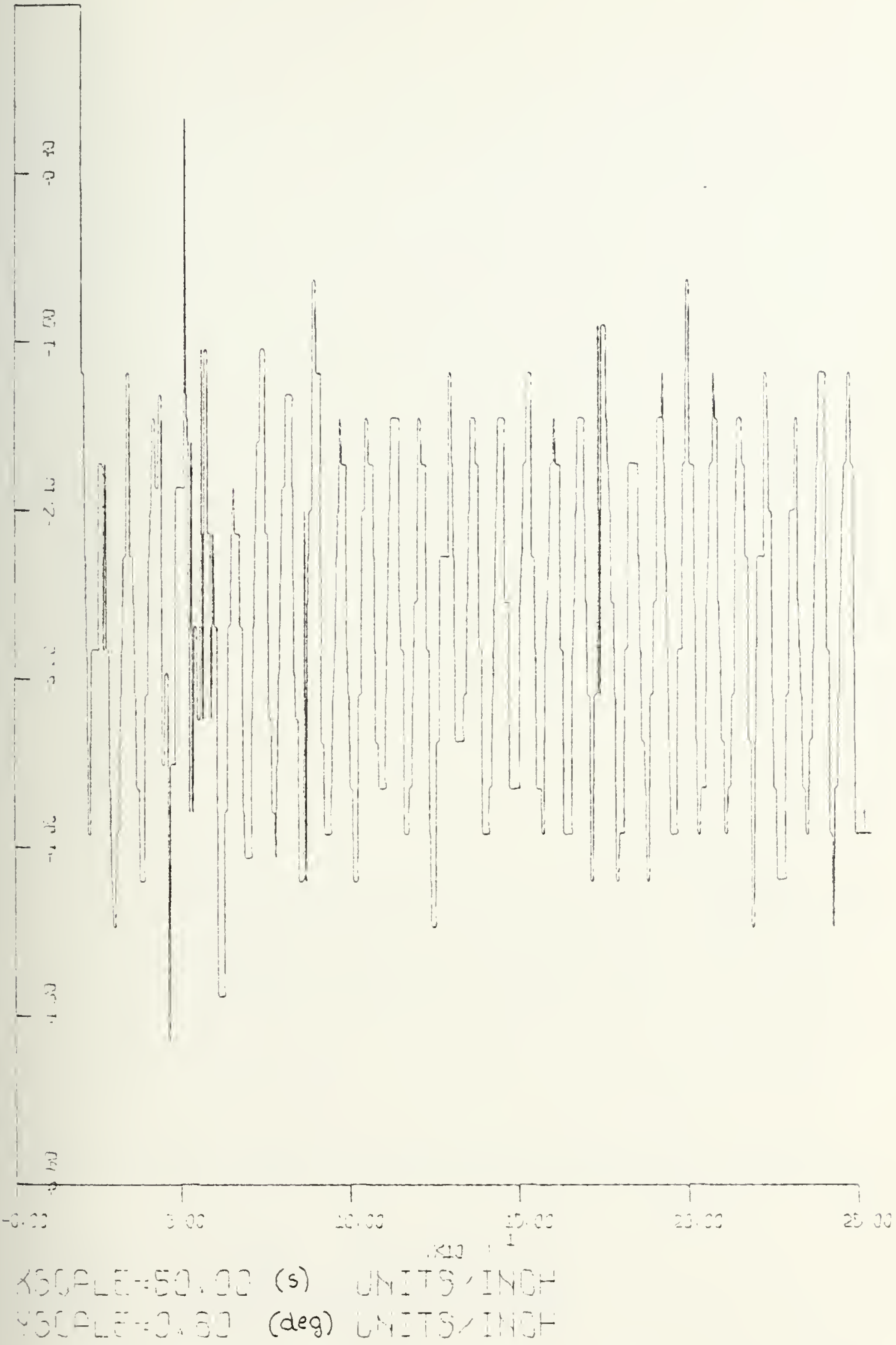


Fig. III-3e. Fairwater Plane Angle vs. Time. Response to a sinusoidal force at AU. Bounded controller without noise filters. Submarine not "in trim."

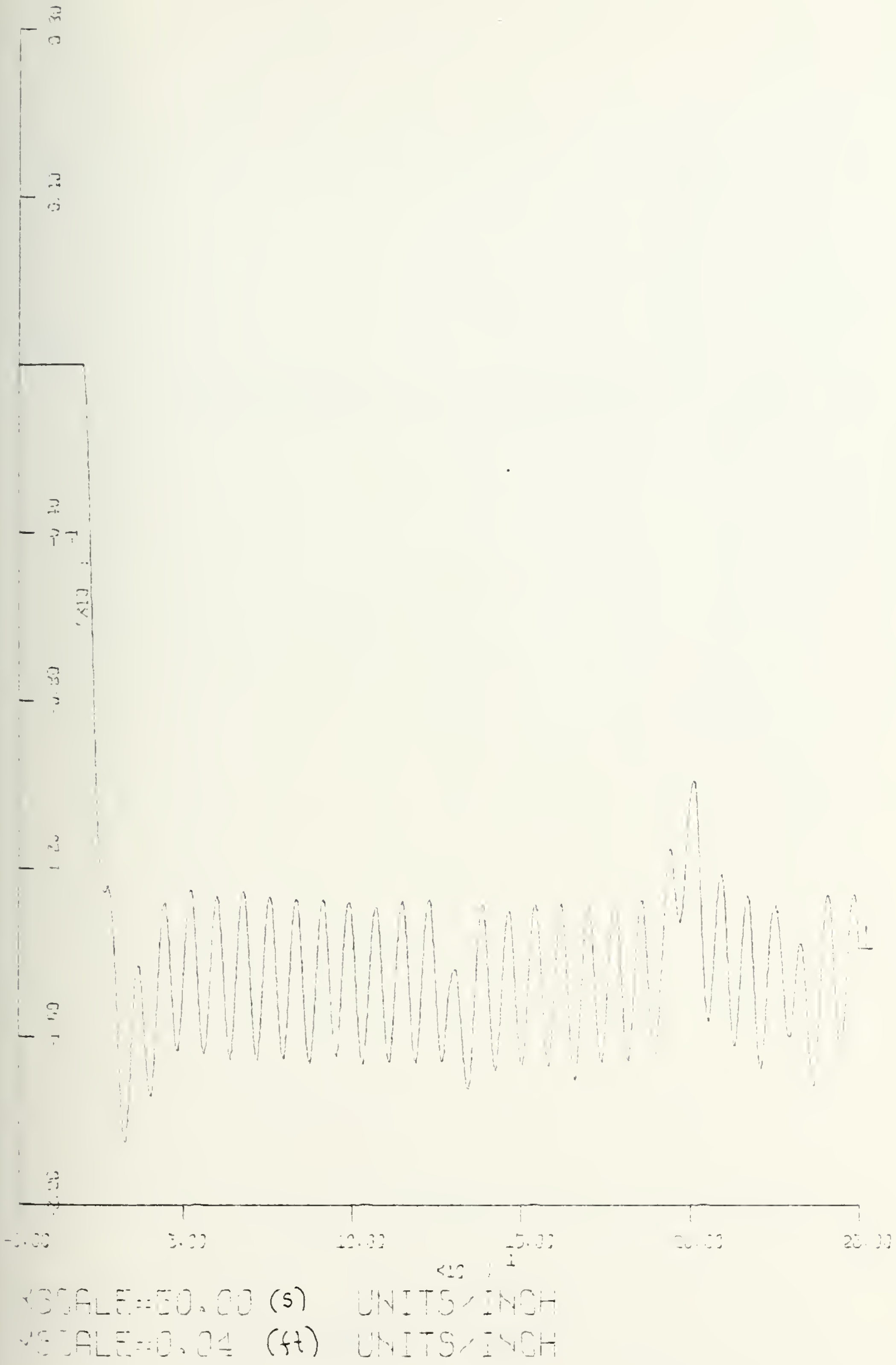


Fig. III-4a. Depth vs. Time. Response to a sinusoidal input at AU. Unbounded controller with noise filters. Submarine not "in trim."

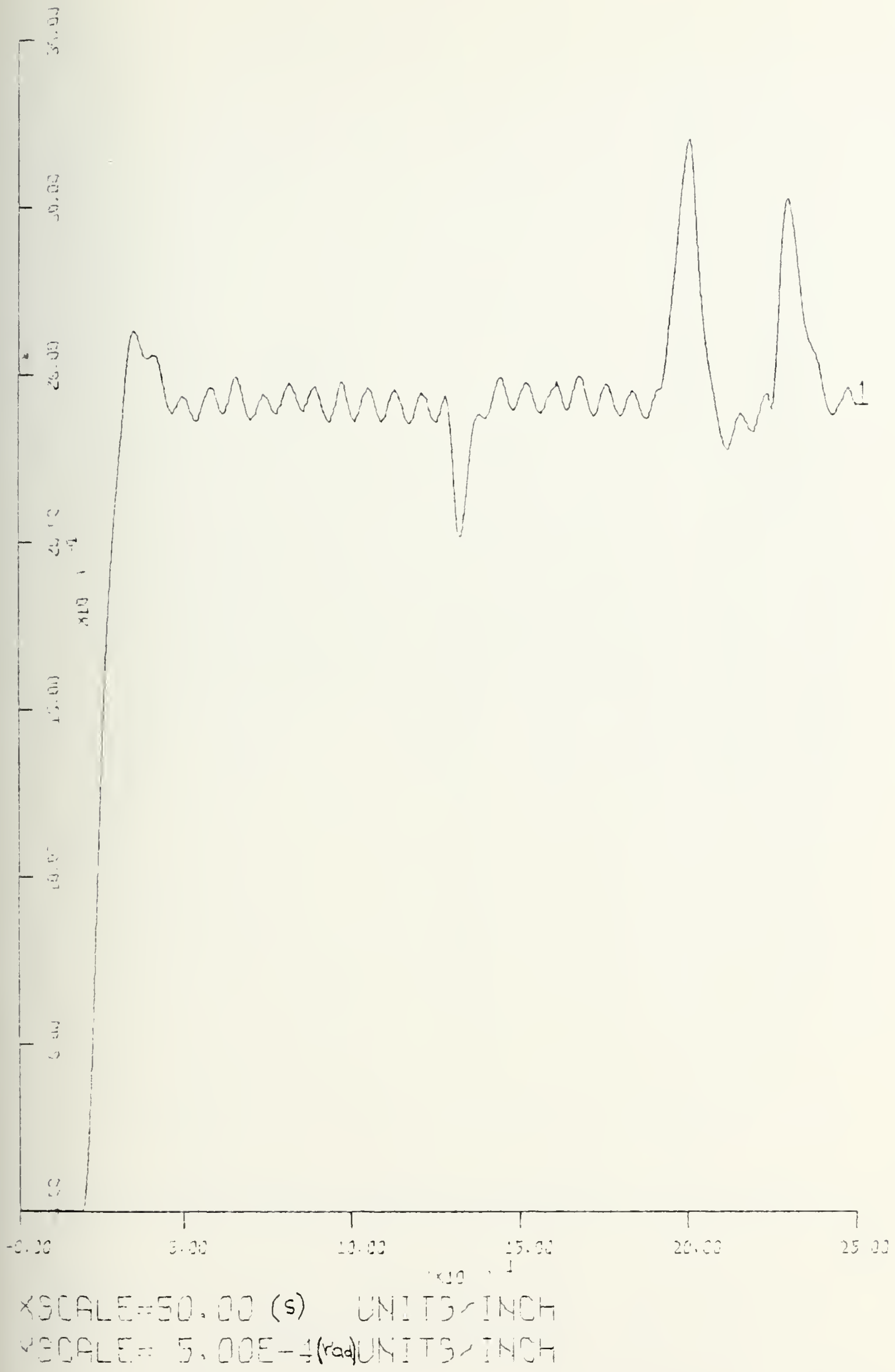


Fig. III-4b. Pitch vs. Time. Response to a sinusoidal input at AU. Unbounded controller with noise filters. Submarine not "in trim."

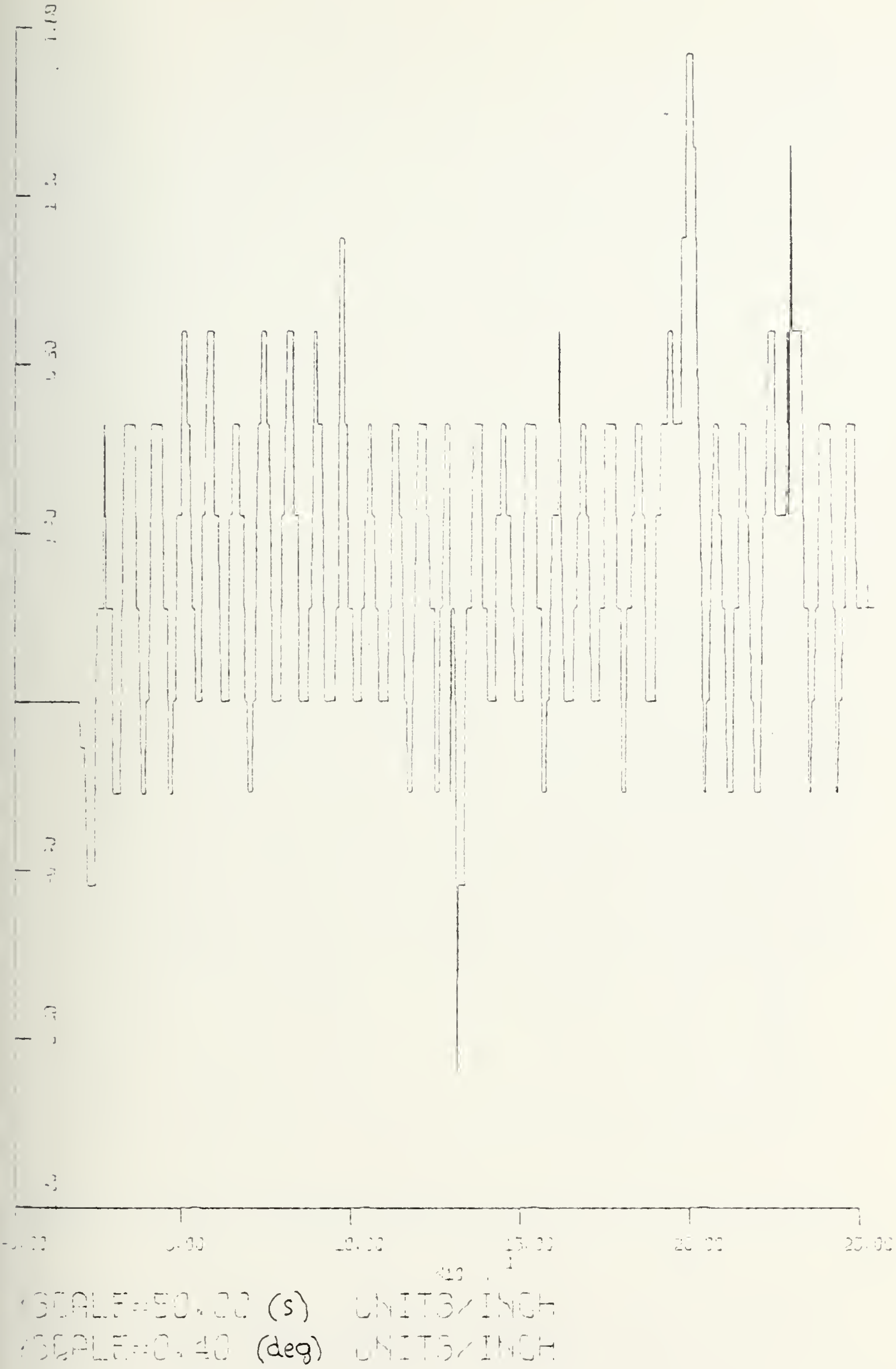


Fig. III-4c. Stern Plane Angle vs. Time. Response to a sinusoidal input at AU. Unbounded controller with noise filters. Submarine not "in trim."

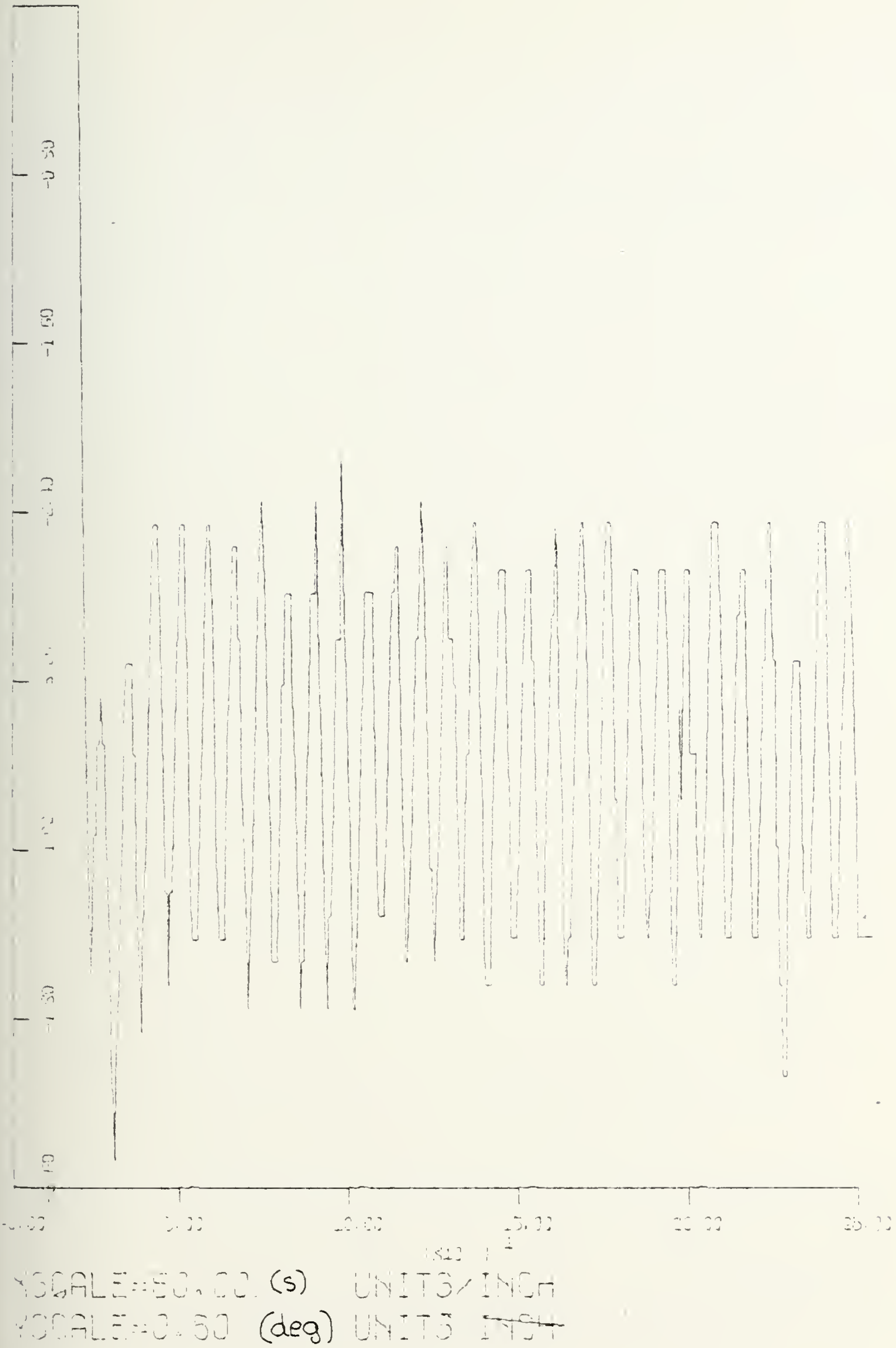


Fig. III-4d. Fairwater Plane Angle vs. Time. Response to a sinusoidal input at AU. Unbounded controller with noise filters. Submarine not "in trim."

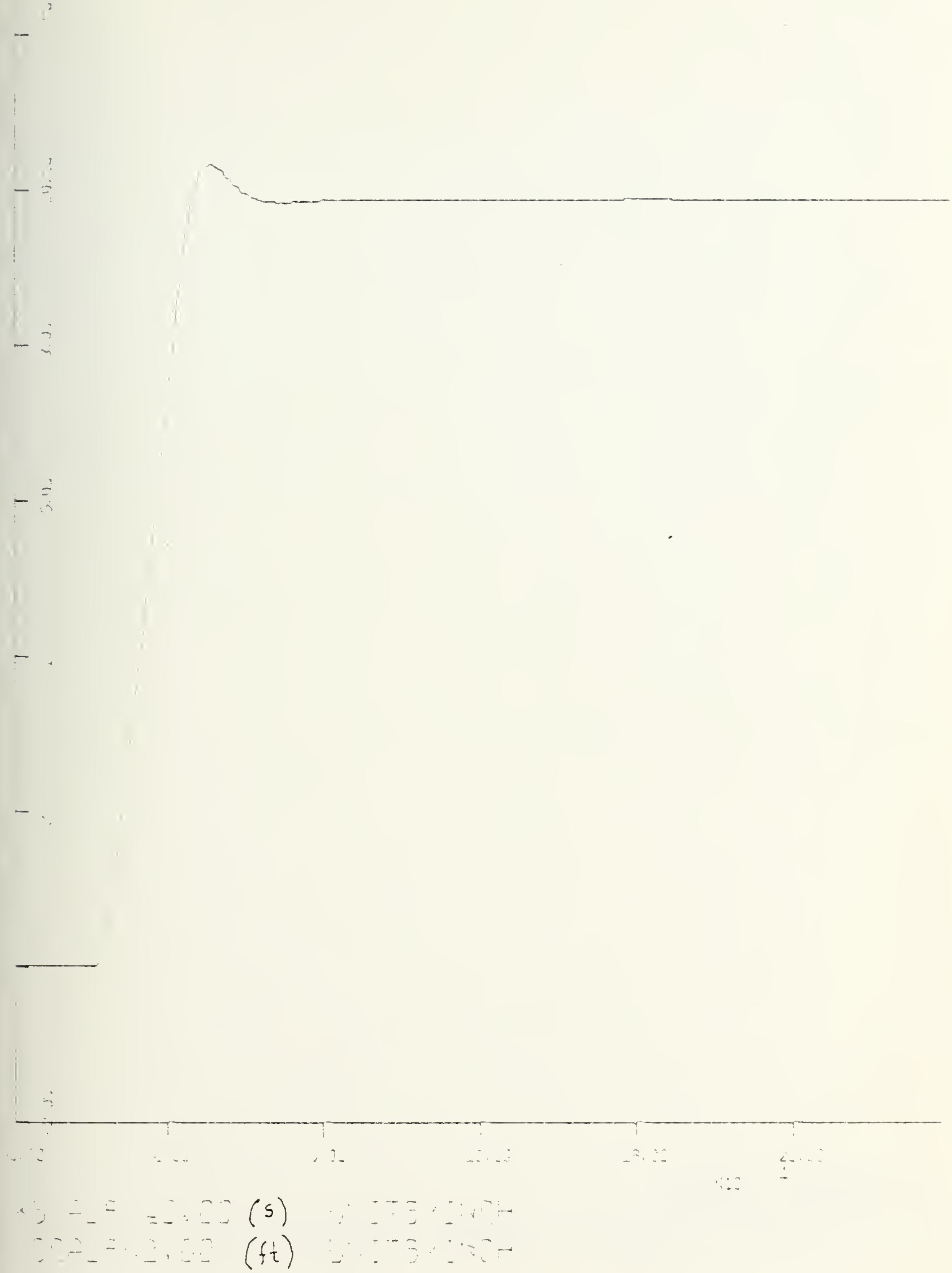


Fig. III-5a. Depth vs. Time. Response to a depth order of 10 ft. Unbounded controller with error limiters and noise filters.

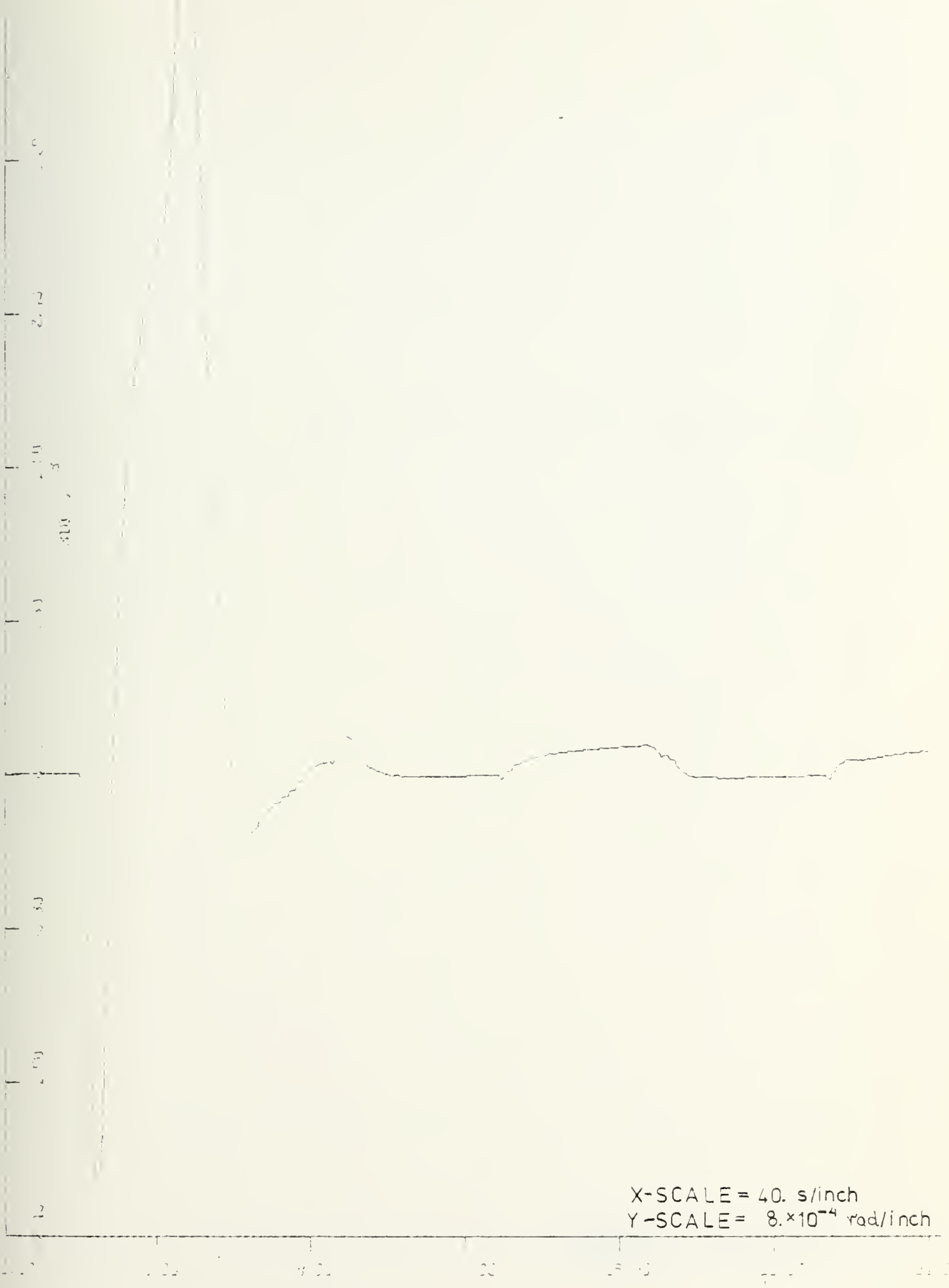


Fig. III-5b. Pitch vs. Time. Response to a depth order of 10 ft. Unbounded controller with error limiters and noise filters.

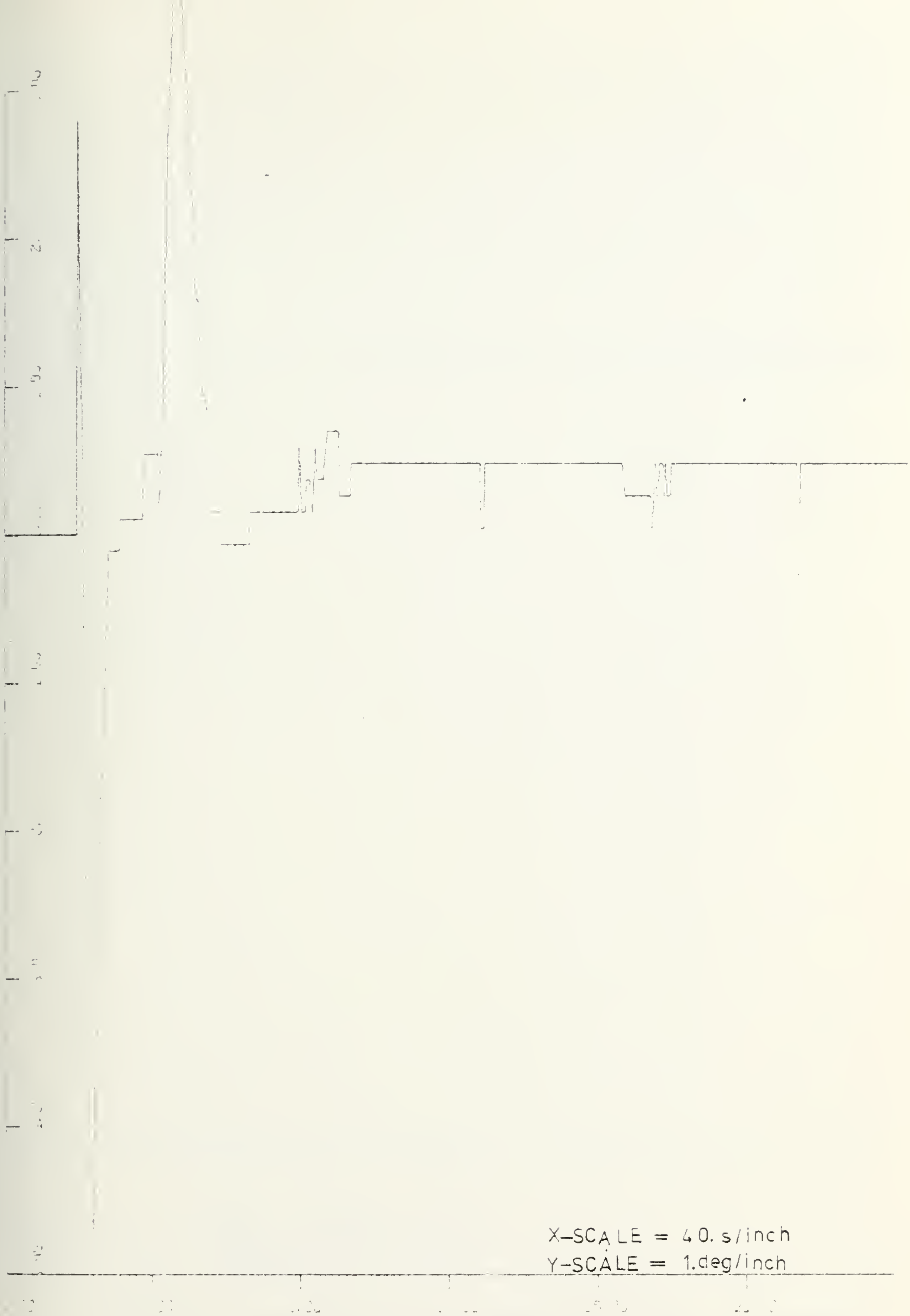


Fig. III-5c. Stern Plane Angle vs. Time. Response to a depth order of 10 ft. Unbounded controller with error limiters and noise filters.

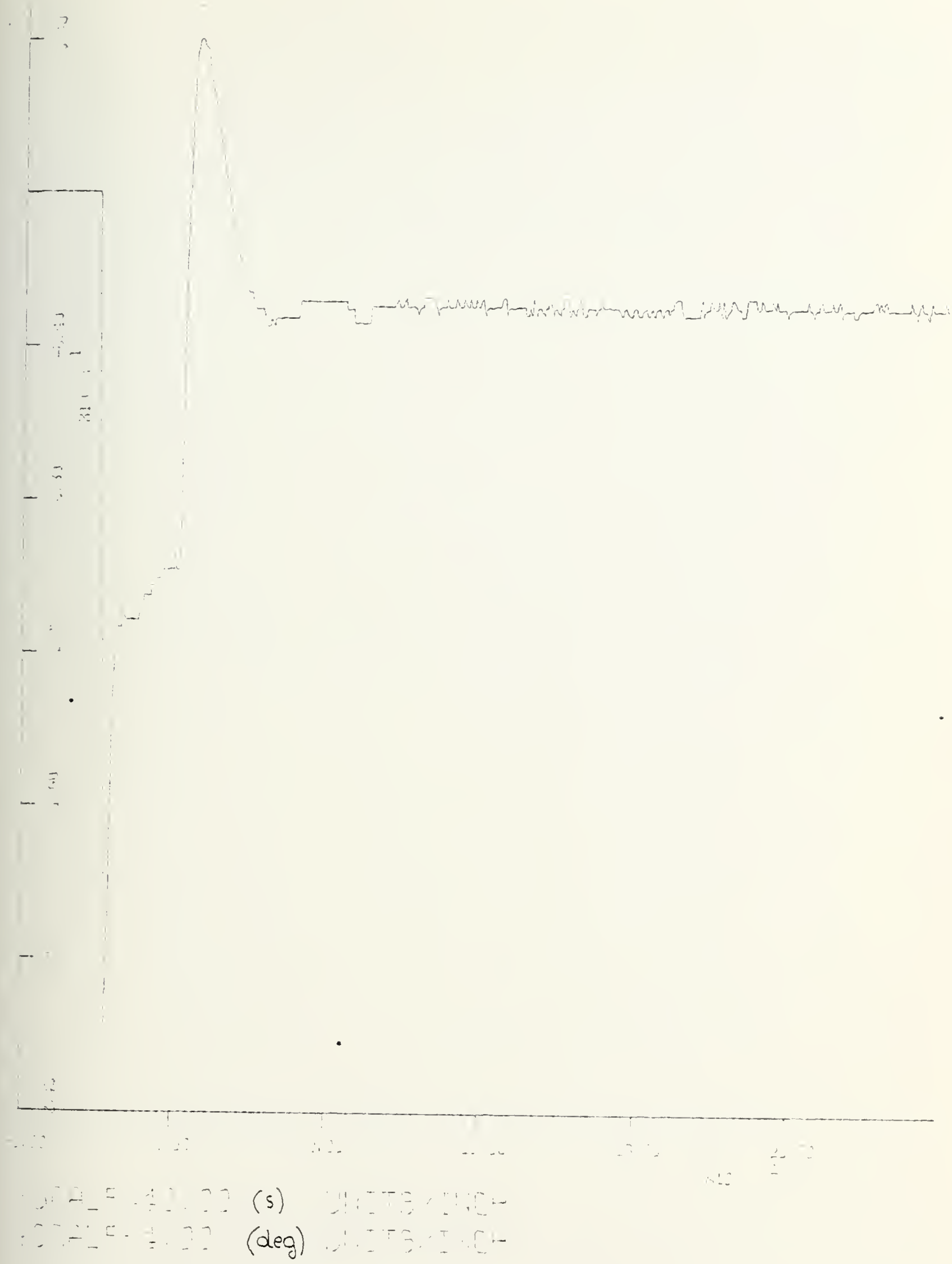


Fig. III-5d. Fairwater Plane Angle vs. Time. Response to a depth order of 10 ft. Unbounded controller with error limiters and noise filters.

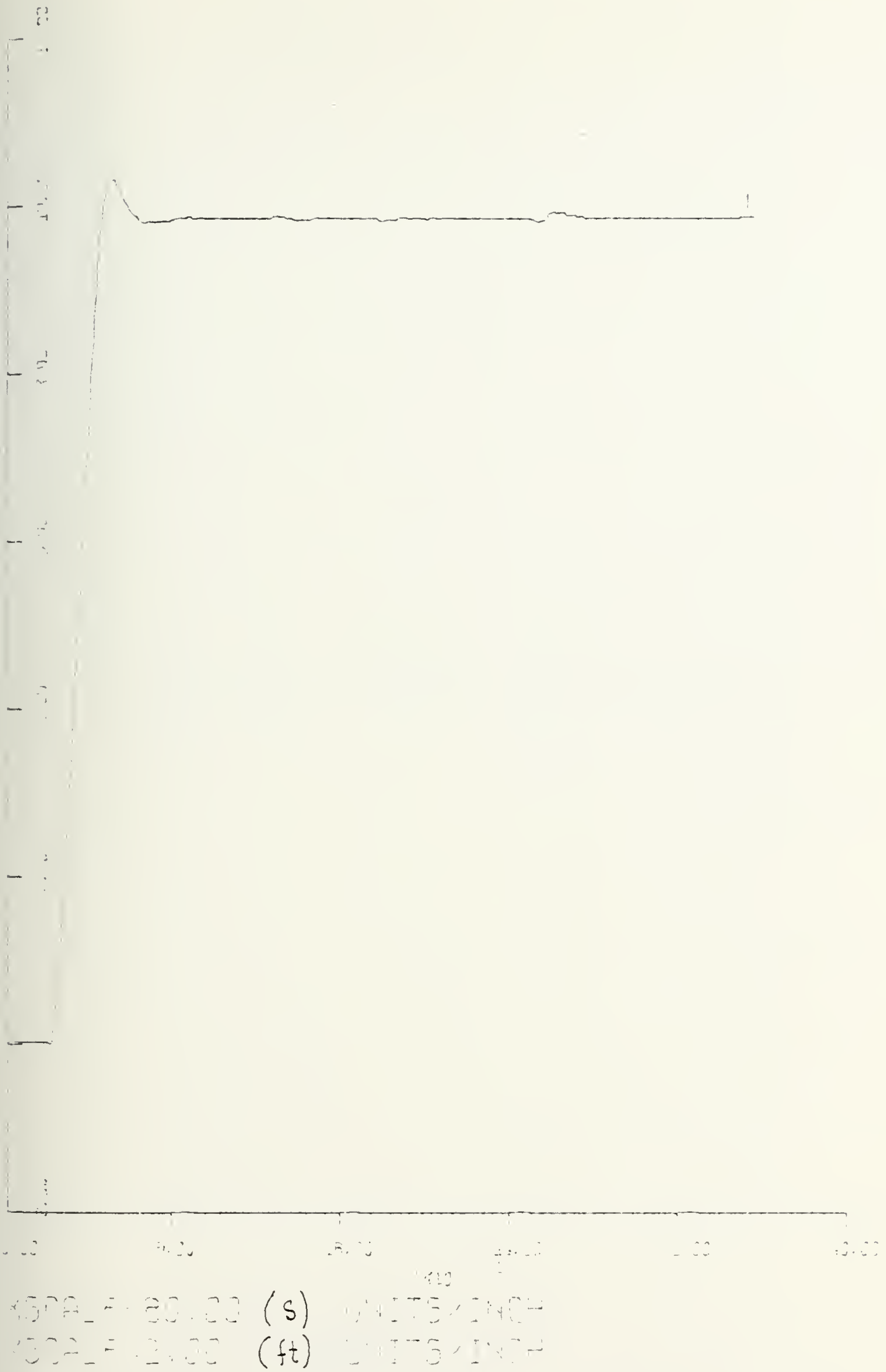


Fig. III-6a. Depth vs. Time. Response to a depth order of 10 ft. Bounded controller without noise filters.

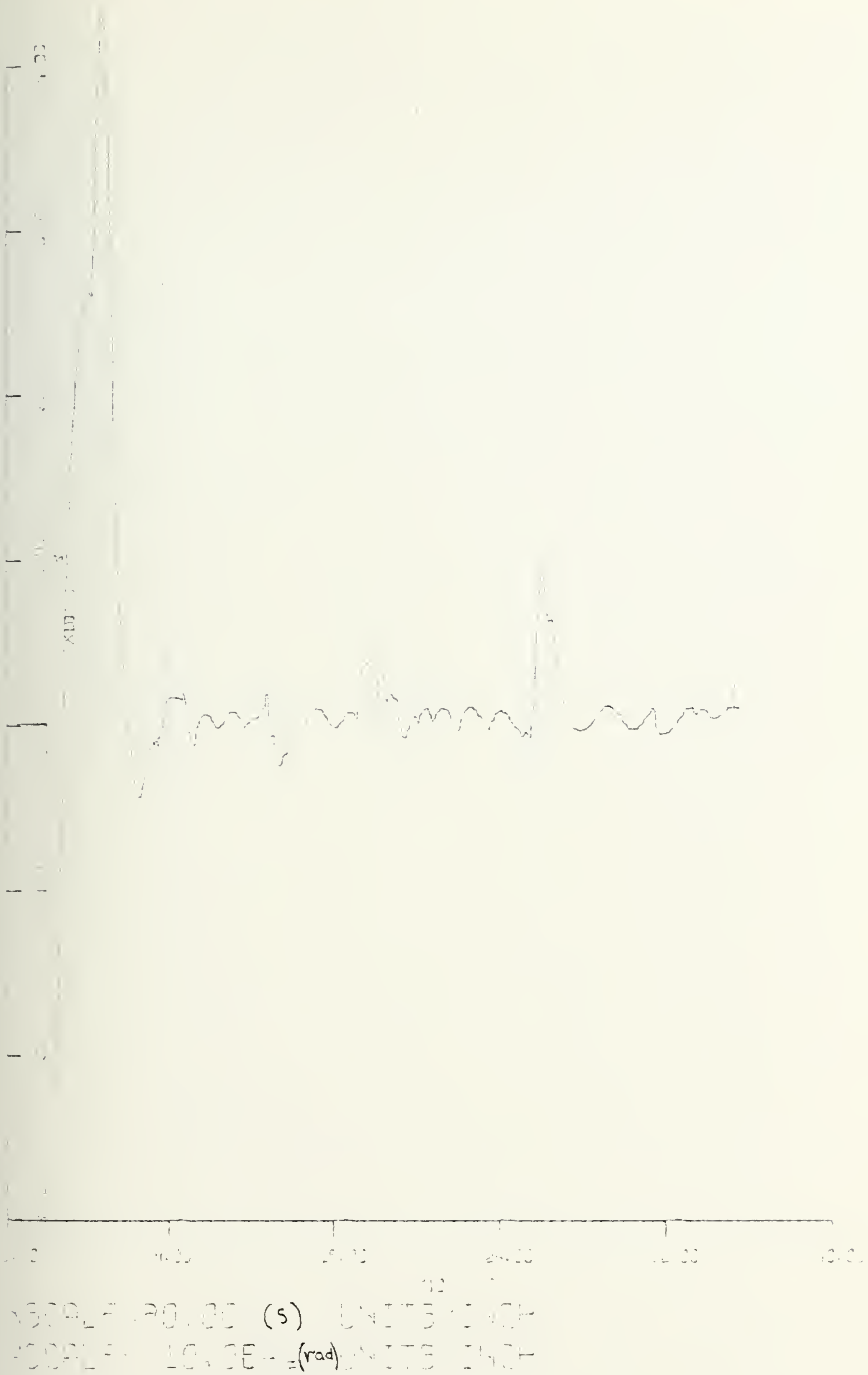


Fig. III-6b. Pitch vs. Time. Response to a depth order of 10 ft. Bounded controller without noise filters.

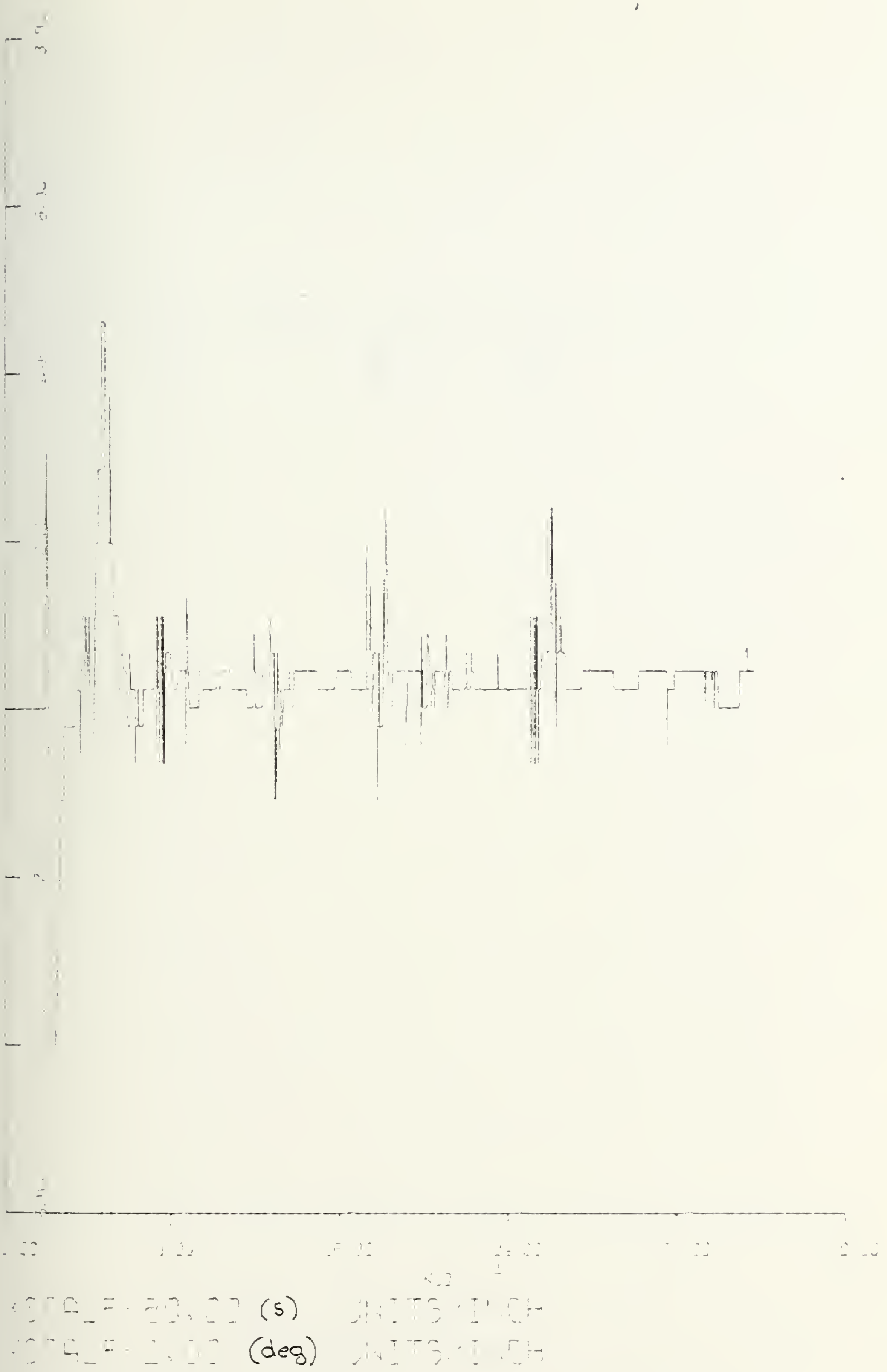


Fig. III-6c. Stern Plane Angle vs. Time. Response to a depth order of 10 ft. Bounded controller without noise filters.

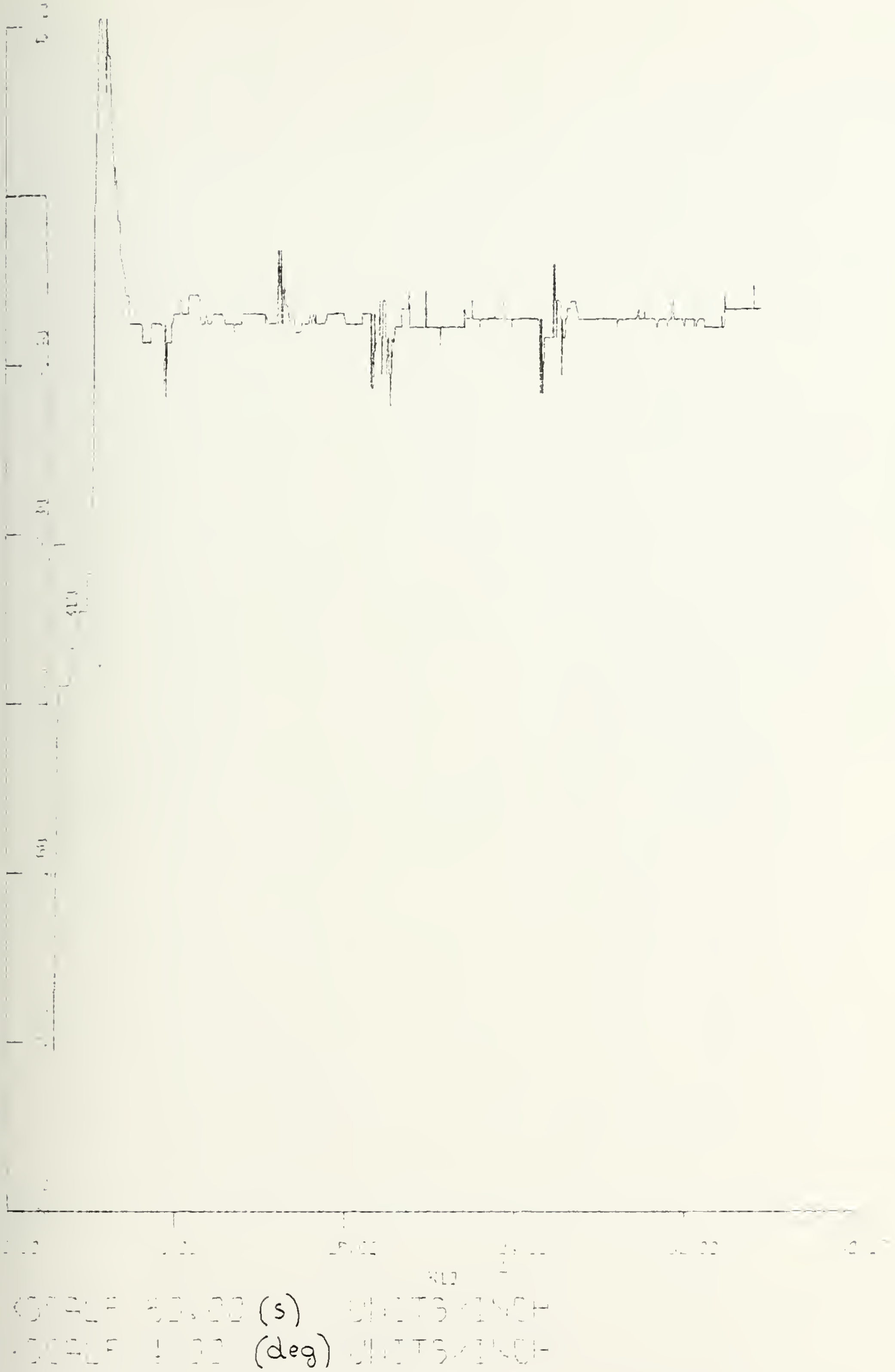


Fig. III-6d. Fairwater Plane Angle vs. Time. Response to a depth order of 10 ft. Bounded controller without noise filters.

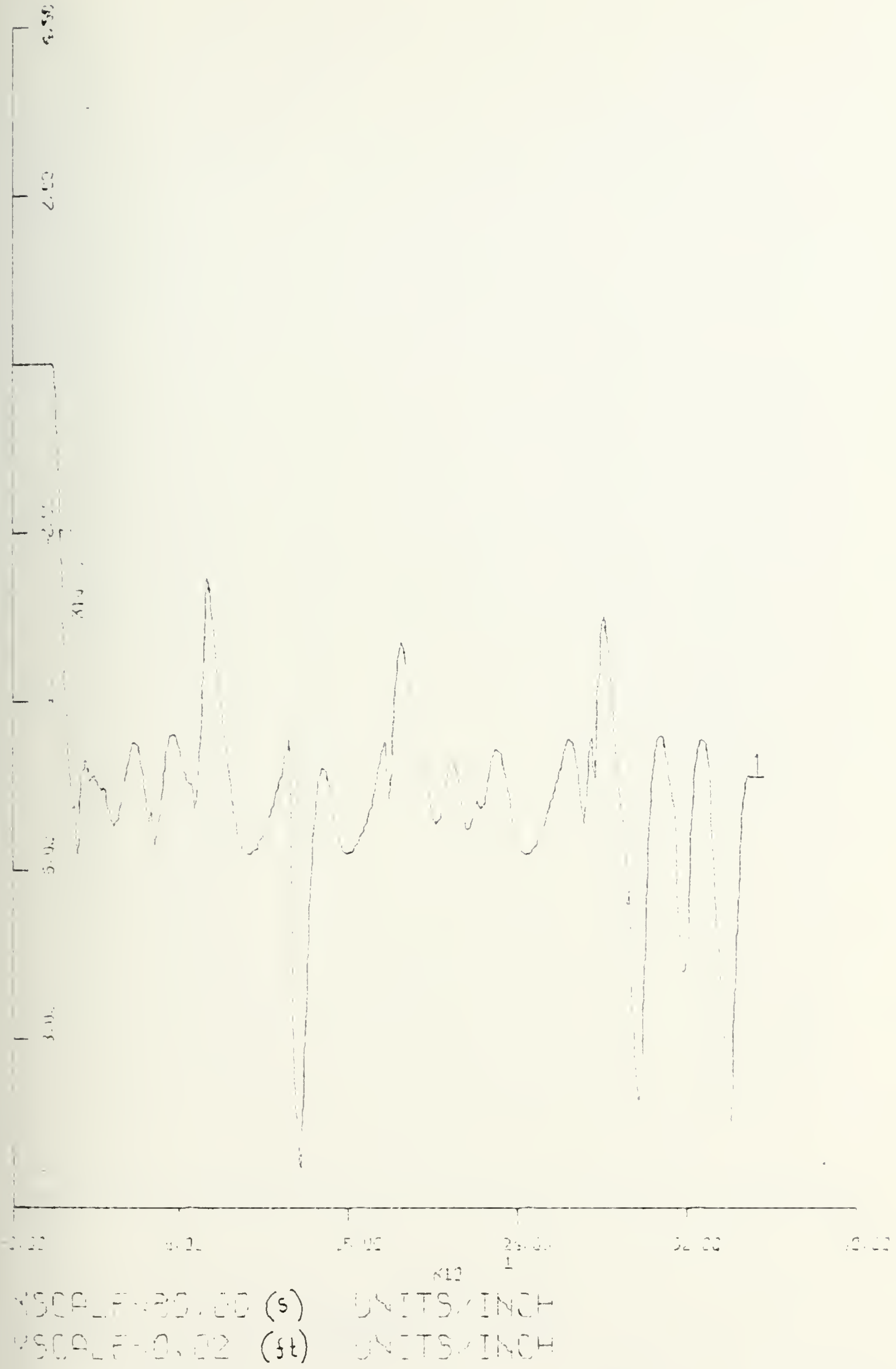


Fig. III-7a. Depth vs. Time. Submarine "in trim." Bounded controller without noise filters. Zero ordered depth.

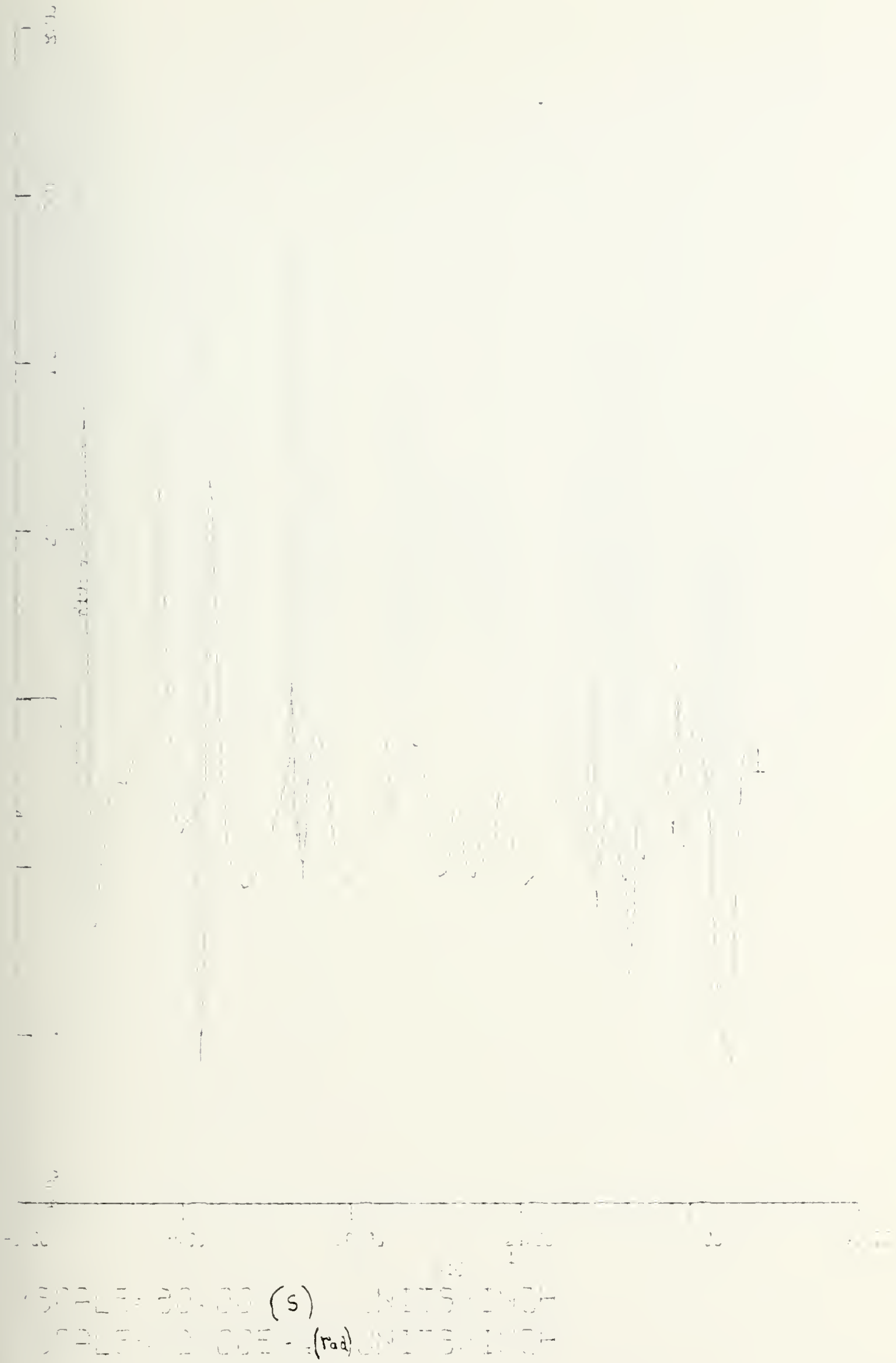


Fig. III-7b. Pitch vs. Time. Submarine "in trim." Bounded controller without noise filters. Zero ordered depth.

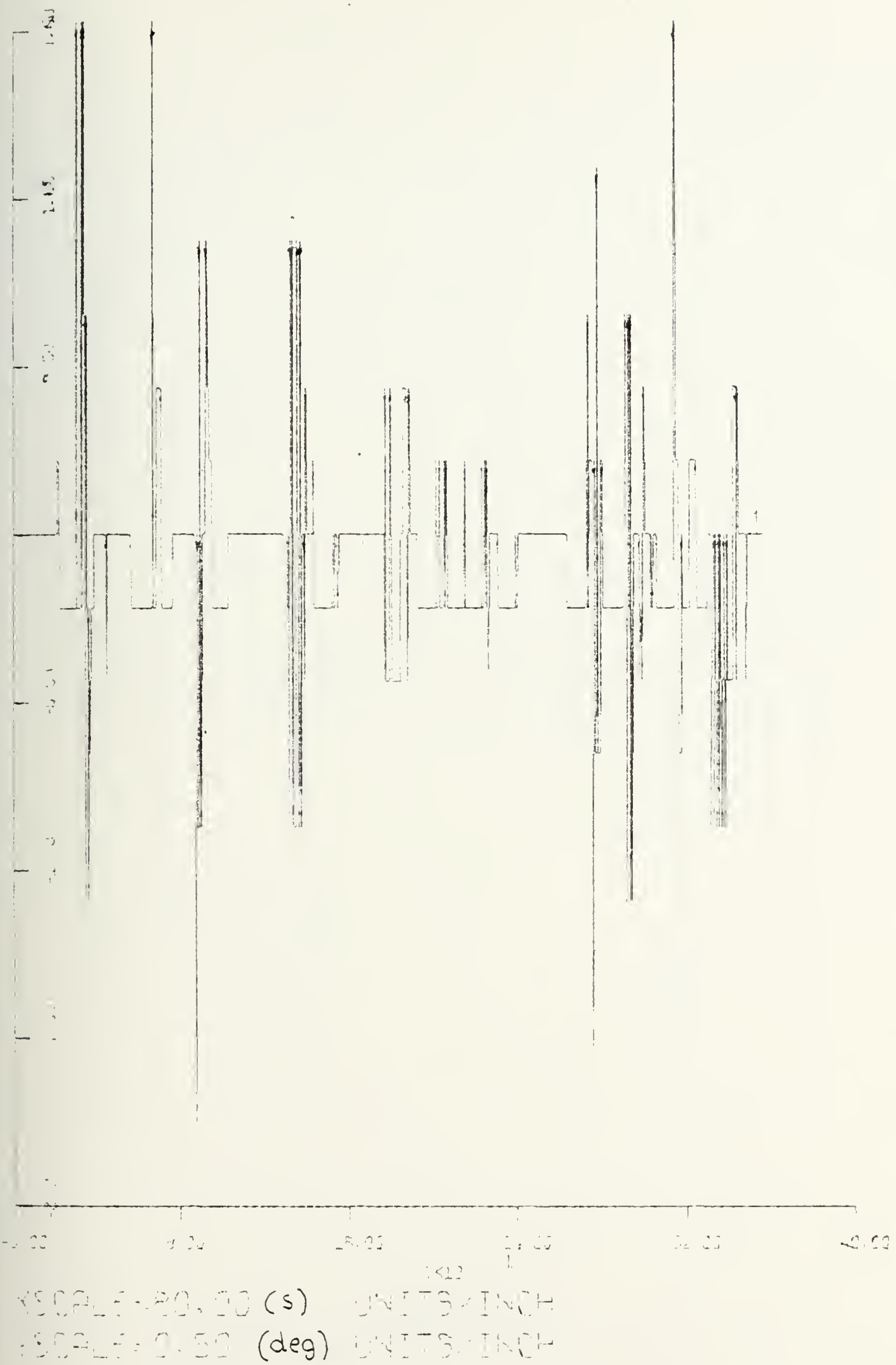


Fig. III-7c. Stern Plane Angle vs. Time. Submarine "in trim." Bounded controller without noise filters. Zero ordered depth.

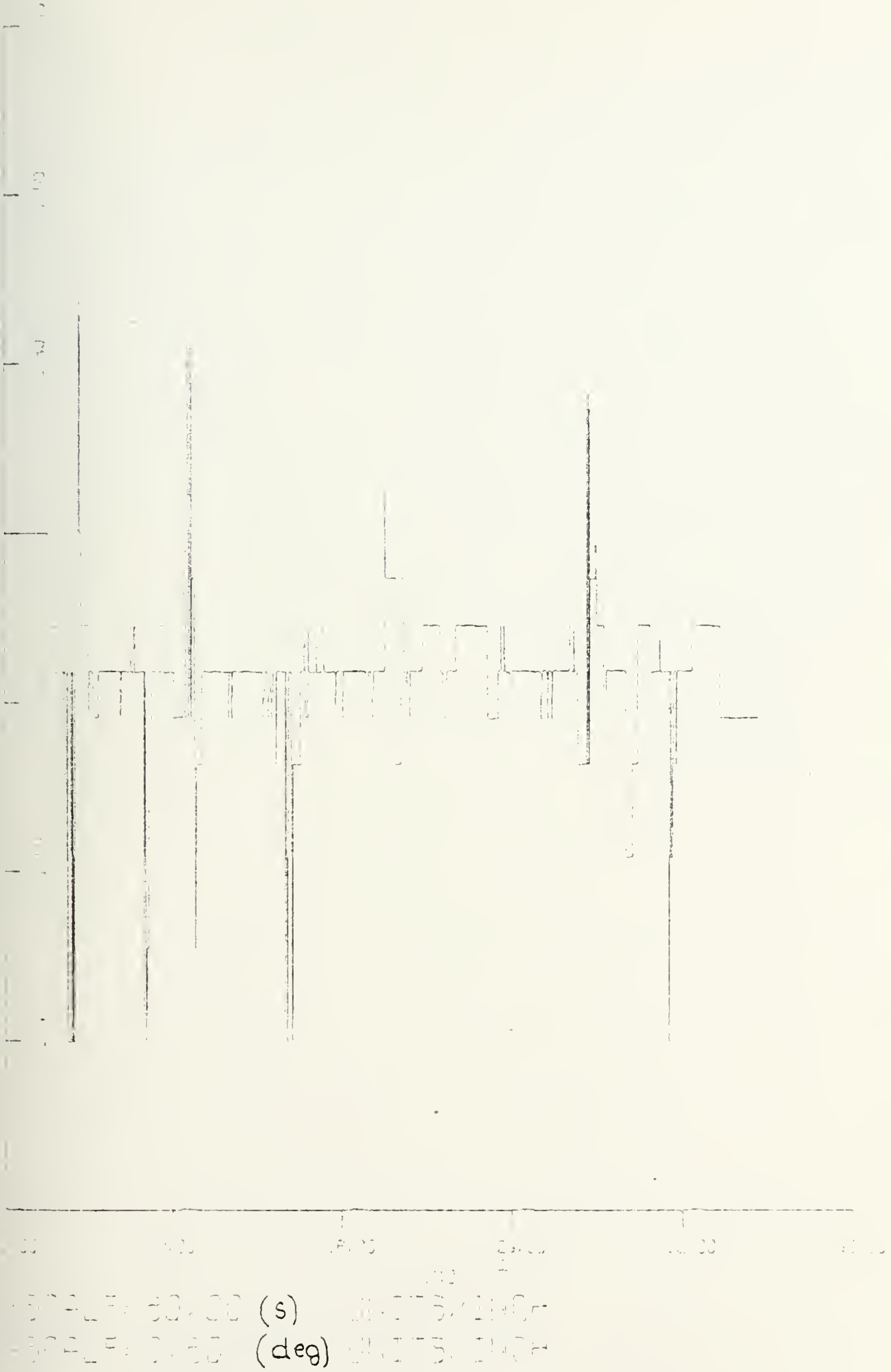


Fig. III-7d. Fairwater Plane Angle vs. Time. Submarine "in trim." Bounded controller without noise filters. Zero ordered depth.

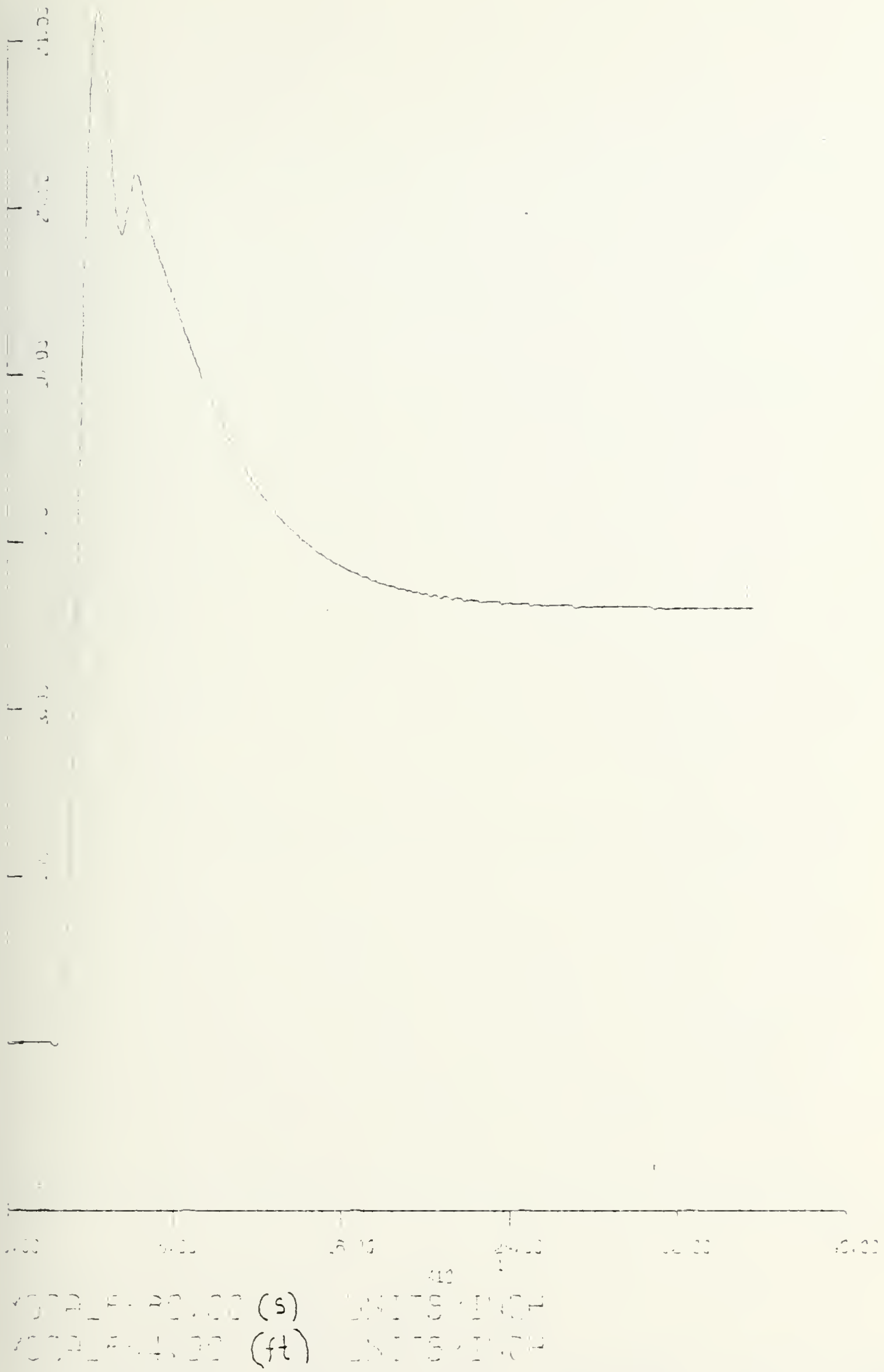


Fig. III-8a. Depth vs. Time. Submarine not "in trim." Response to depth and pitch orders. Tracking controller without error limiters noise filters.

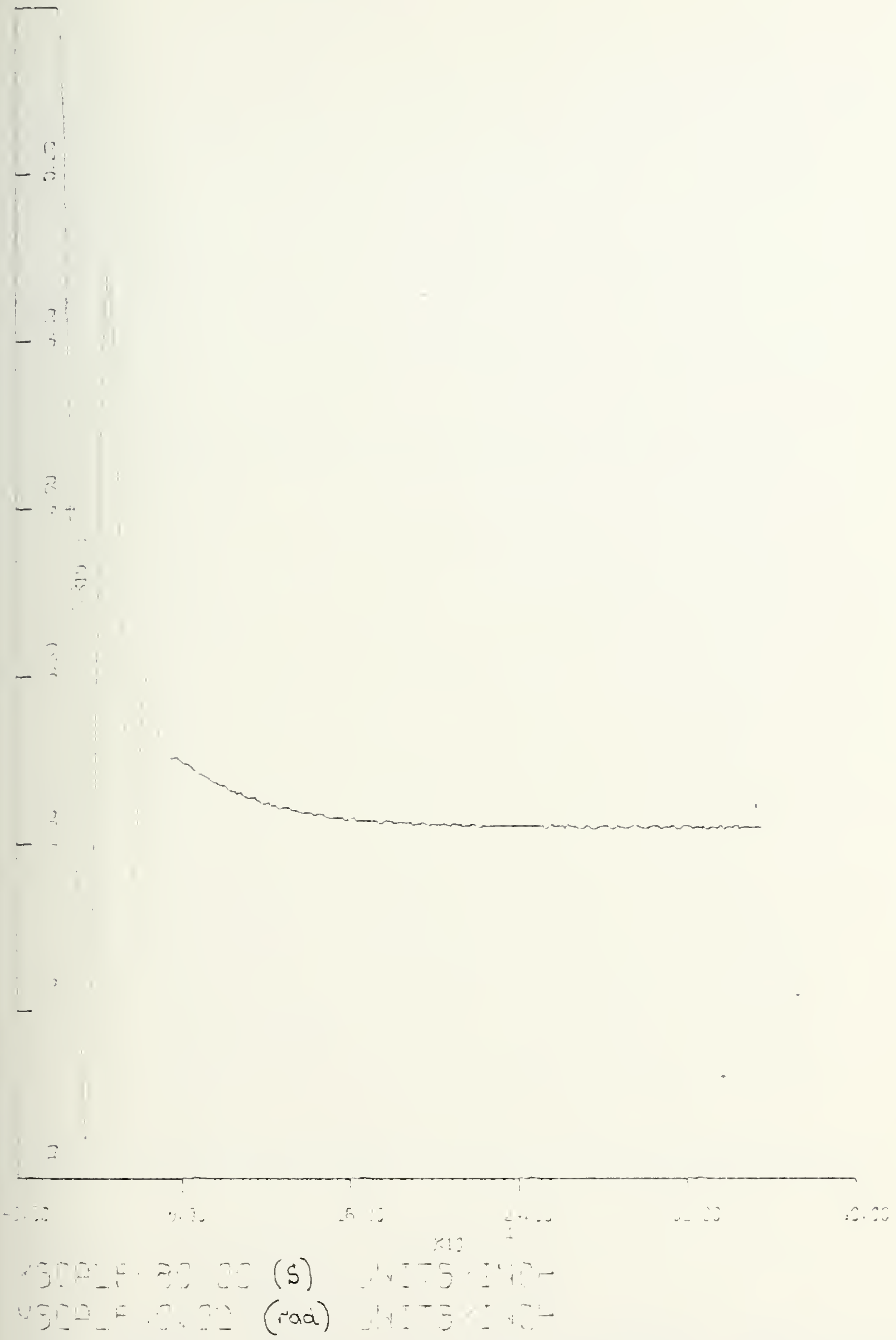


Fig. III-8b. Pitch vs. Time. Submarine not "in trim." Response to depth and pitch orders. Tracking controller without error limiters and noise filters.

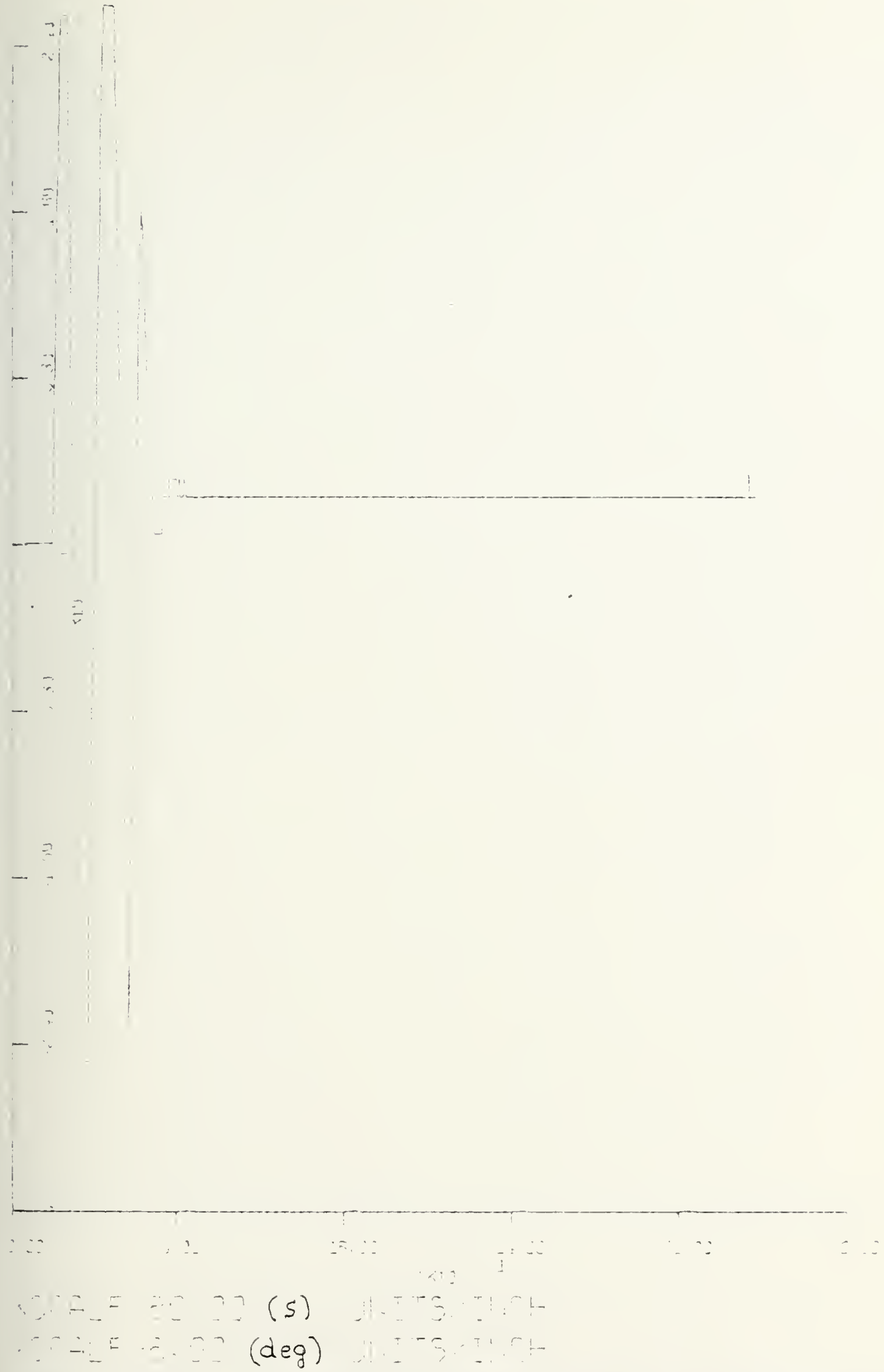


Fig. III-8c. Stern Plane Angle vs. Time. Submarine not "in trim." Response to depth and pitch orders. Tracking controller without error limiters and noise filters.

Fig. III-8d. Fairwater Plane Angle vs. Time. Submarine not "in trim." Response to depth and pitch orders. Tracking controller without error limiters and noise filters.

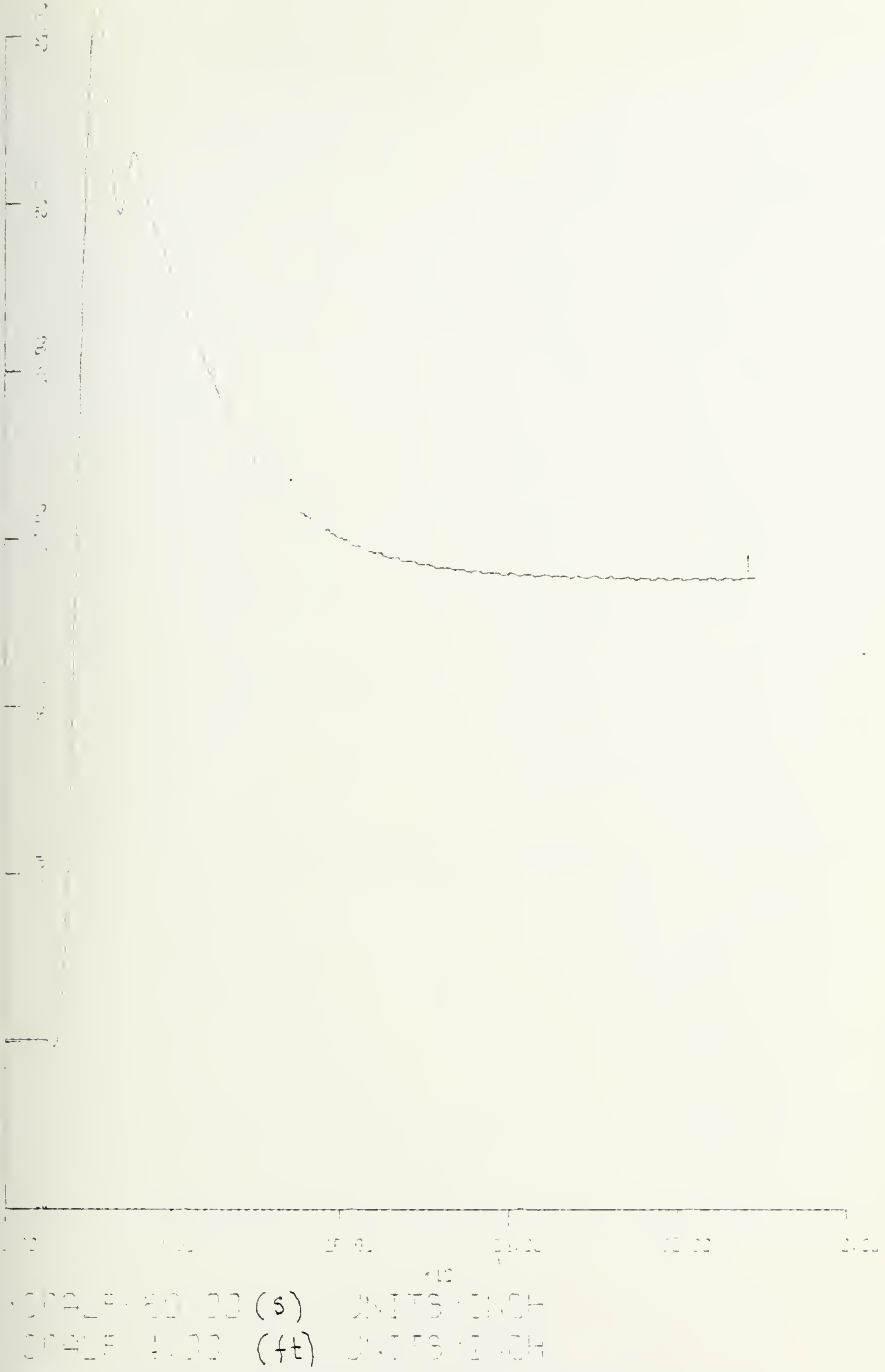


Fig. III-9a. Depth vs. Time. Submarine not "in trim." Response to depth and pitch orders. Sinusoidal input (AU). Tracking controller without error limiters and plane noise filters.

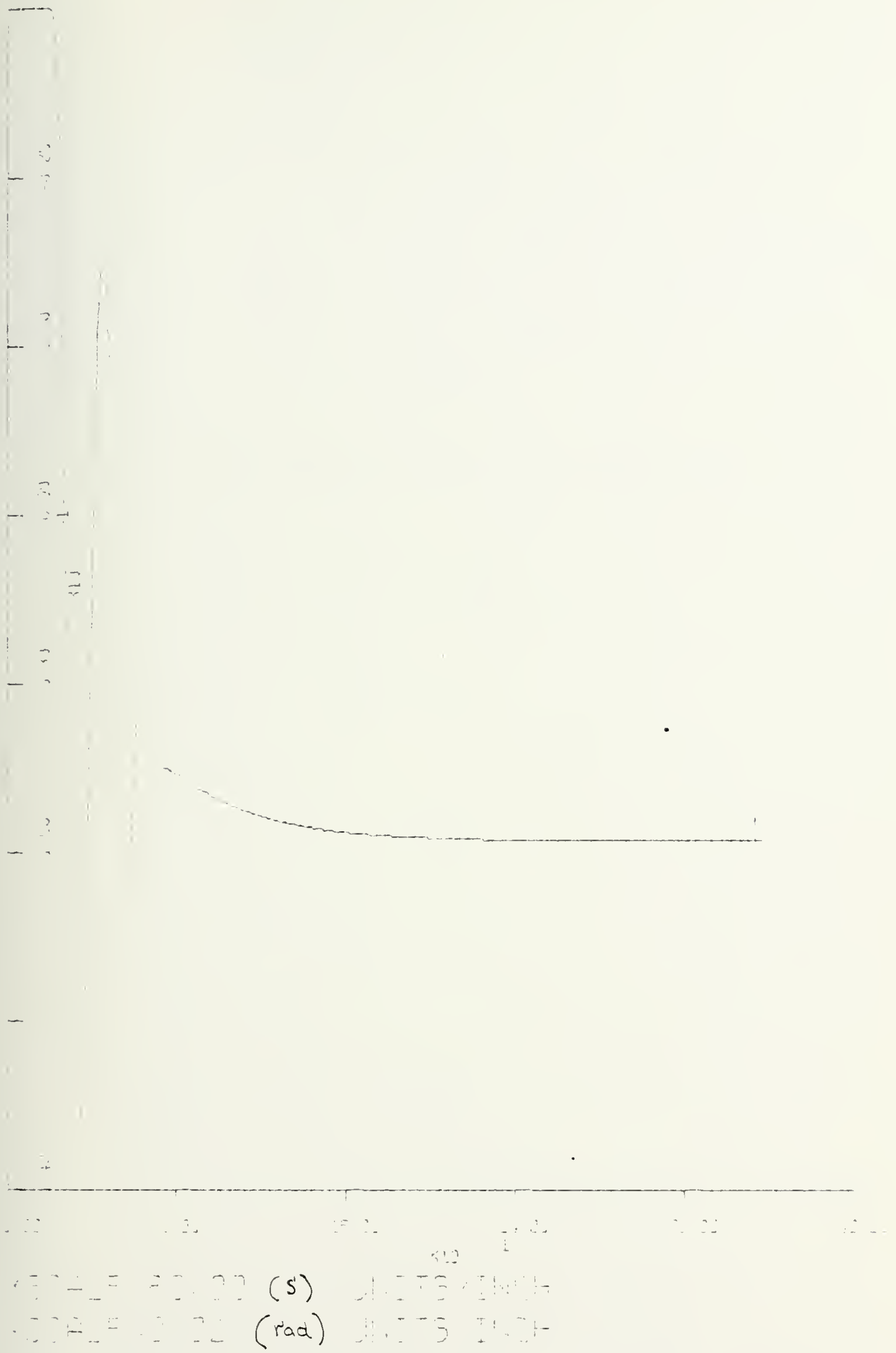


Fig. III-9b. Pitch vs. Time. Submarine not "in trim." Response to depth and pitch orders. Sinusoidal input(AU). Tracking controller without error limiters and plane noise filters.

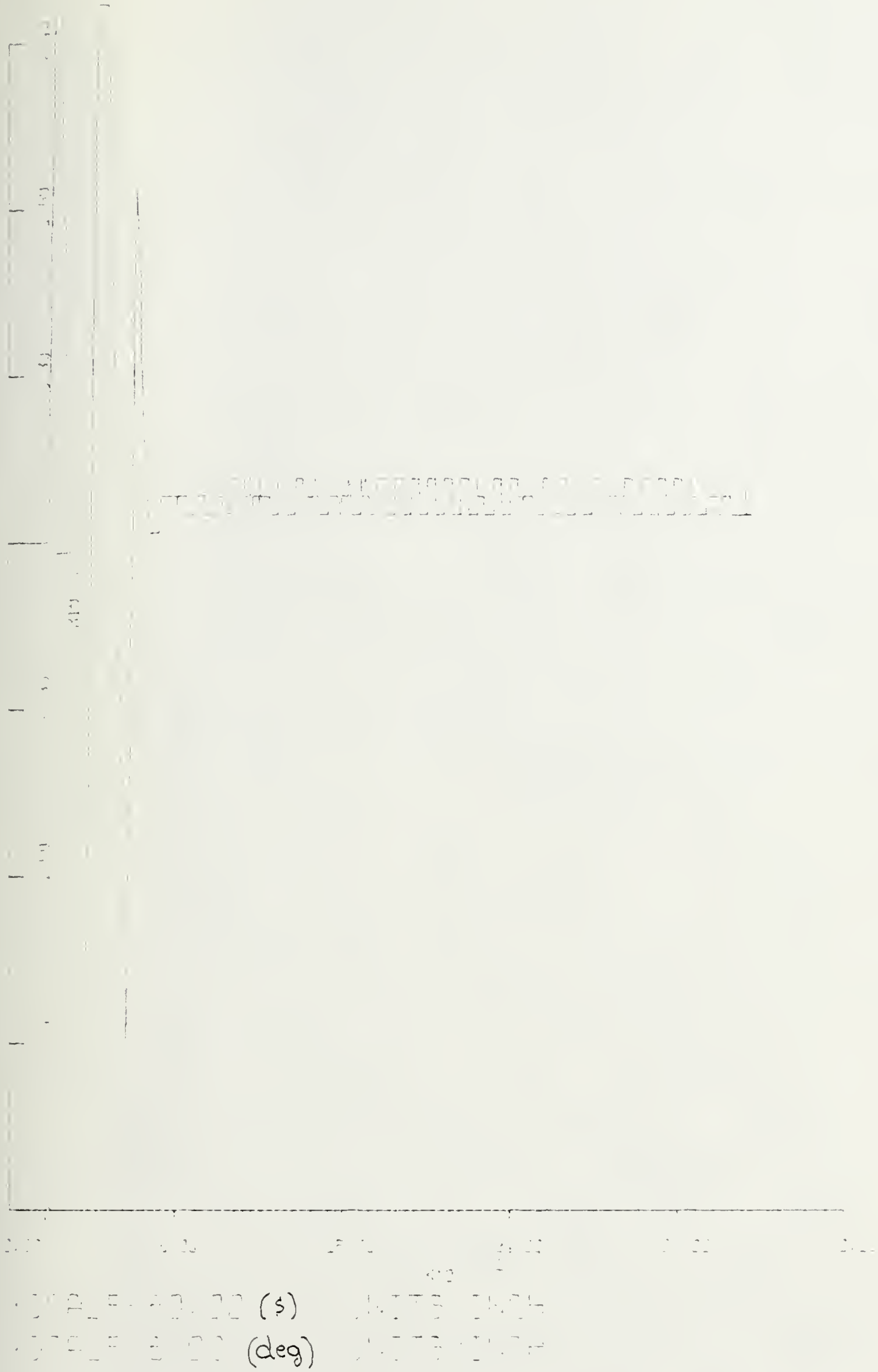


Fig. III-9c. Stern Plane Angle vs. Time. Submarine not "in trim." Response to depth and pitch orders. Sinusoidal input (AU). Tracking controller without error limiters and plane noise filters.

Fig. III-9d. Fairwater Plane Angle vs. Time. Submarine not "in trim." Response to depth and pitch orders. Sinusoidal input (AU). Tracking controller without error limiters and plane noise filters.

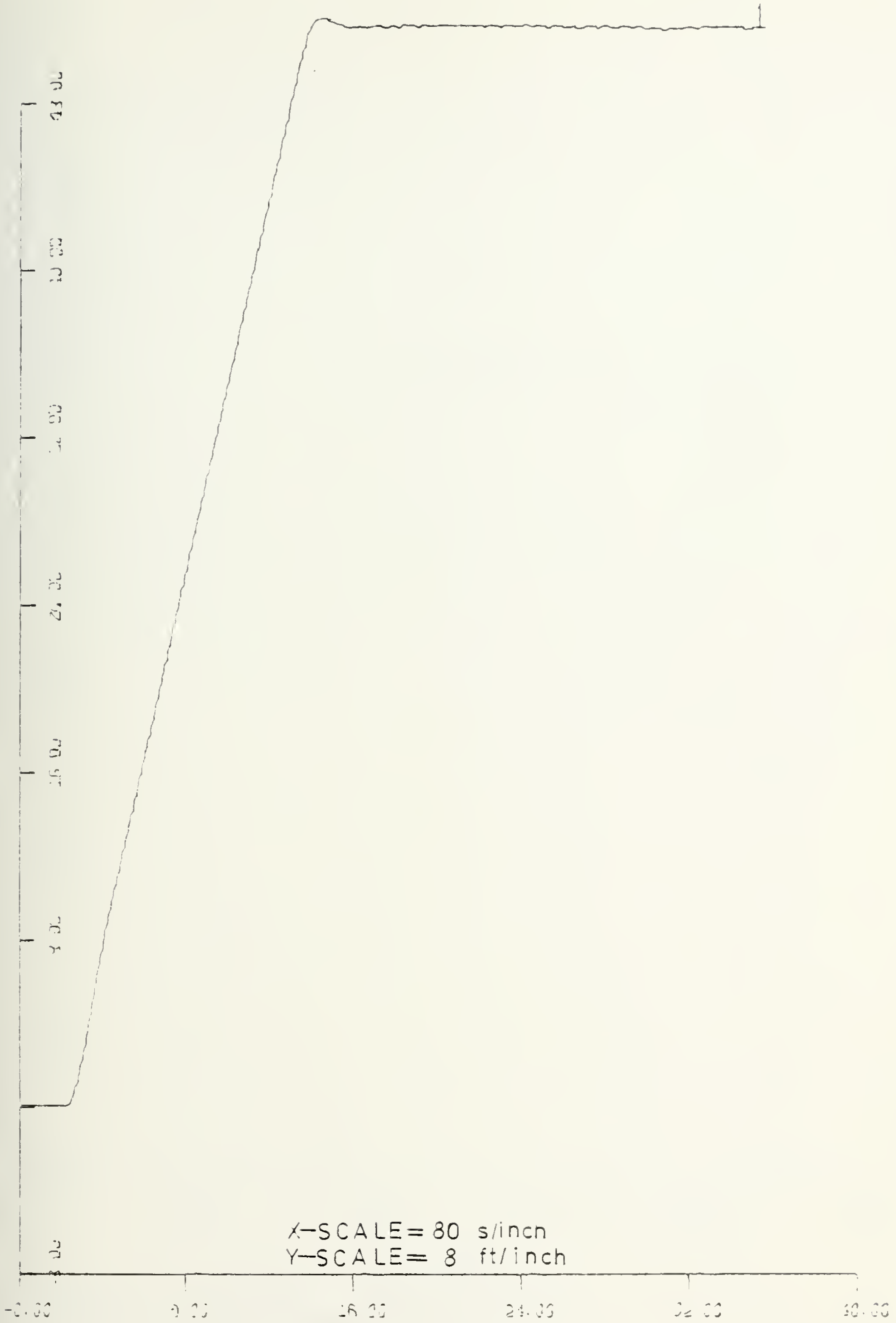


Fig. III-10a. Depth vs. Time. Submarine "in trim." Response to a depth order of 50 ft. Tracking controller (final) without error limiters and noise filters.

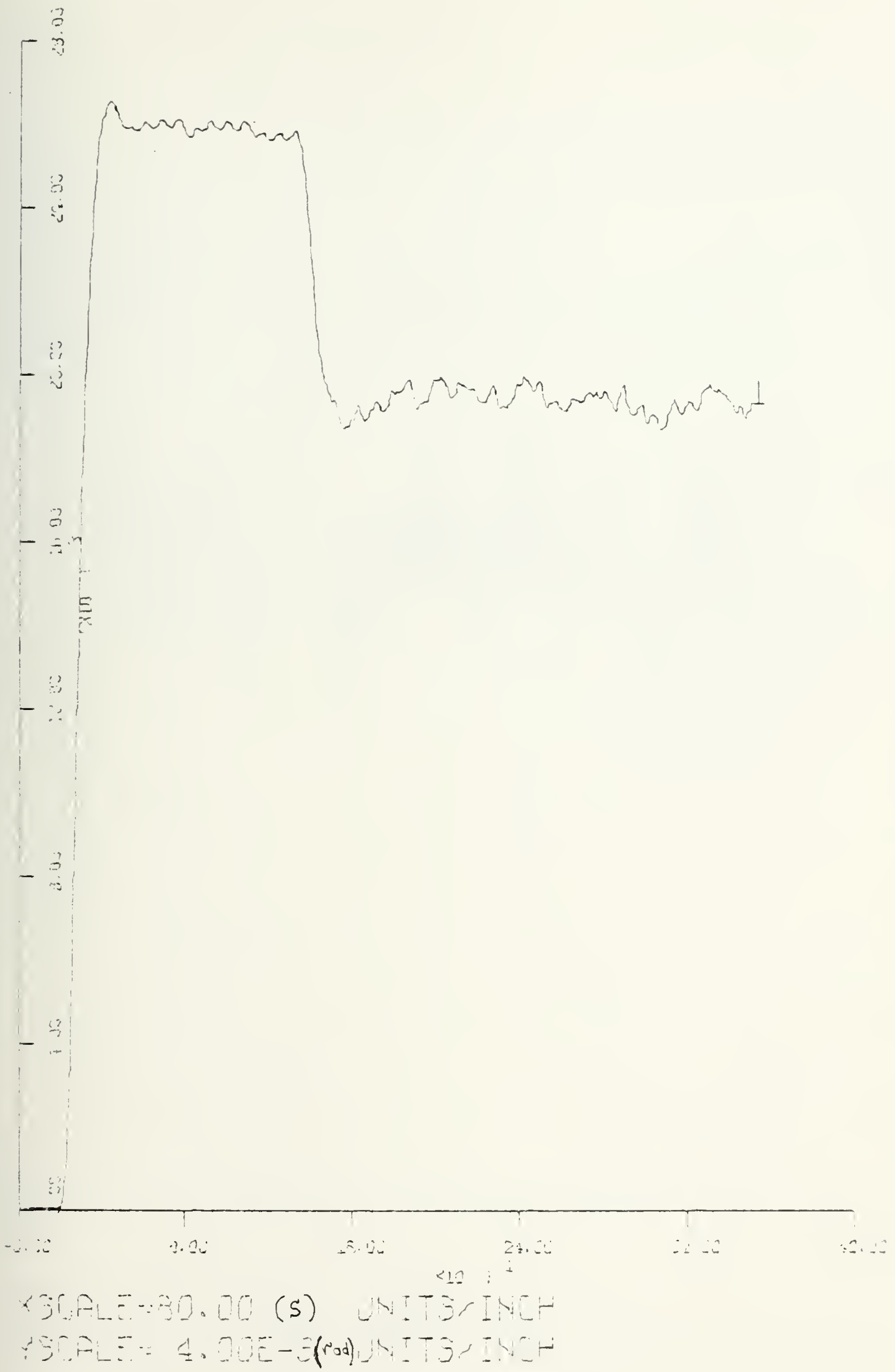


Fig. III-10b. Pitch vs. Time. Submarine "in trim." Response to a depth order of 50 ft. Tracking controller (final) without error limiters and noise filters.

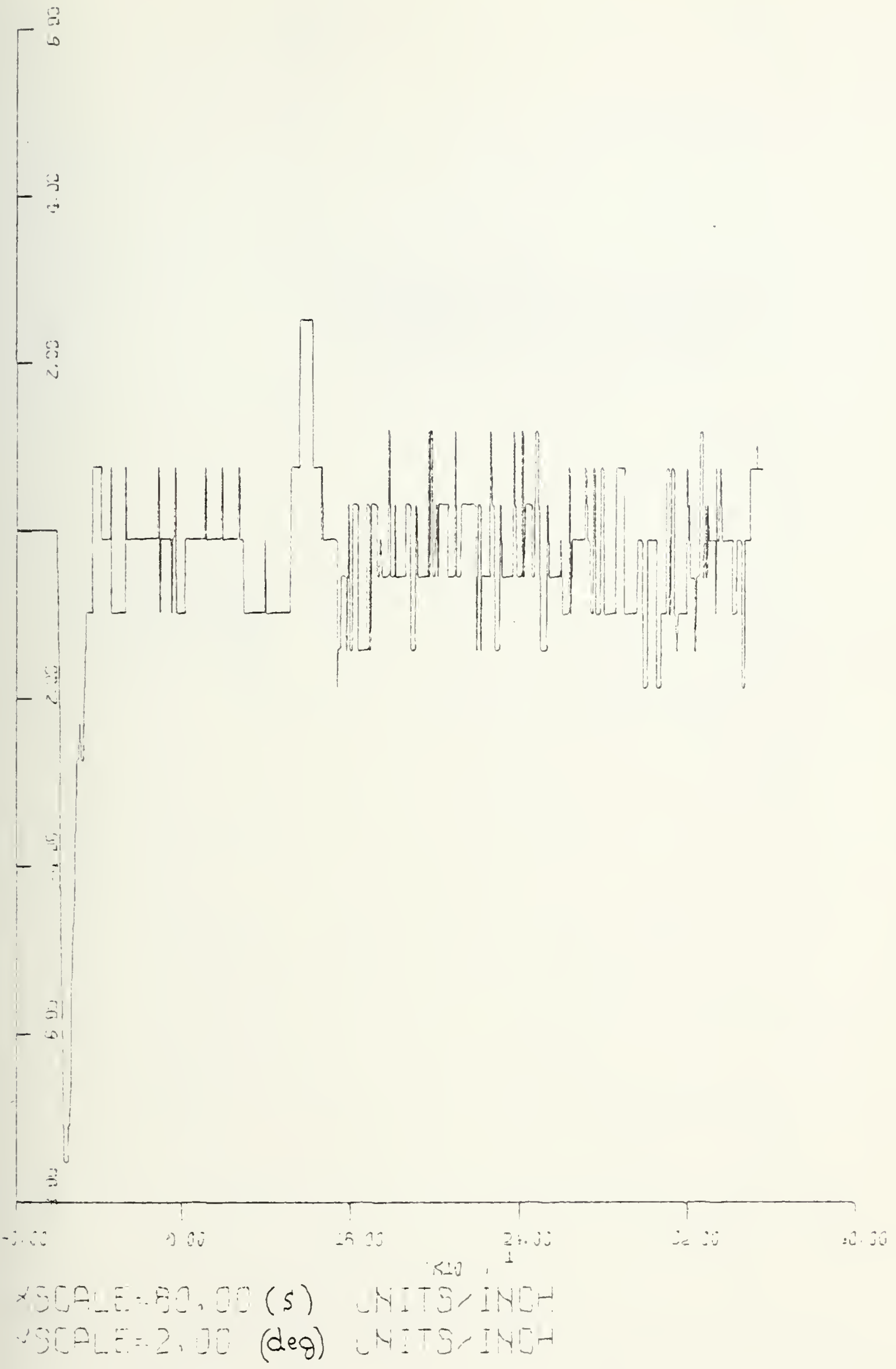


Fig. III-10c. Stern Plane Angle vs. Time. Submarine "in trim." Response to a depth order of 50 ft. Tracking controller (final) without error limiters and noise filters.

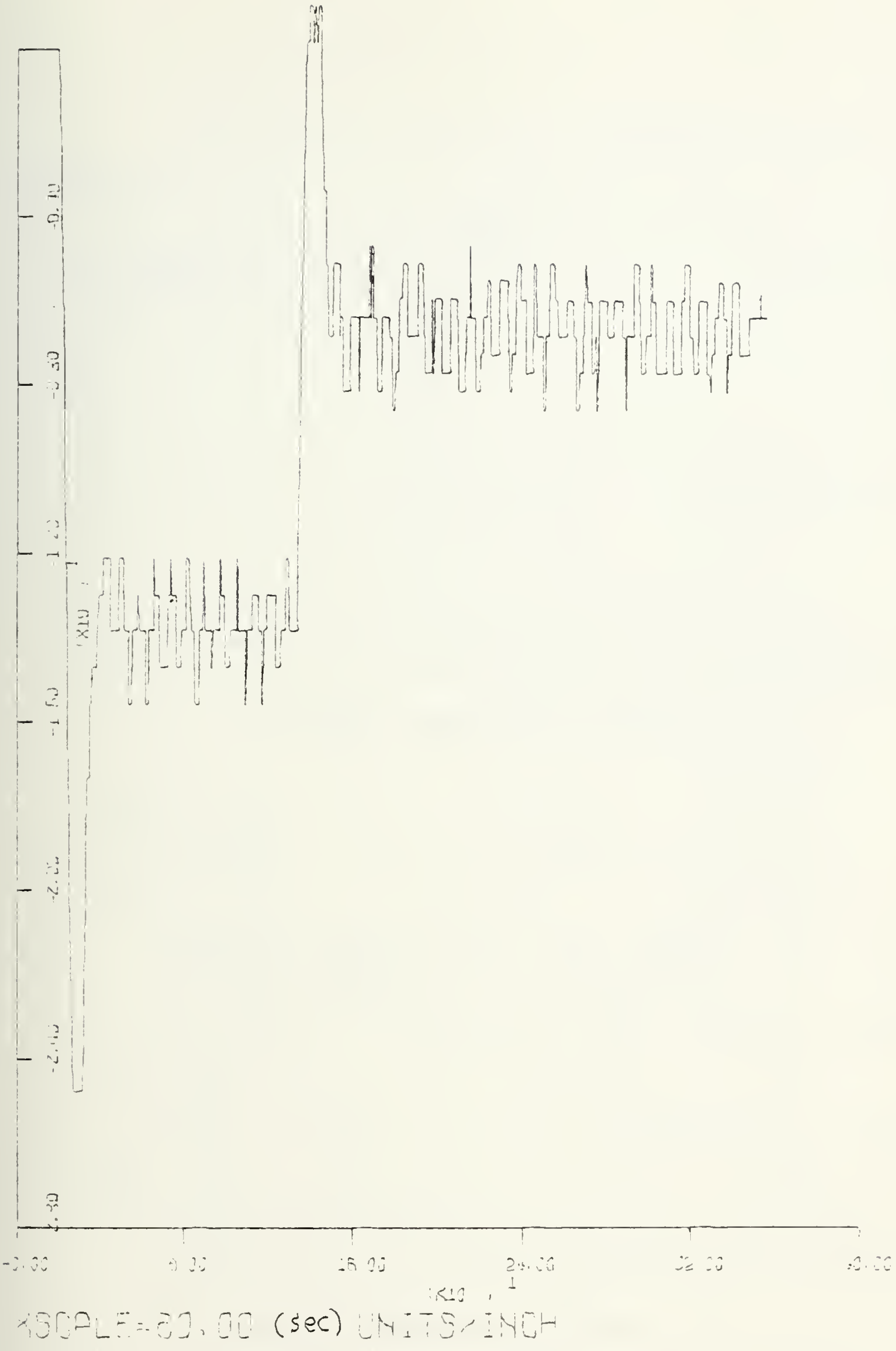
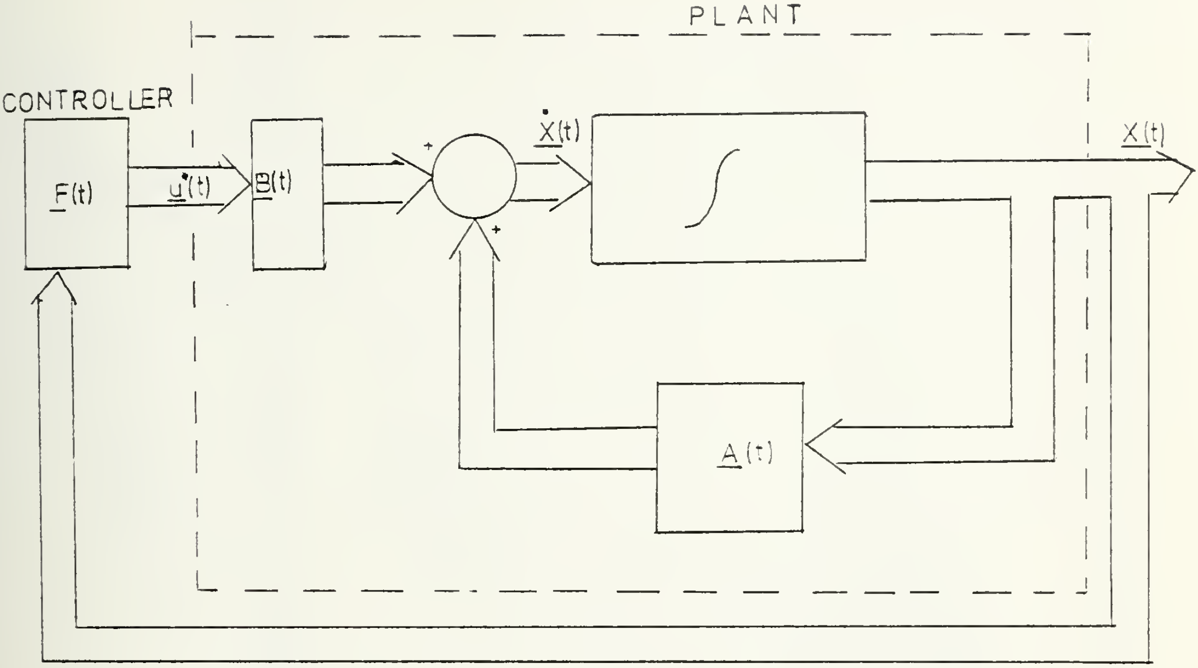
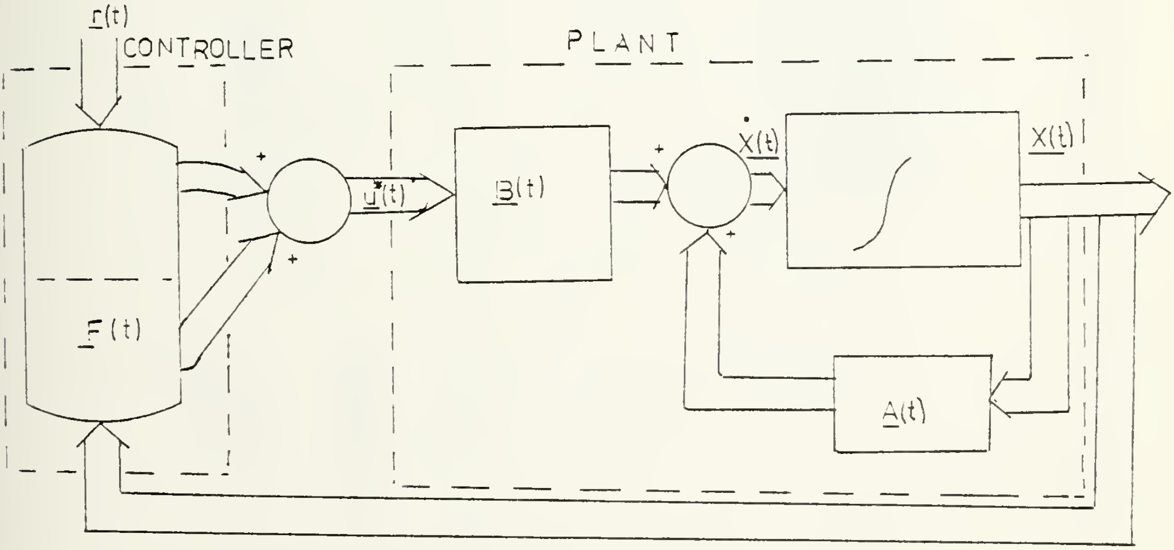


Fig. III-10d. Fairwater Plane Angle vs. Time. Response to a depth order of 50 ft. Tracking controller (final) without error limiters and noise filters.



(a) Linear regulator problem



(b)Linear tracking problem

Fig. III-11. Block diagrams for optimal feedback controllers.

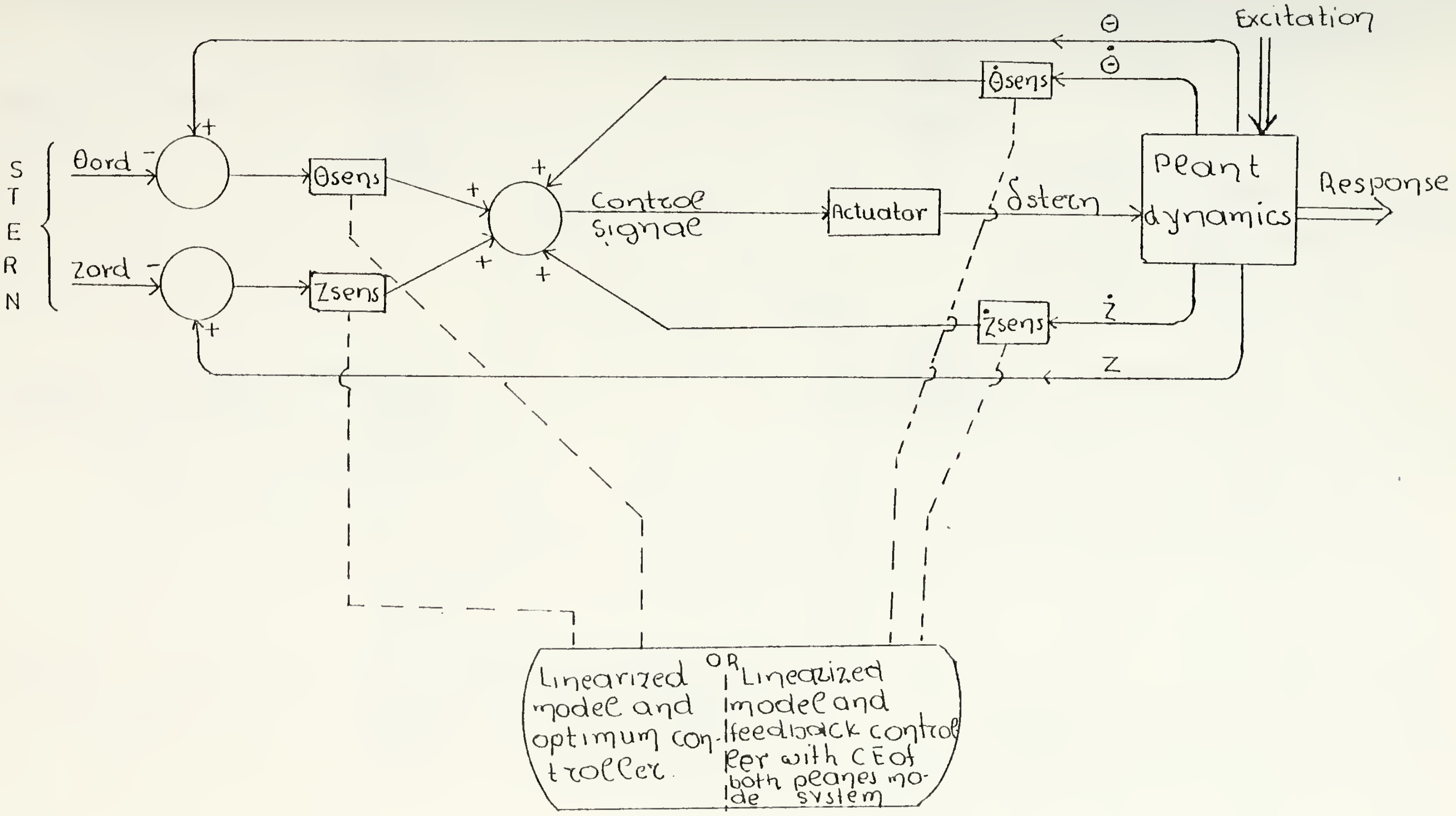

S
T
E
R
N
θord
Zord
θsens
Zsens
Control signal
Actuator
δstern
Plant dynamics
Excitation
Response
θ
θ̇
θ̇sens
ż
żsens
Z
Linearized model and optimum controller.
OR
Linearized model and feedback controller with CE of both planes mode system

Fig. IV-1. Block diagram for stern planes only controller

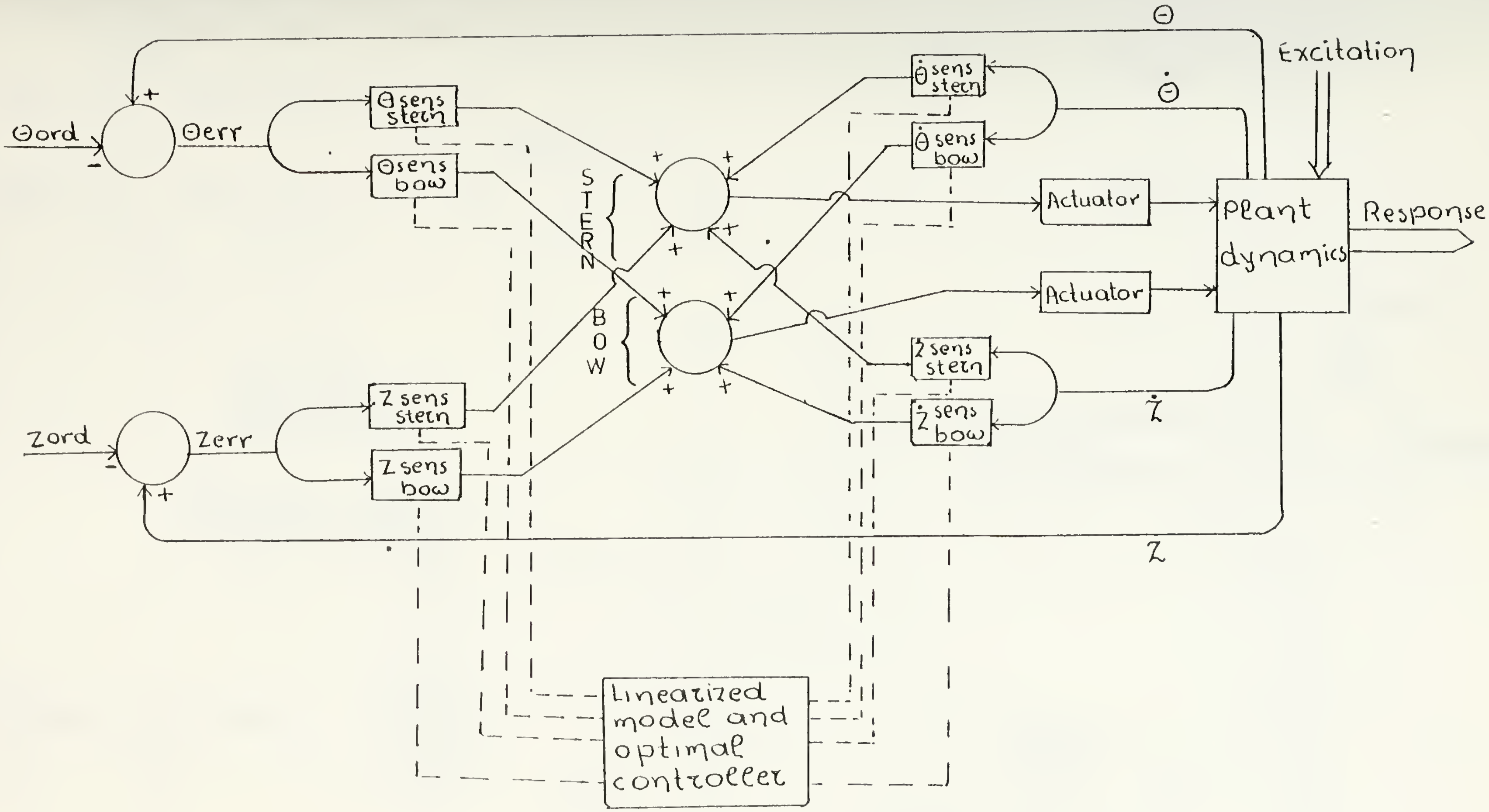


Fig. IV-2. Block diagram for both planes controller

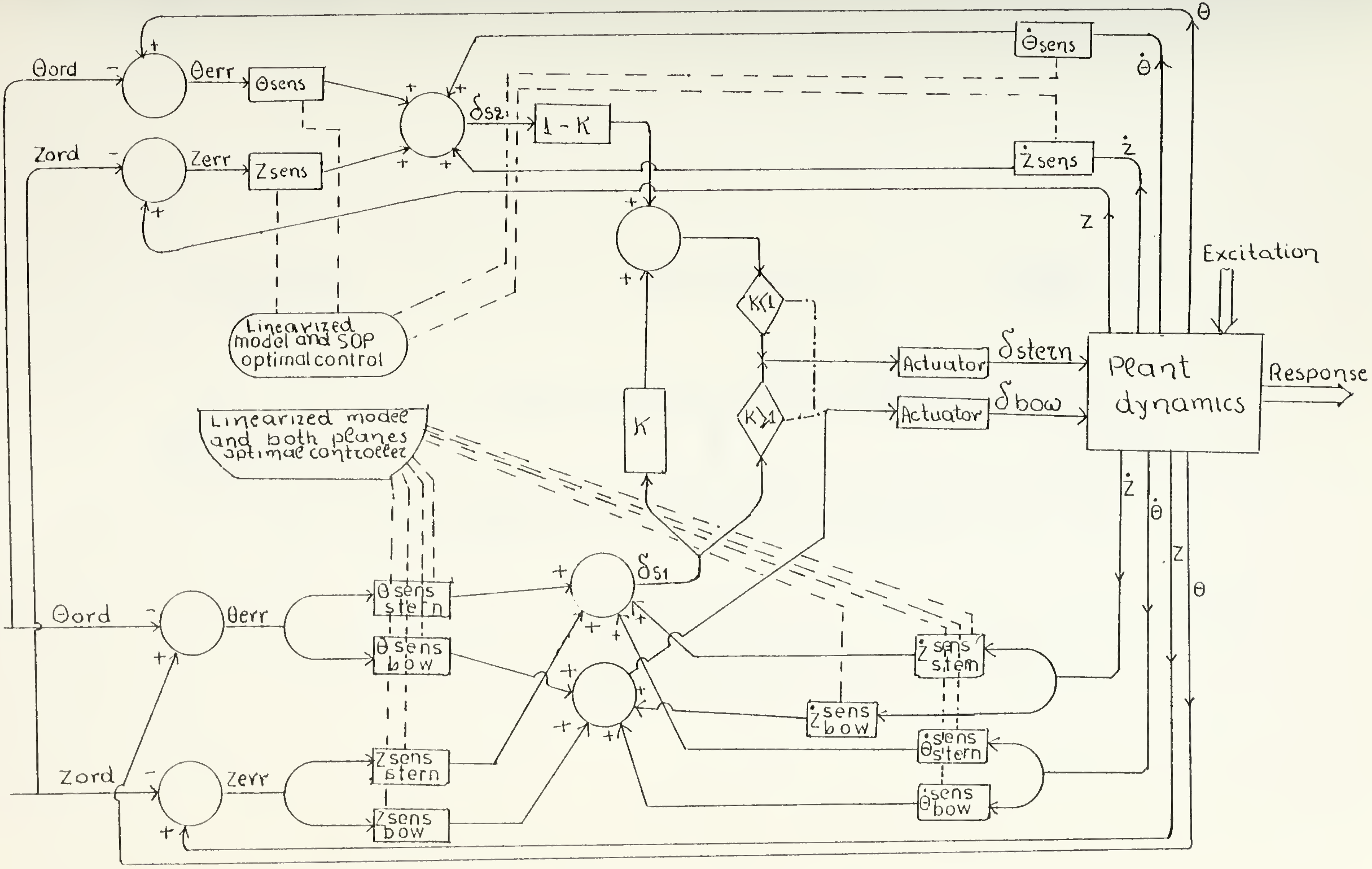


Fig. IV-3. Block diagram for the combined controller

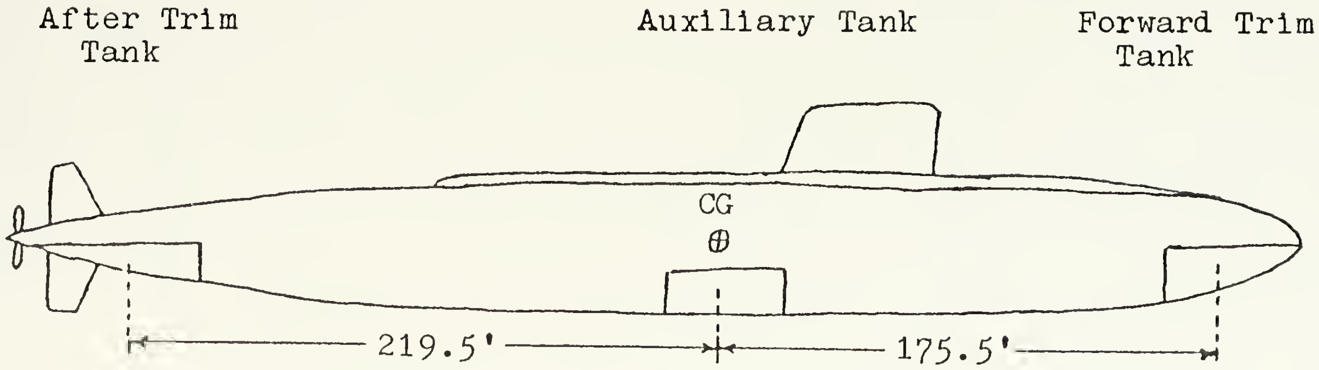


Figure IV-4

Tank Locations

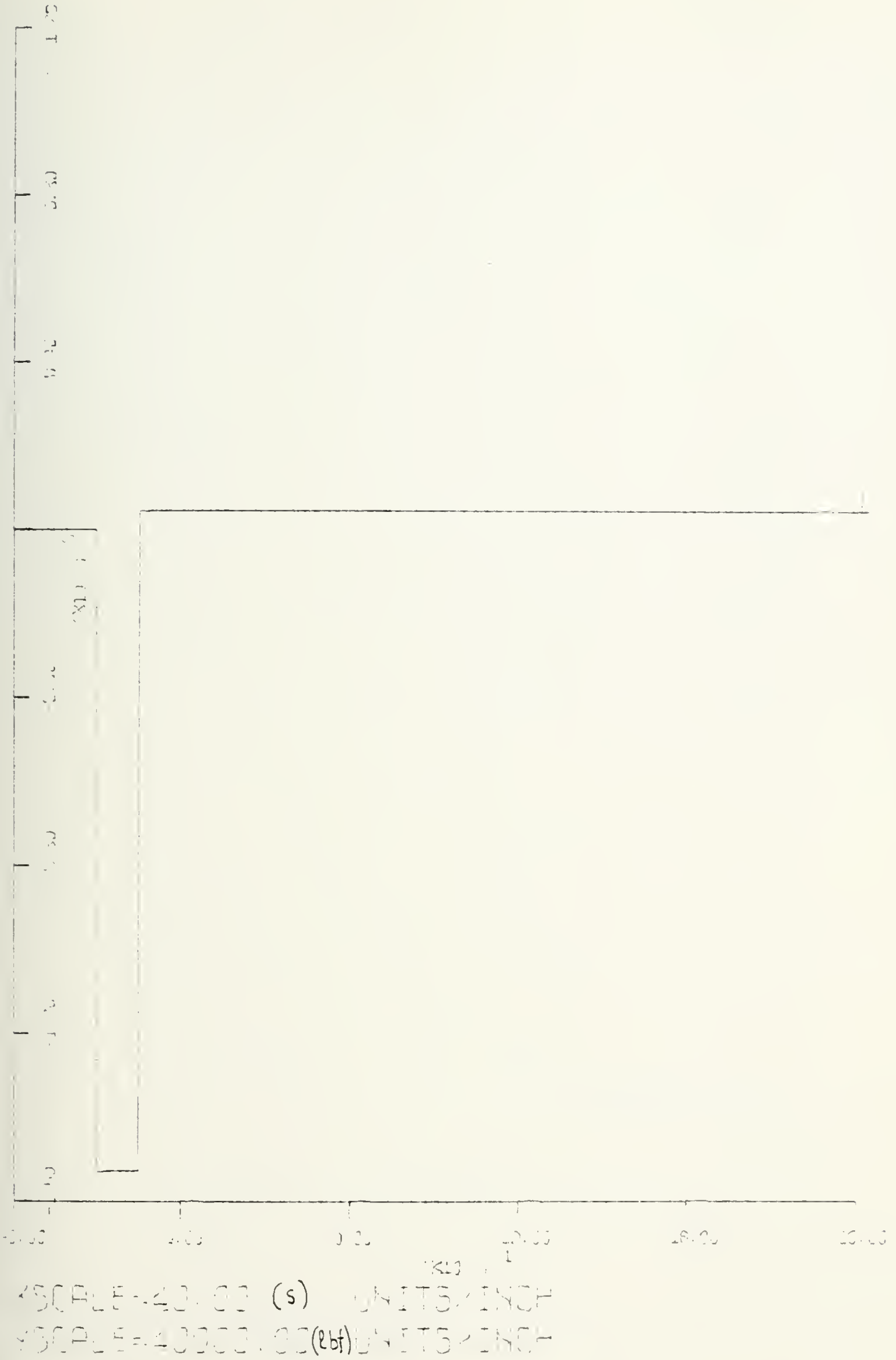


Fig. IV-5a. Pulse force at FT

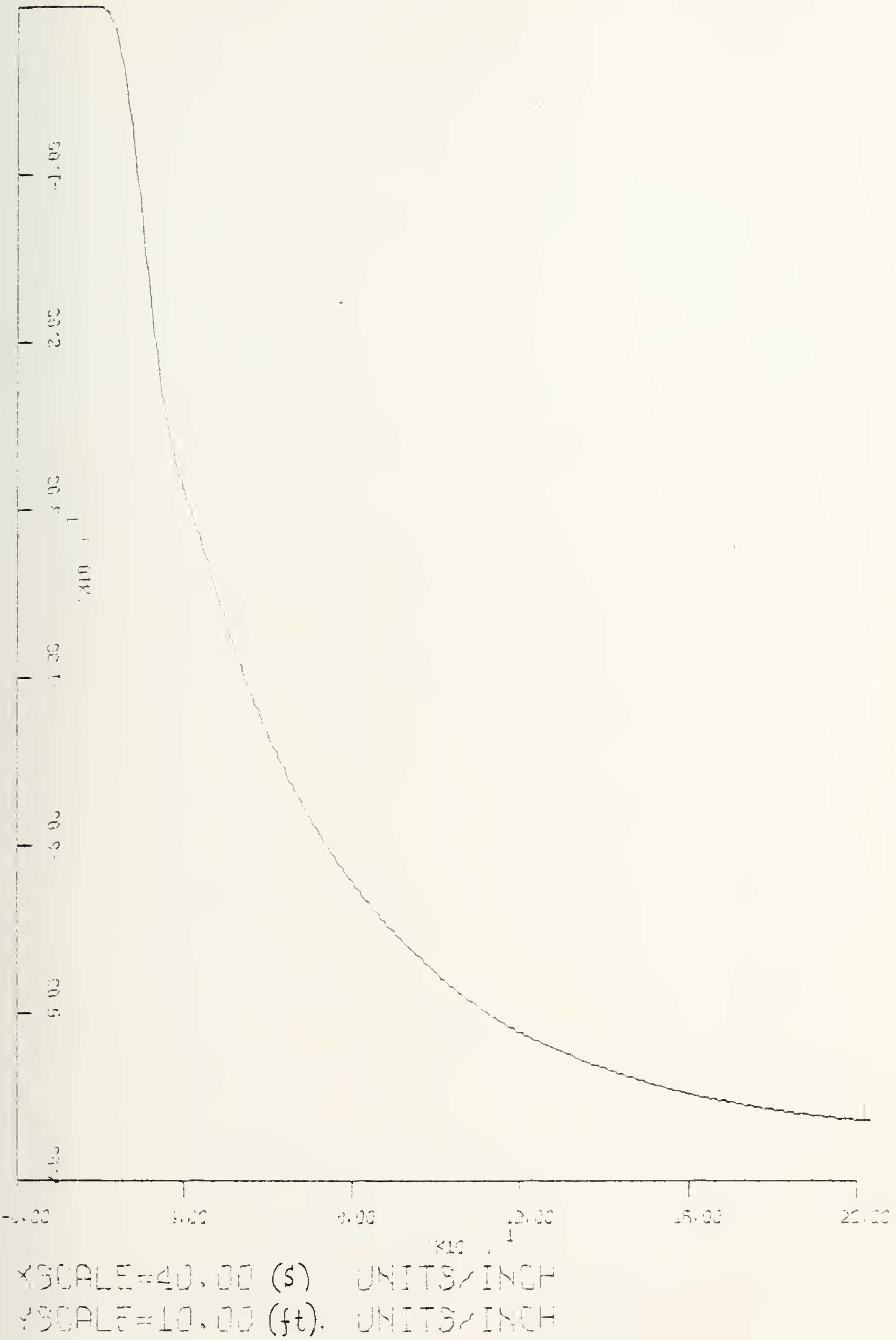


Fig IV-5b. Depth vs. Time. Response to a pulse force at FT. Bounded controller without error limiters. (B=800, C-10, E=1)

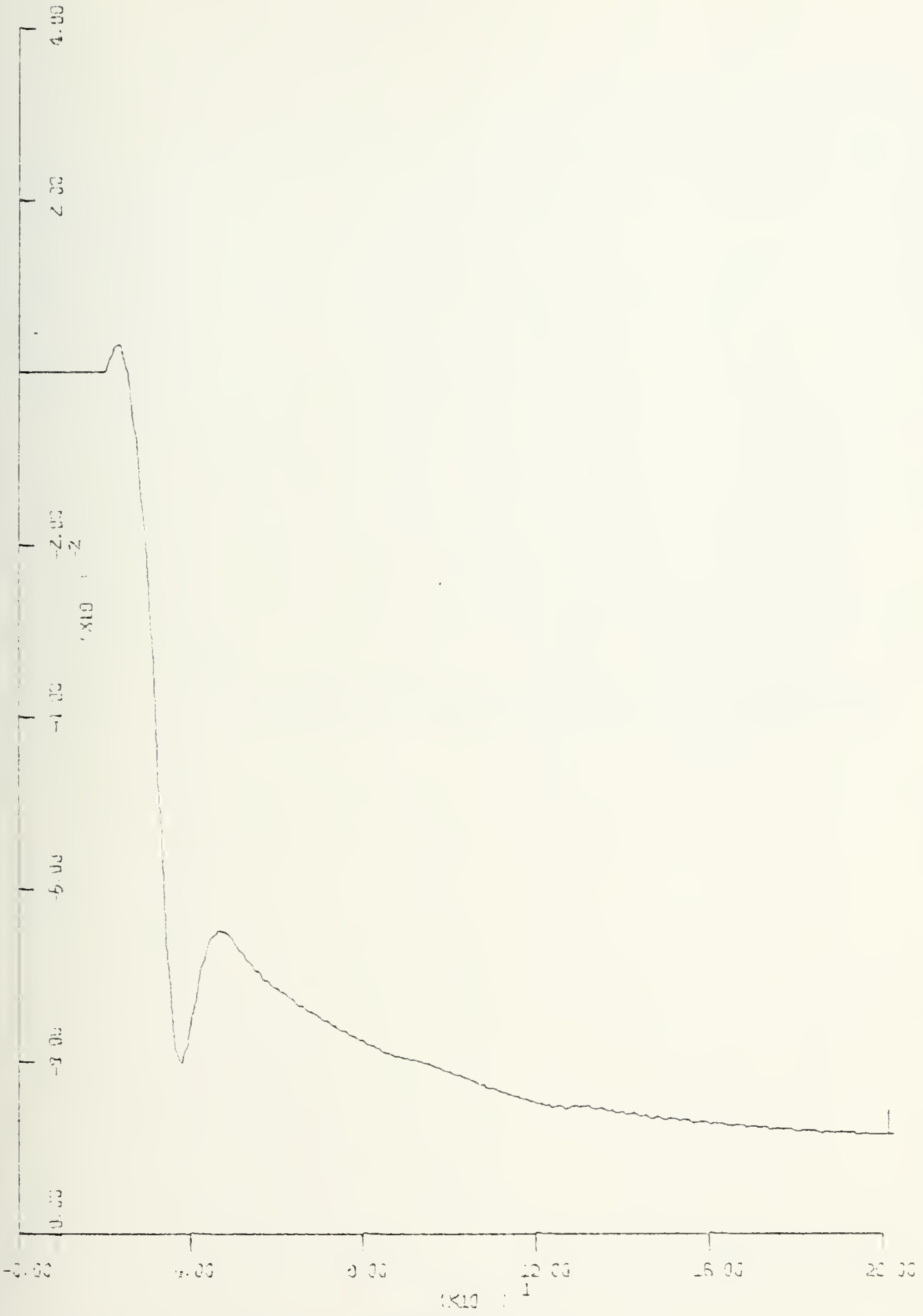


Fig. IV-5c. Pitch vs. Time. Response to a pulse force at FT. Bounded controller without error limiters. (B=800, C=10, E=1)

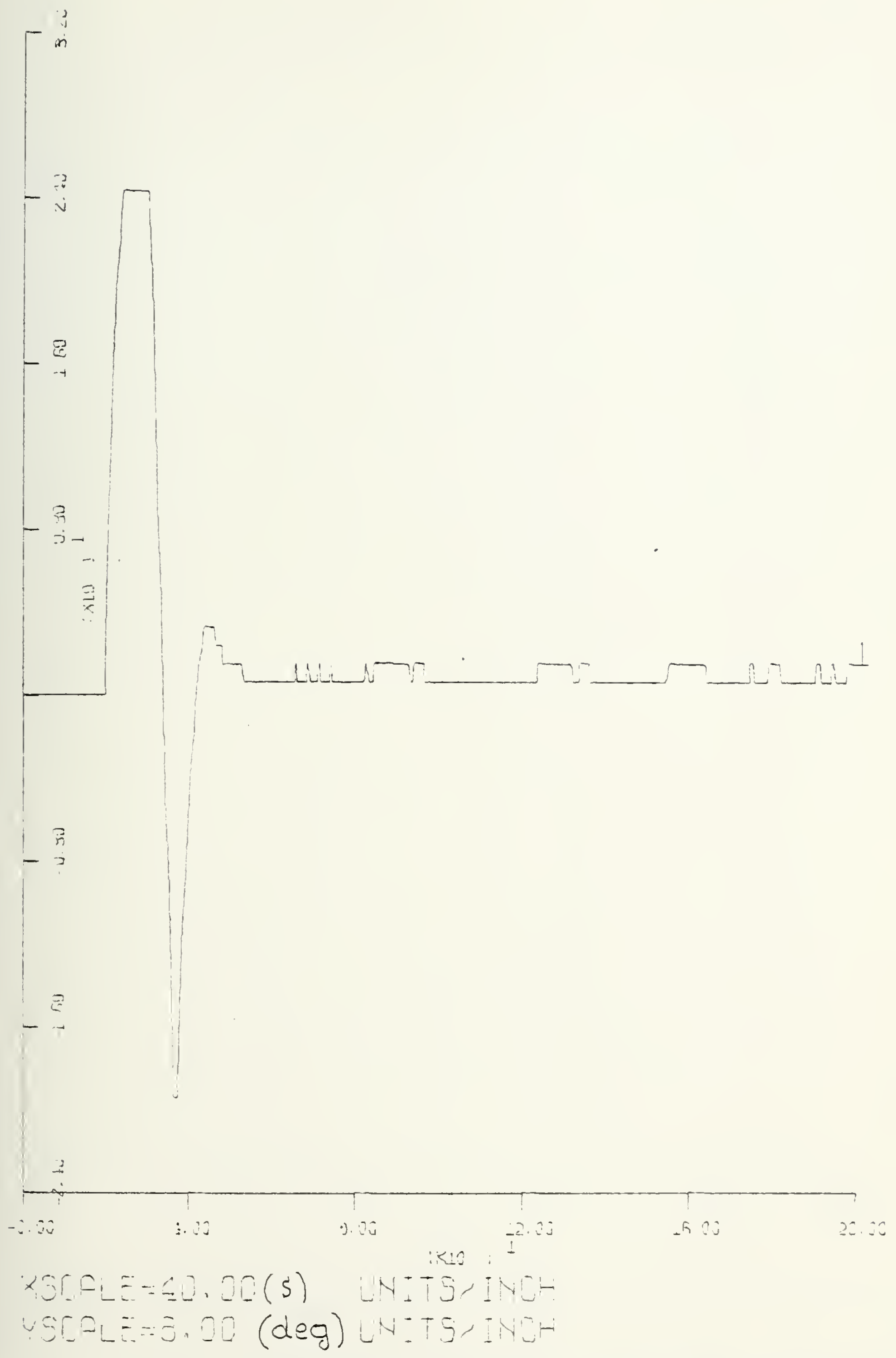


Fig. IV-5d. Stern Plane Angle vs. Time. Response to a pulse force at FT. Bounded controller without limiters. (B=800, C=10, E=1)

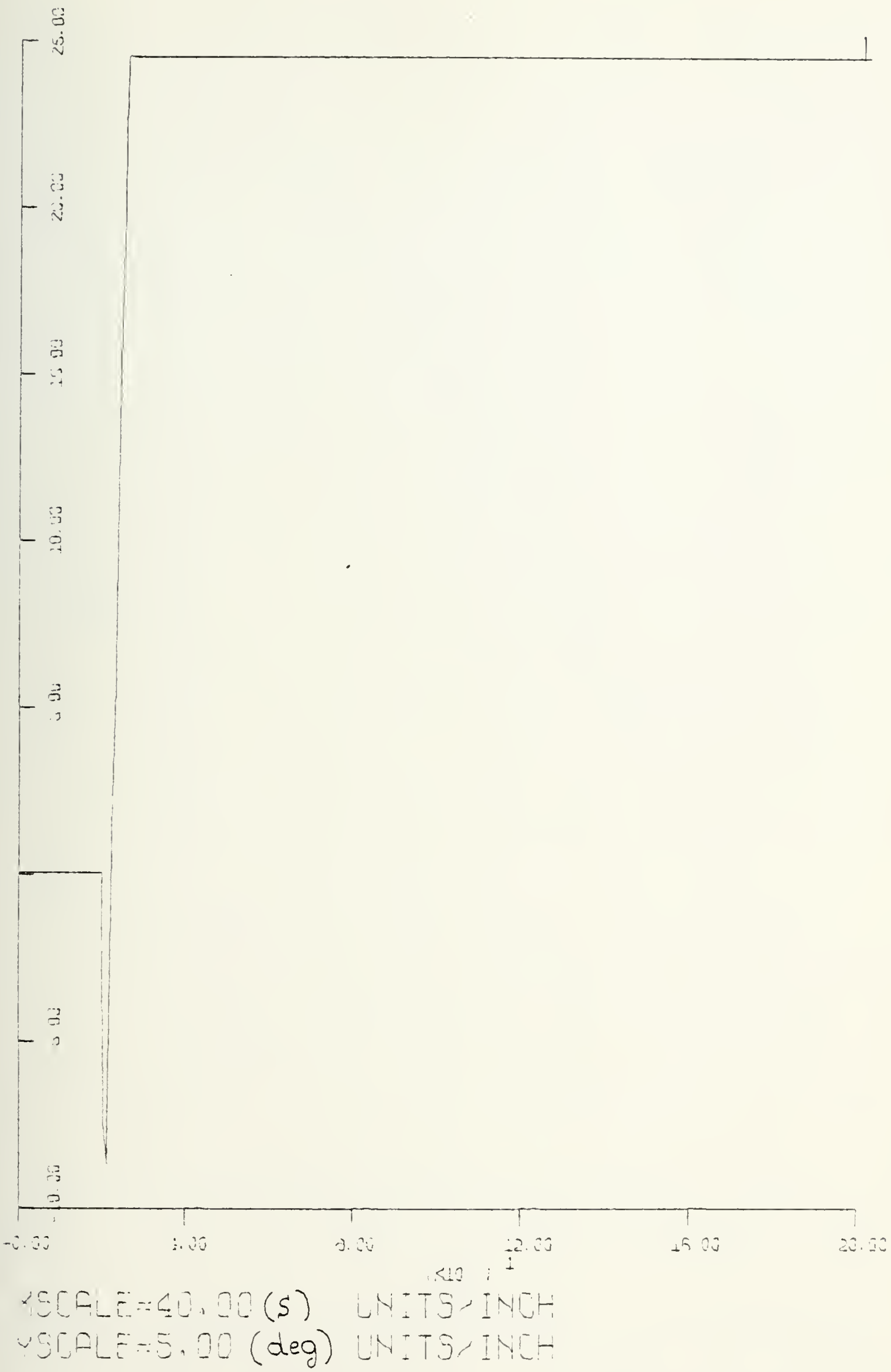


Fig. IV-5e. Fairwater Plane Angle vs. Time. Response to a pulse force of FT. Bounded controller without error limiters. (B=800, C=10, E=1)

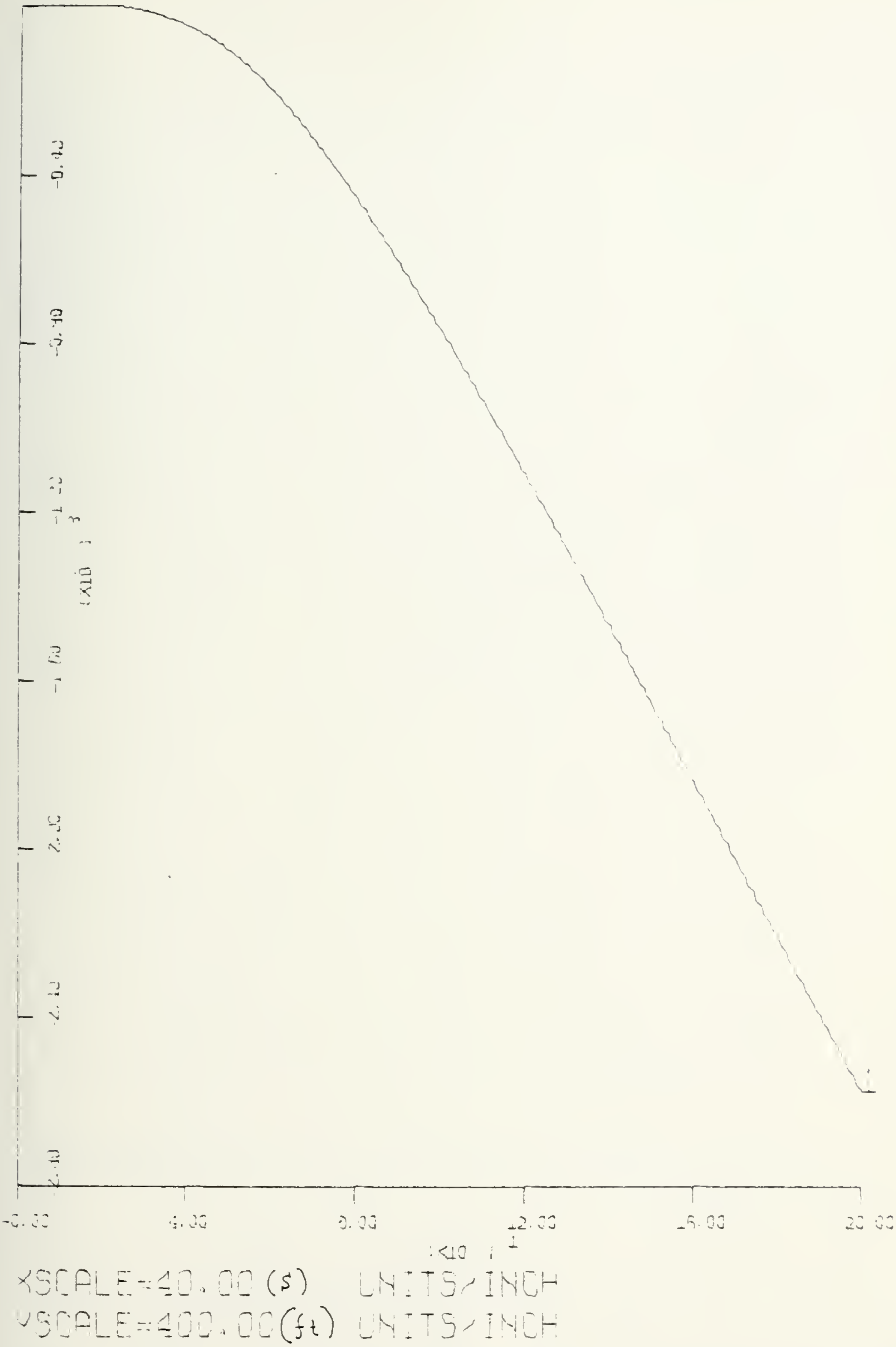
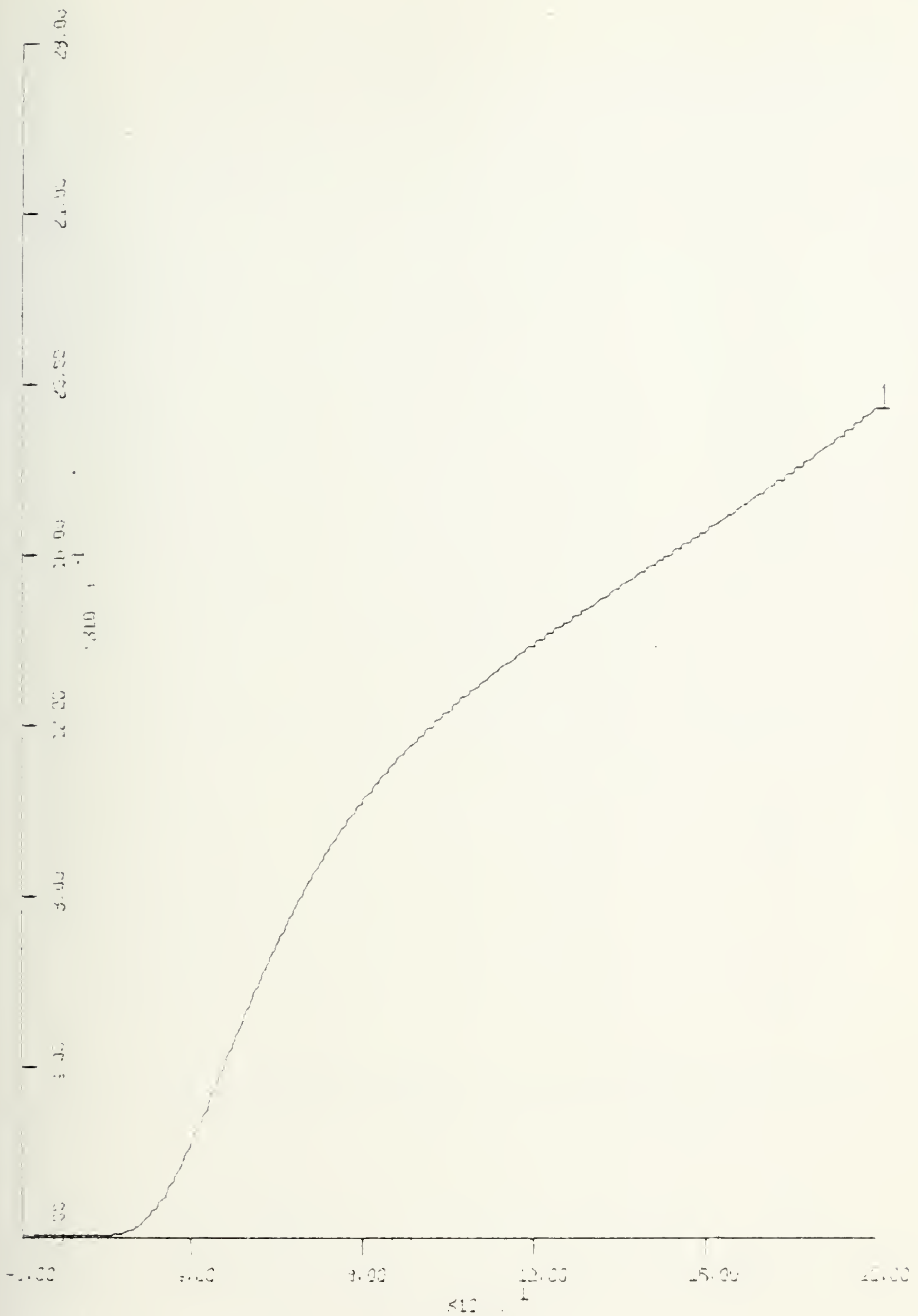


Fig. IV-5f. Depth vs. Time. Response to a pulse force at FT. Bounded controller without error limiters. (B=164, C=0.001, E=0.001)



XSCALE=40.00 (s) UNITS/INCH
YSCALE=0.40 (rad) UNITS/INCH

Fig. IV-5g. Pitch vs. Time. Response to a pulse force at FT. Bounded controller without error limiters. (B=164, C=0.001, E=0.001)

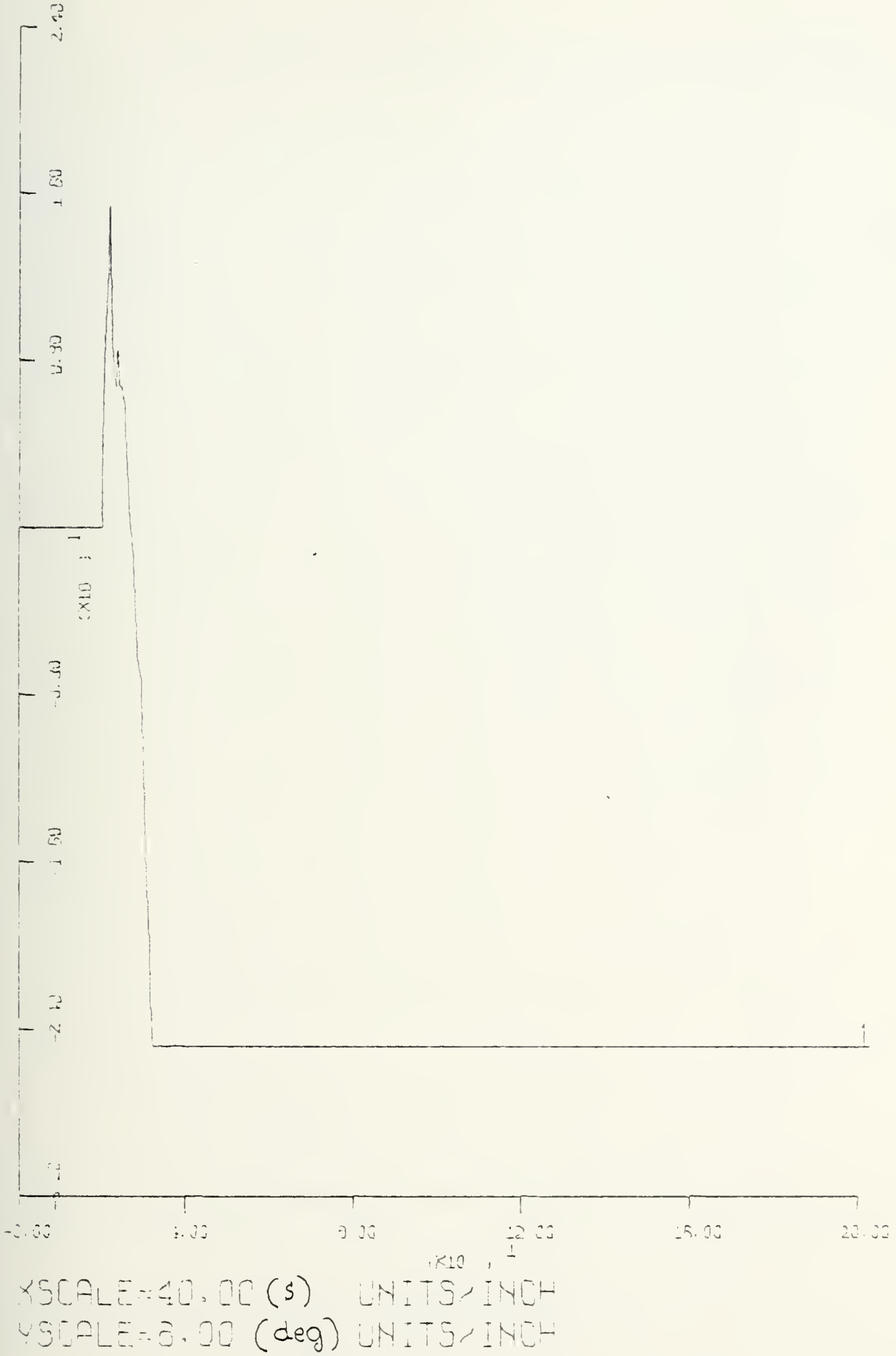


Fig. IV-5h. Stern Plane Angle vs. Time. Response to a pulse force at FT. Bounded controller without error limiters. (B=164, C=0.001, E=0.001)

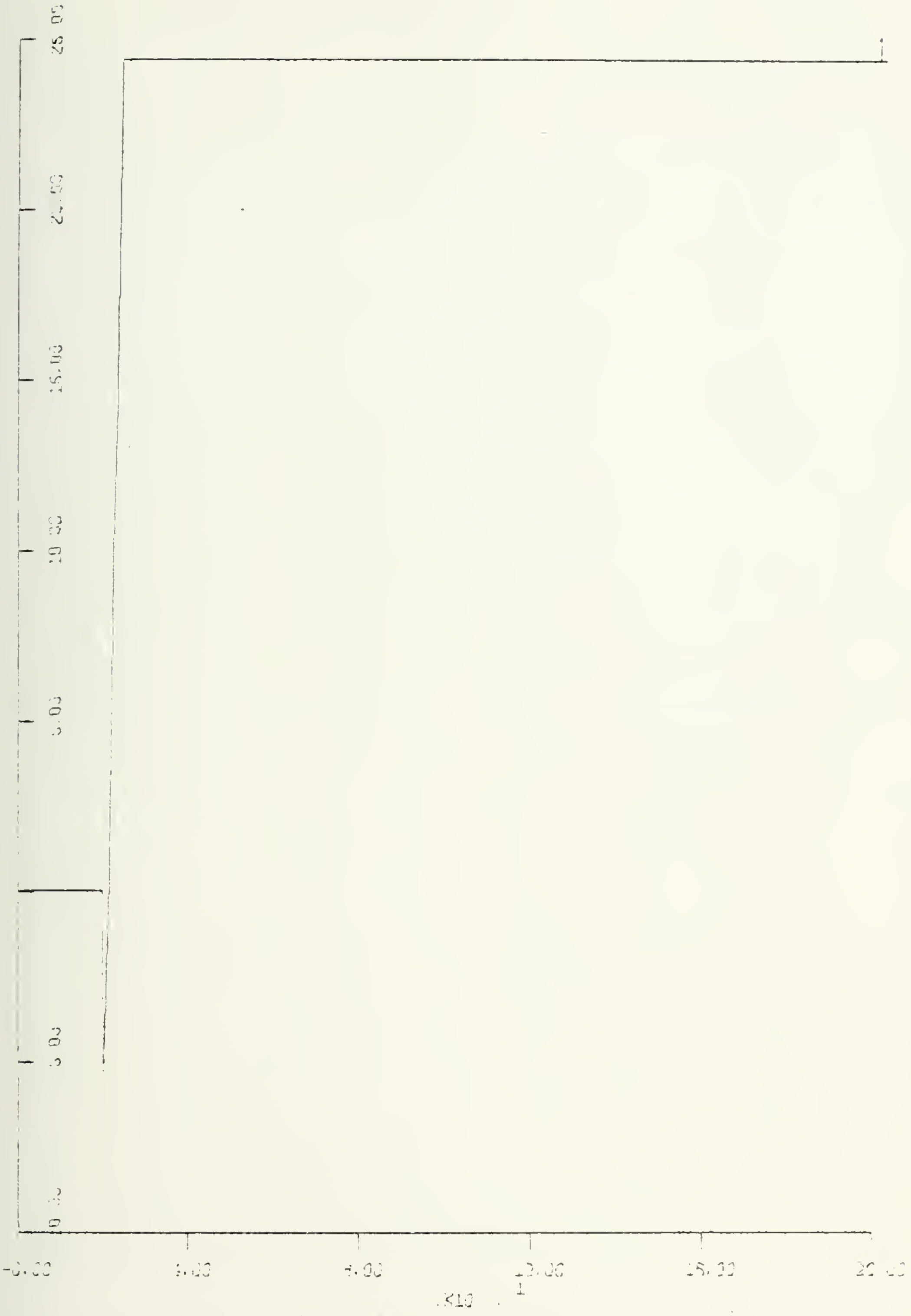


Fig. IV-5i. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. Bounded controller without error limiters. (B=164, C=0.001, E=0.001)

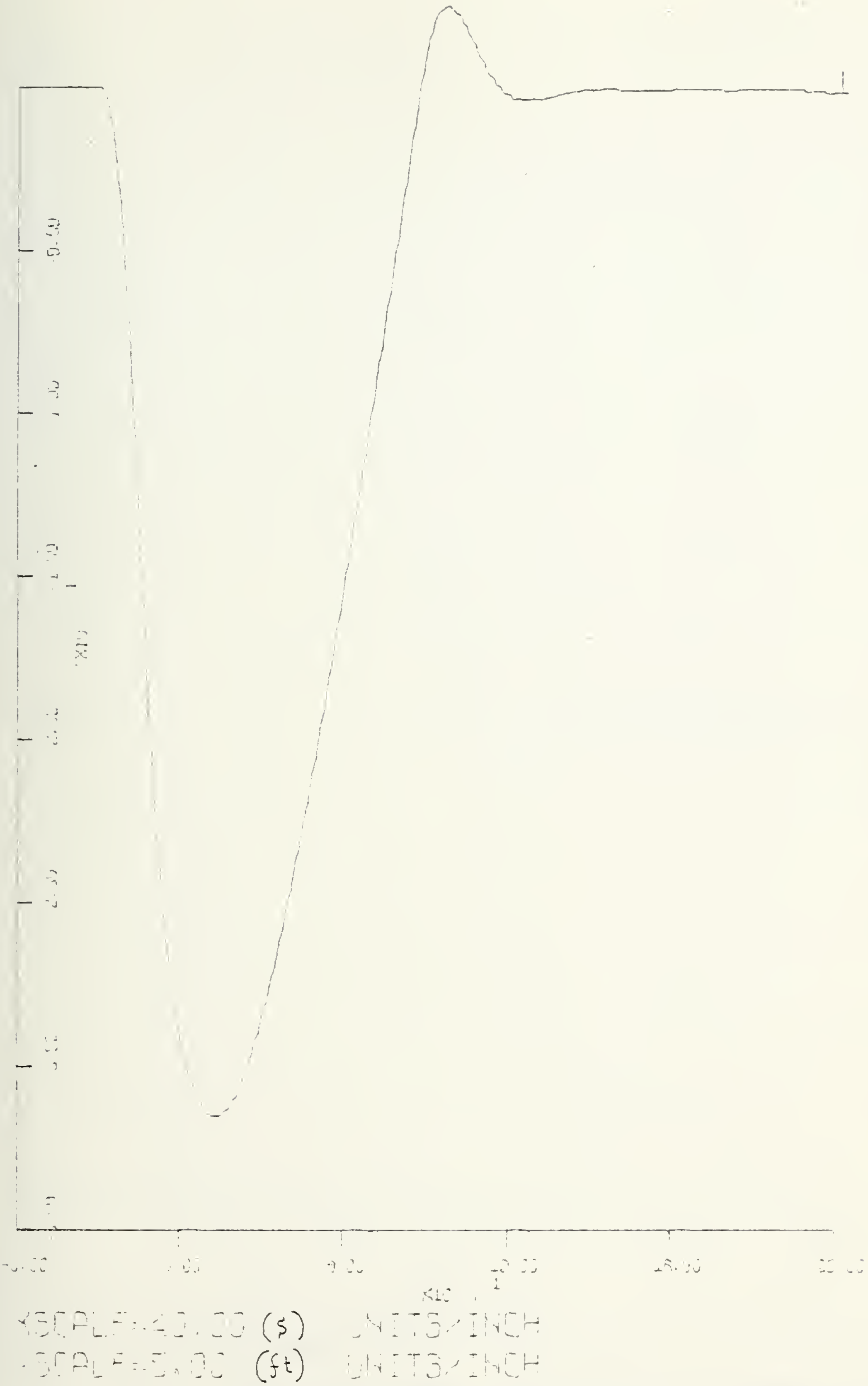


Fig. IV-6a. Depth vs. Time. Response to a pulse force at FT. Bounded controller with error limiters. (B=800, C=10, E=1)

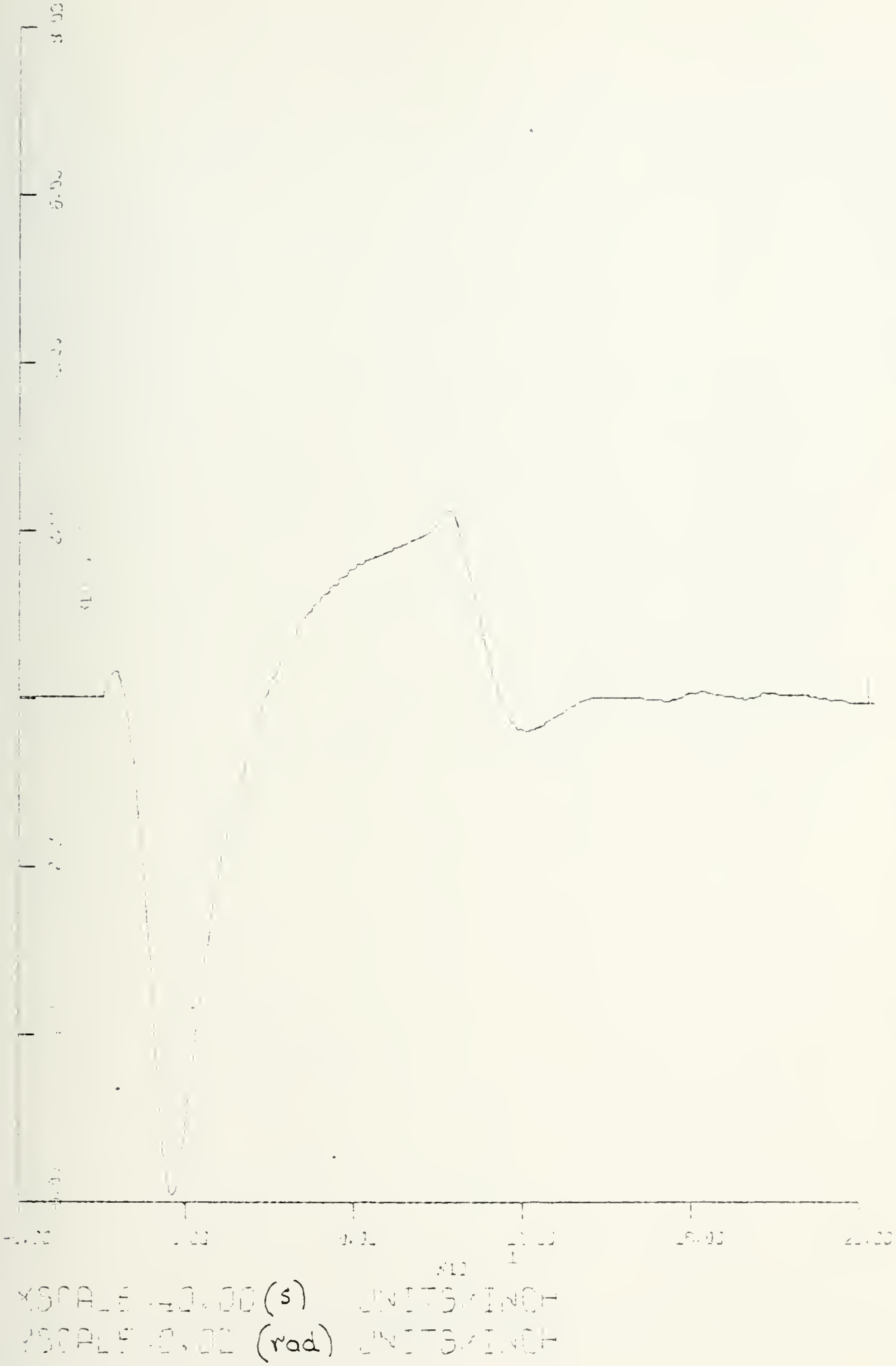


Fig. IV-6b. Pitch vs. Time. Response to a pulse force at FT. Bounded controller with error limiters. (B=800, C=10, E=1)

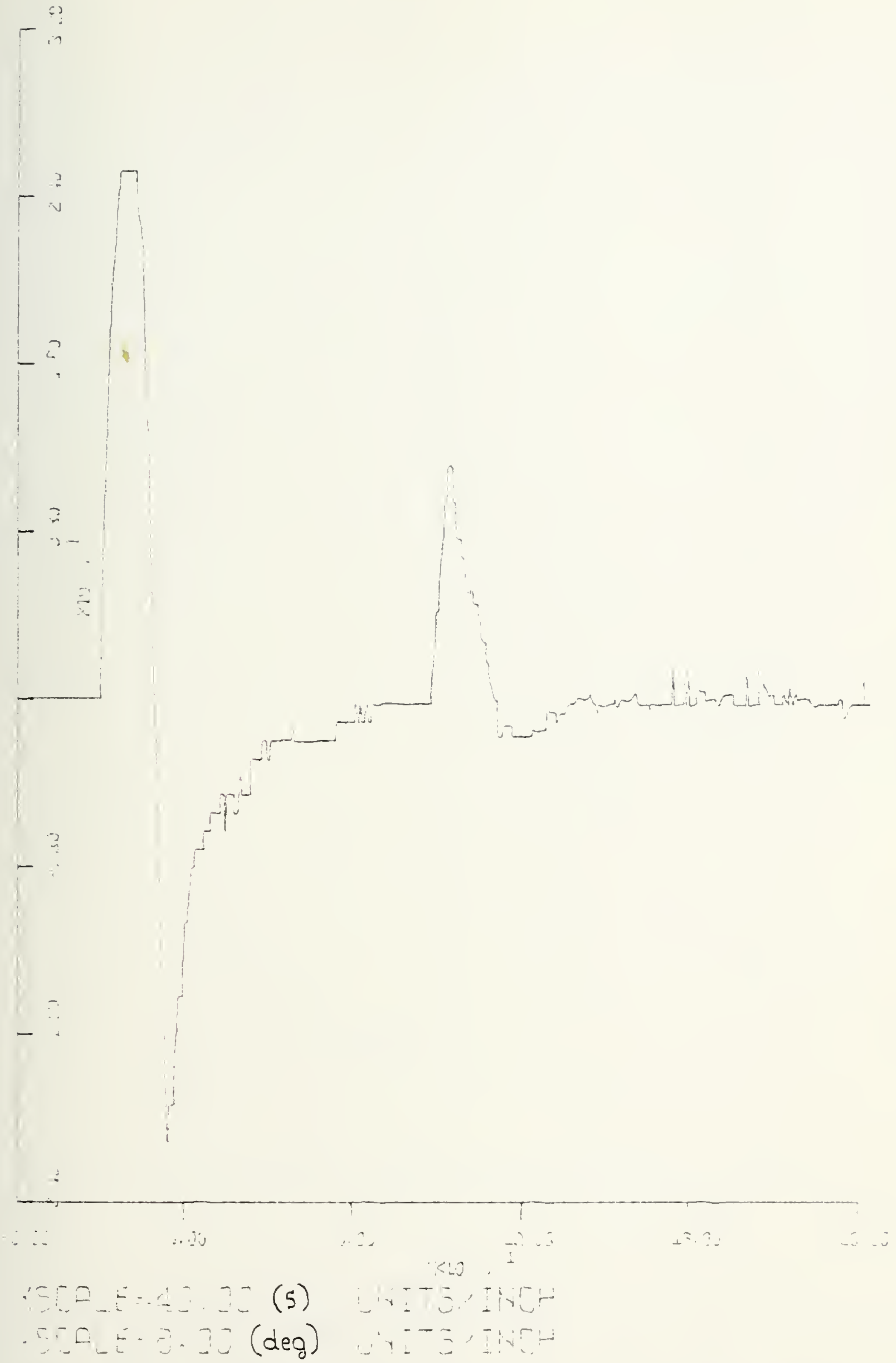


Fig. IV-6c. Stern Plane Angle vs. Time. Response to a pulse force at FT. Bounded controller with error limiters. (B=800, C=10, E=1)

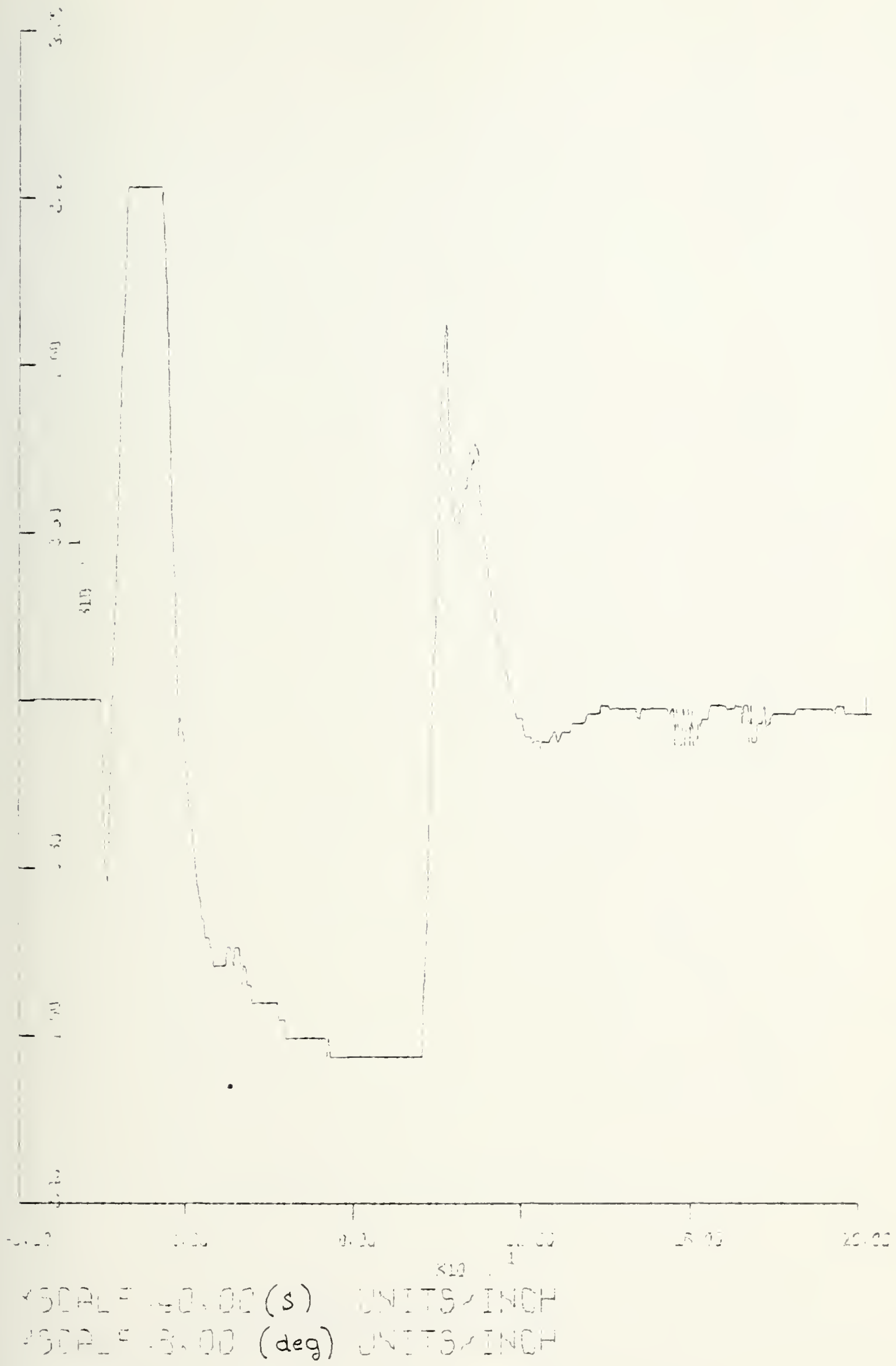


Fig. IV-6d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. Bounded controller with error limiters. (B=800, C=10, E=1)

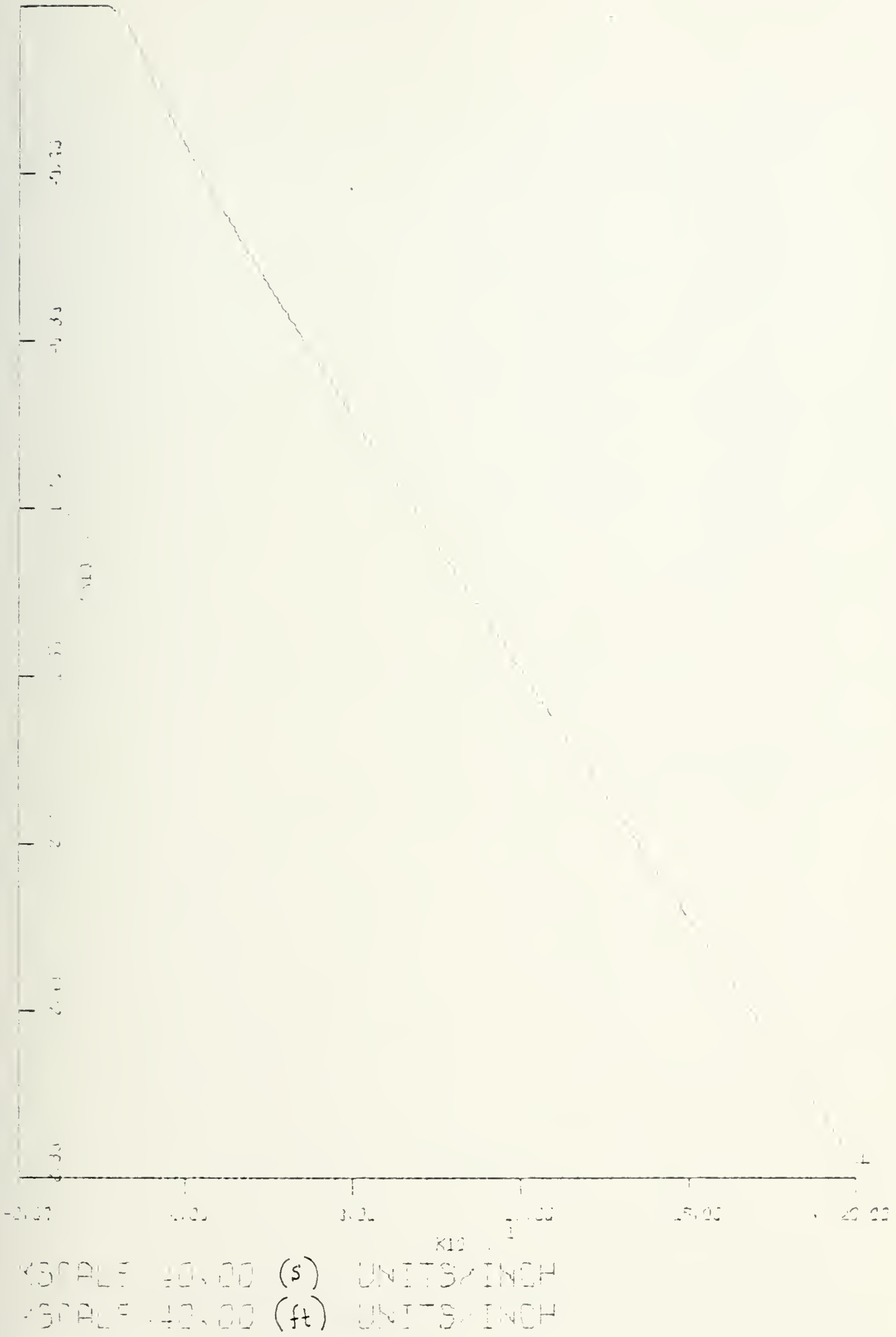


Fig. IV-6e. Depth vs. Time. Response to a pulse force at FT. Bounded controller with error limiters. (C=10, D=3000, E=1)

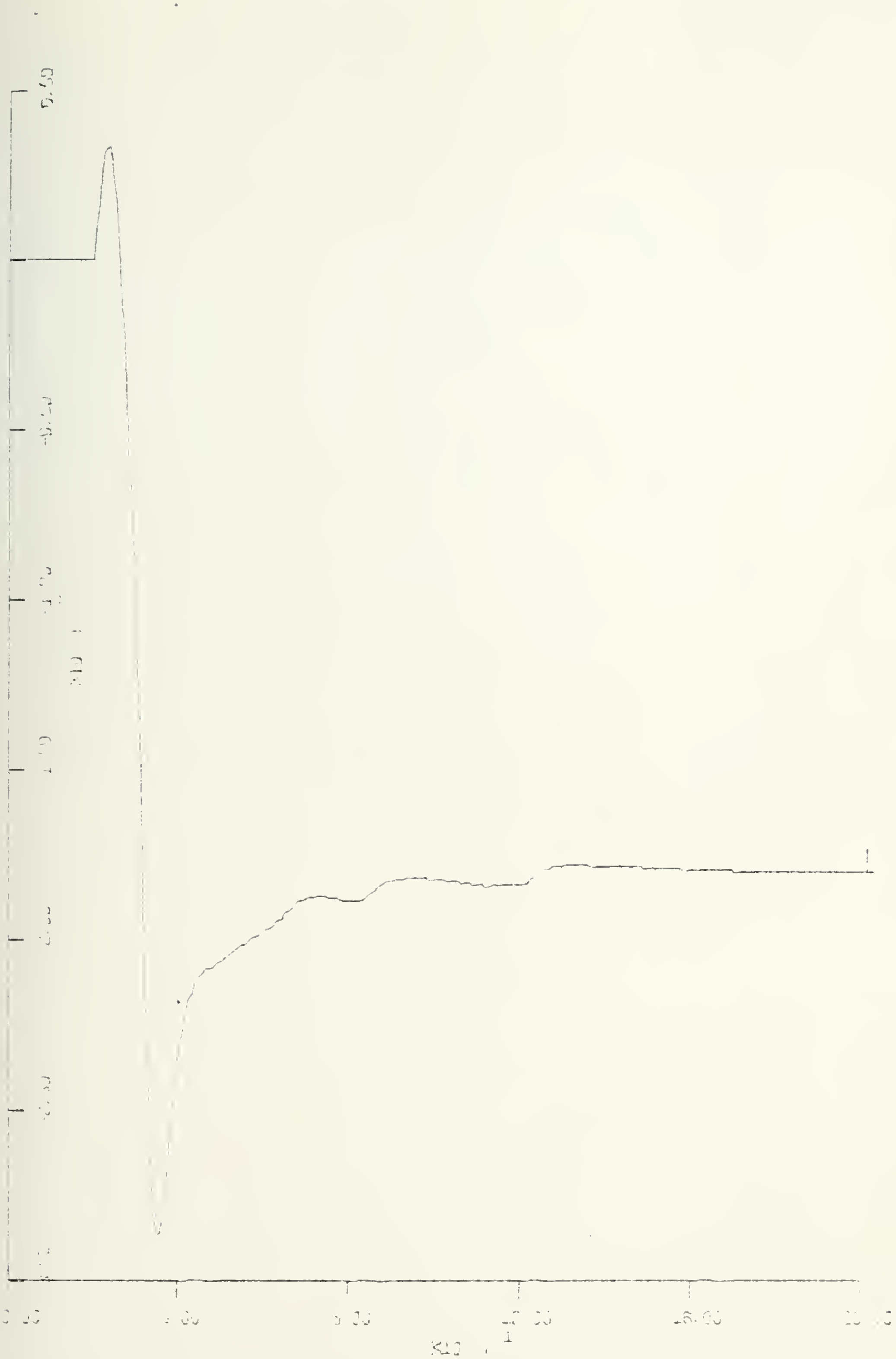


Fig. IV-6f. Pitch vs. Time. Response to a pulse force at FT. Bounded controller with error limiters. (C=10, D=3000, E=1)

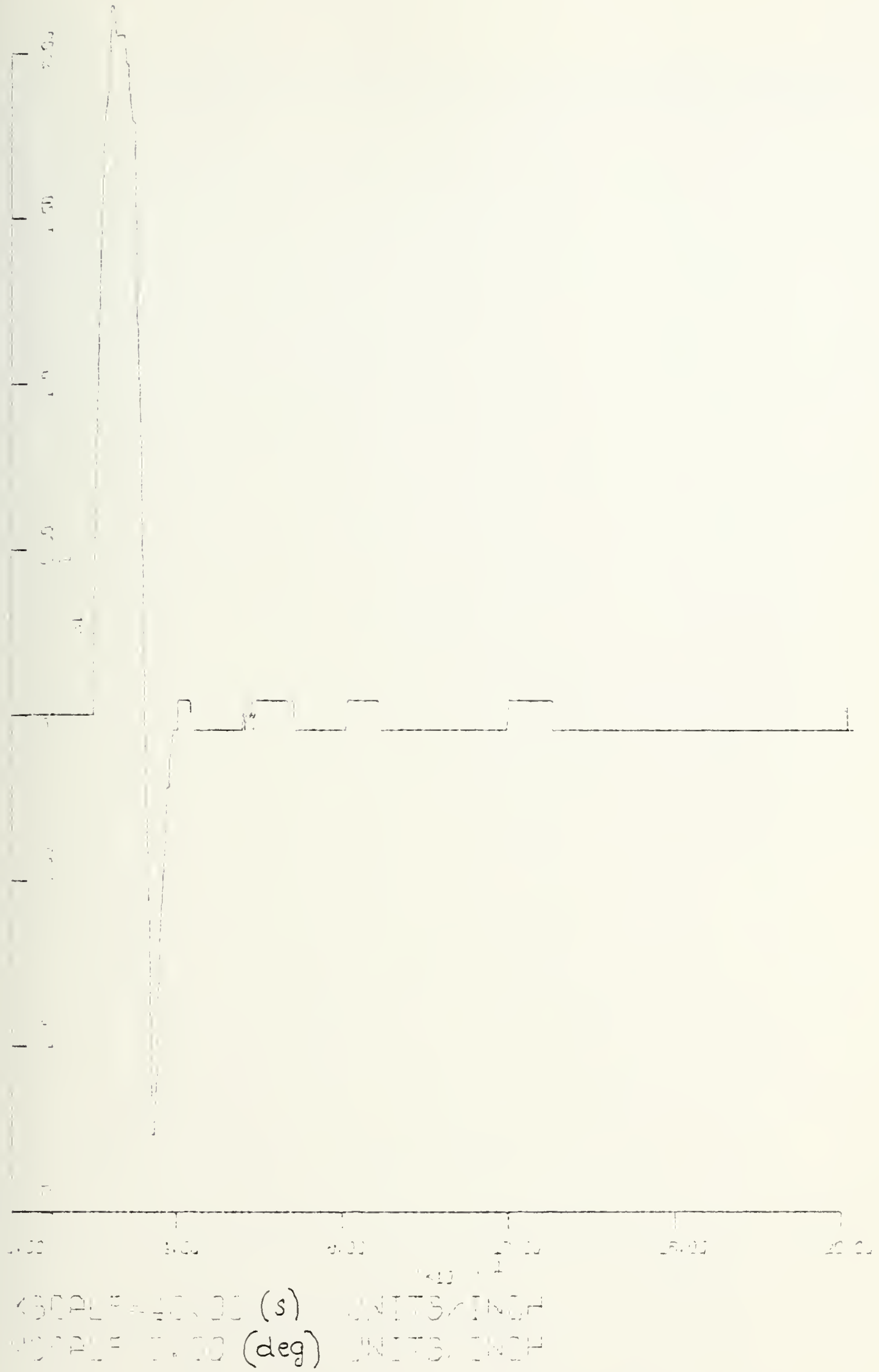


Fig. IV-6g. Stern Plane Angle vs. Time. Response to a pulse force at FT. Bounded controller with error limiters. (C=10, D=3000, E=1)

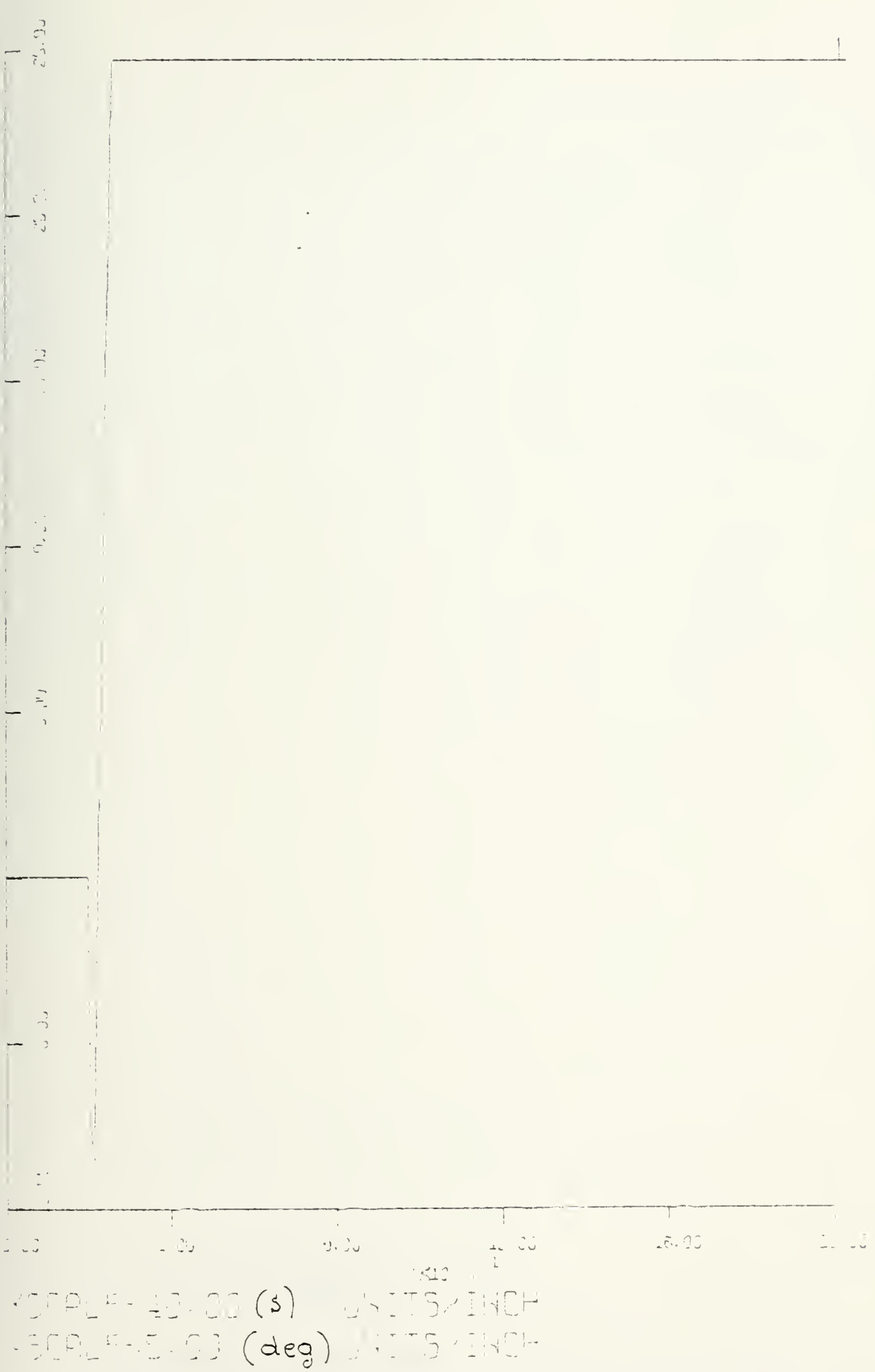


Fig. IV-6h. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. Bounded controller with error limiters. (C=10, D=3000, E=1)

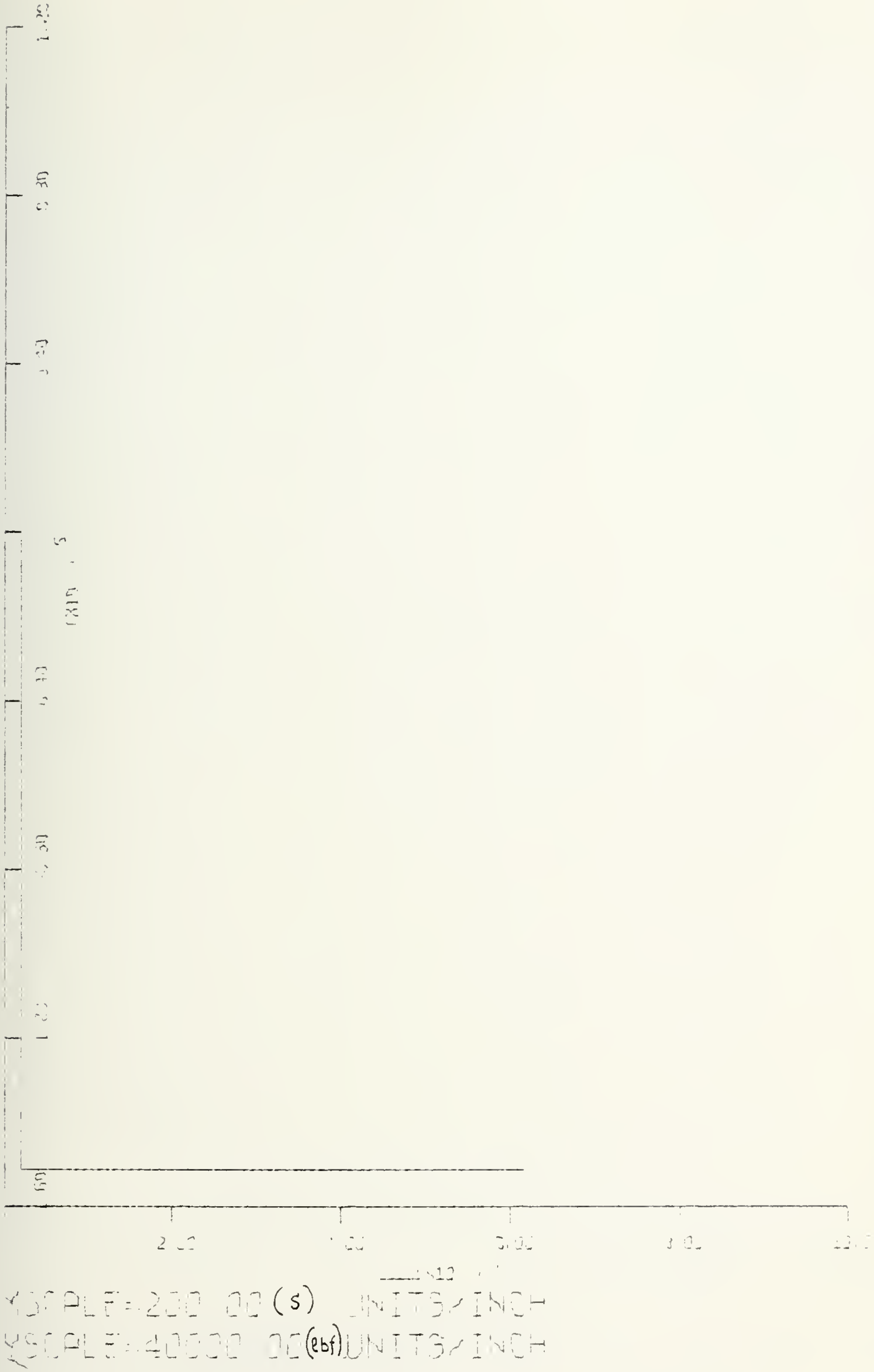


Fig. IV-7a. Step force at AU or FT

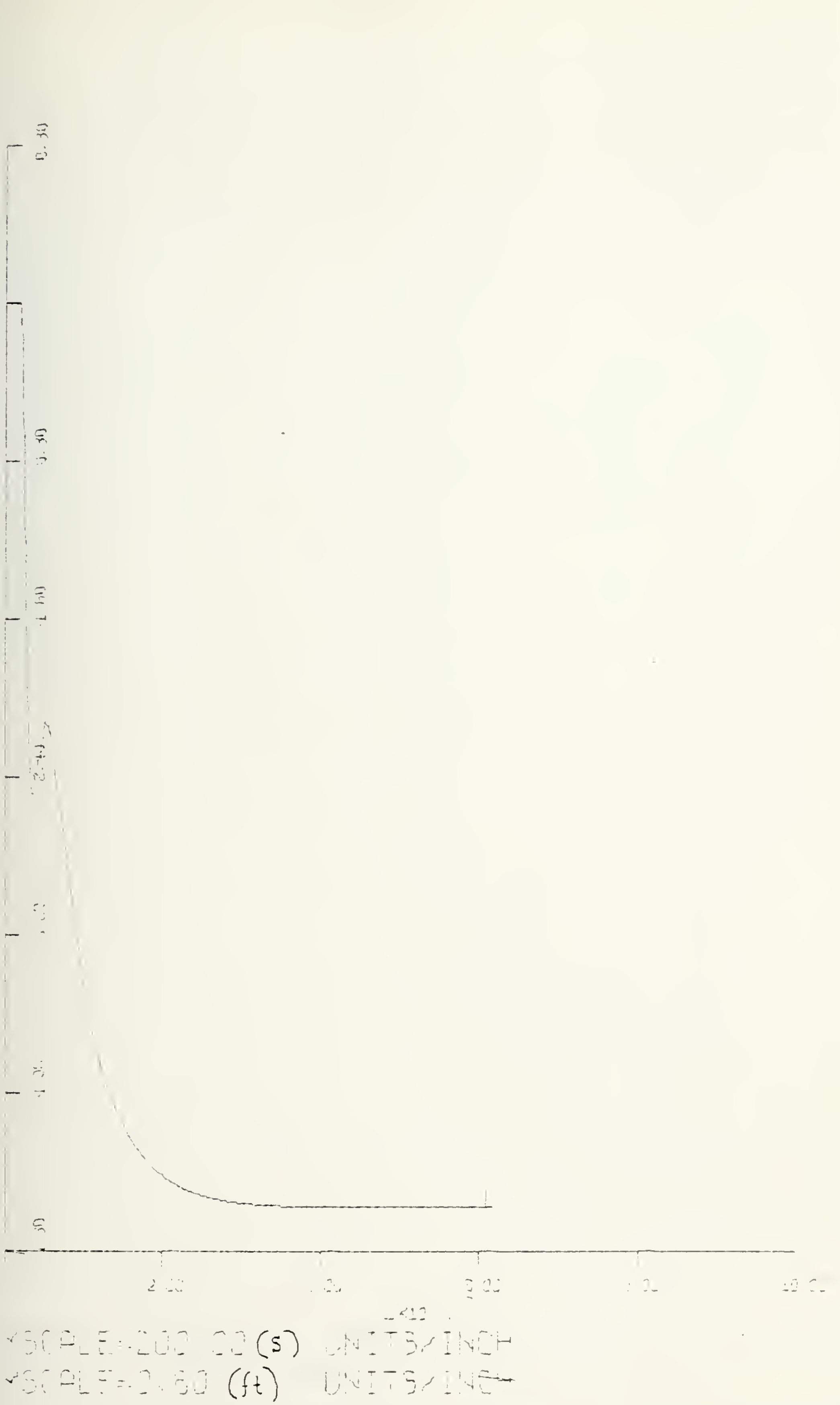
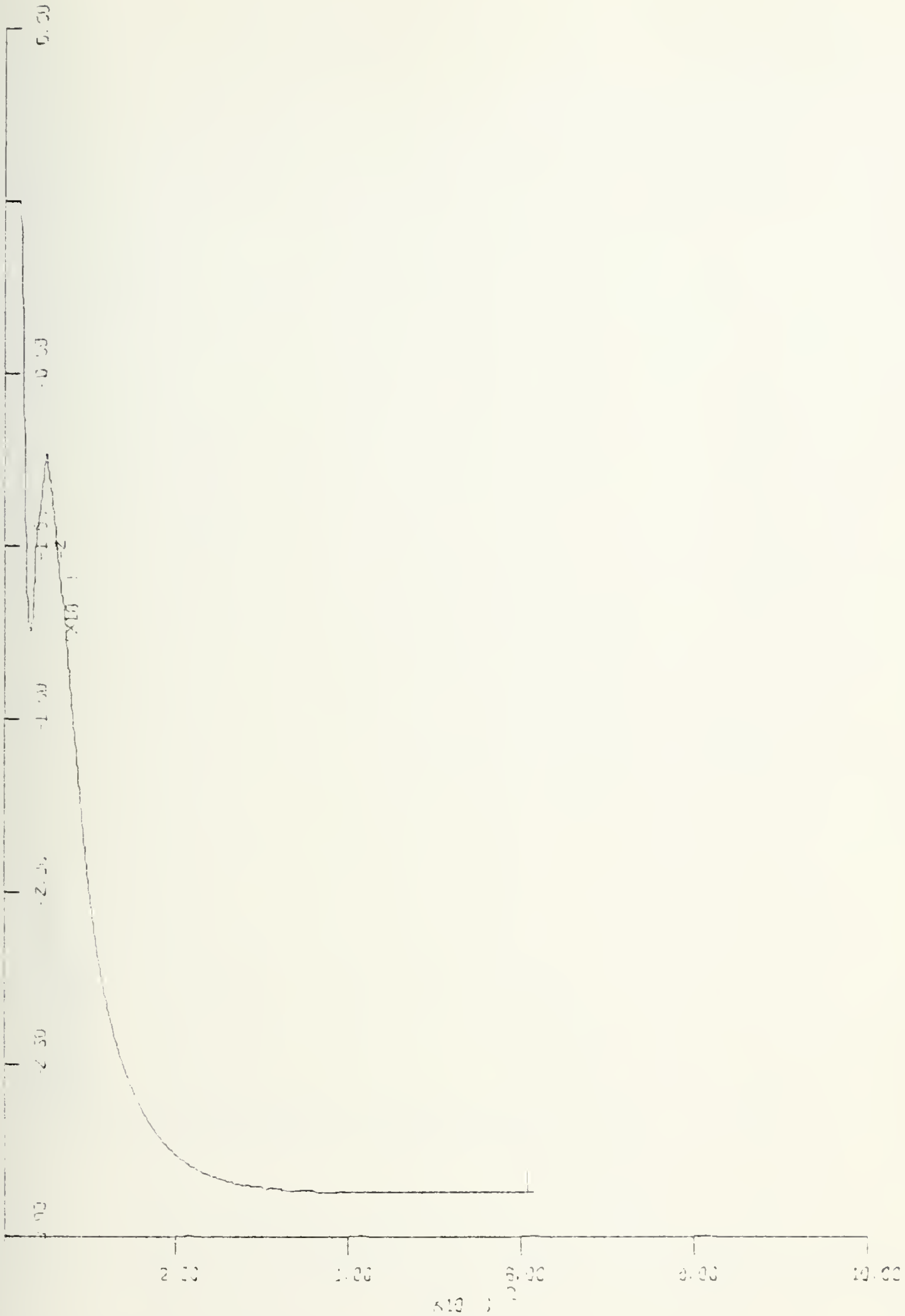


Fig. IV-7b. Depth vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=16



XSCALE=200.00 (s) UNITS/INCH
YSCALE= 5.00E-3(rad)UNITS/INCH

Fig. IV-7c. Pitch vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=16

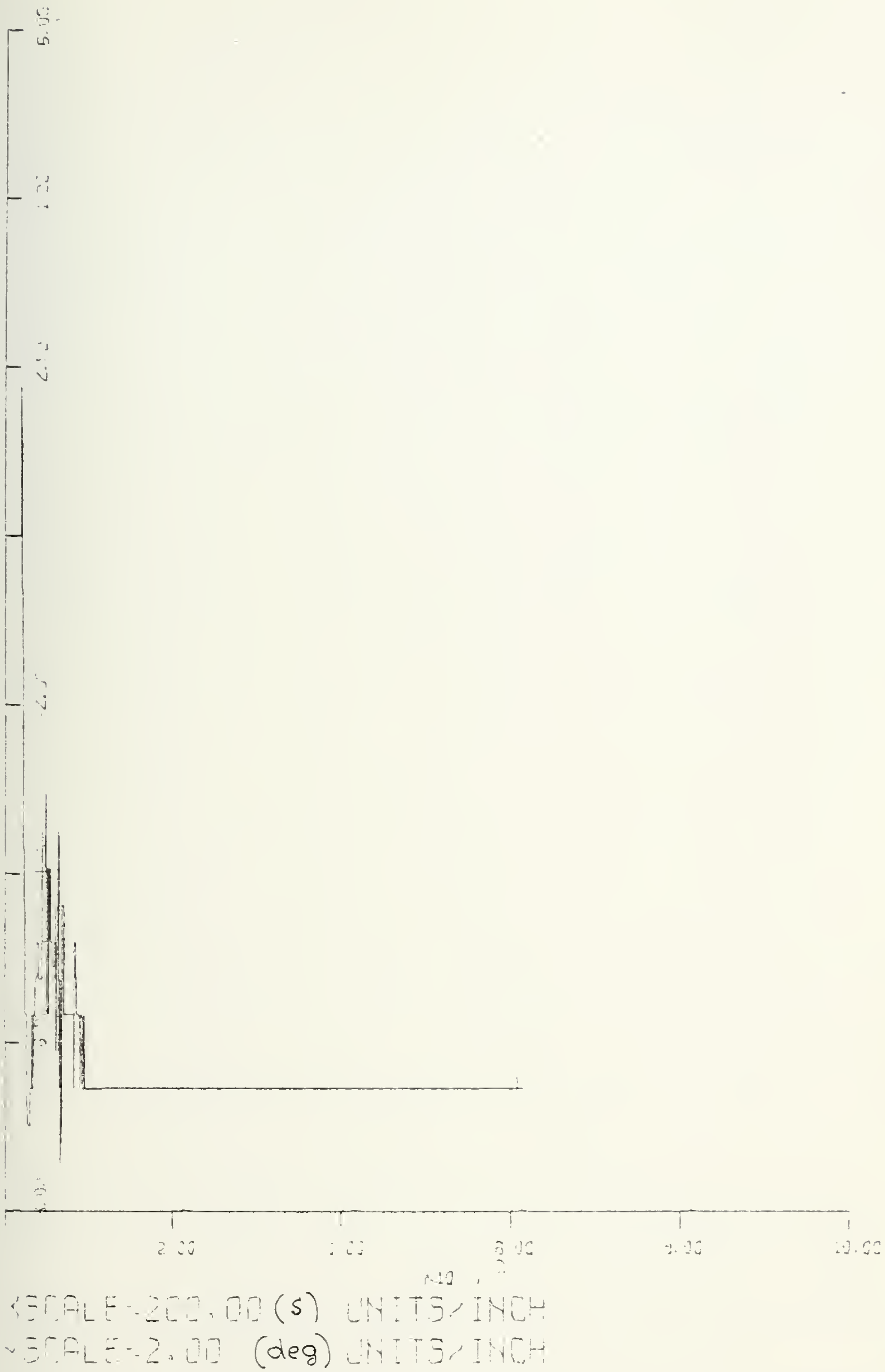


Fig. IV-7d. Stern Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=16

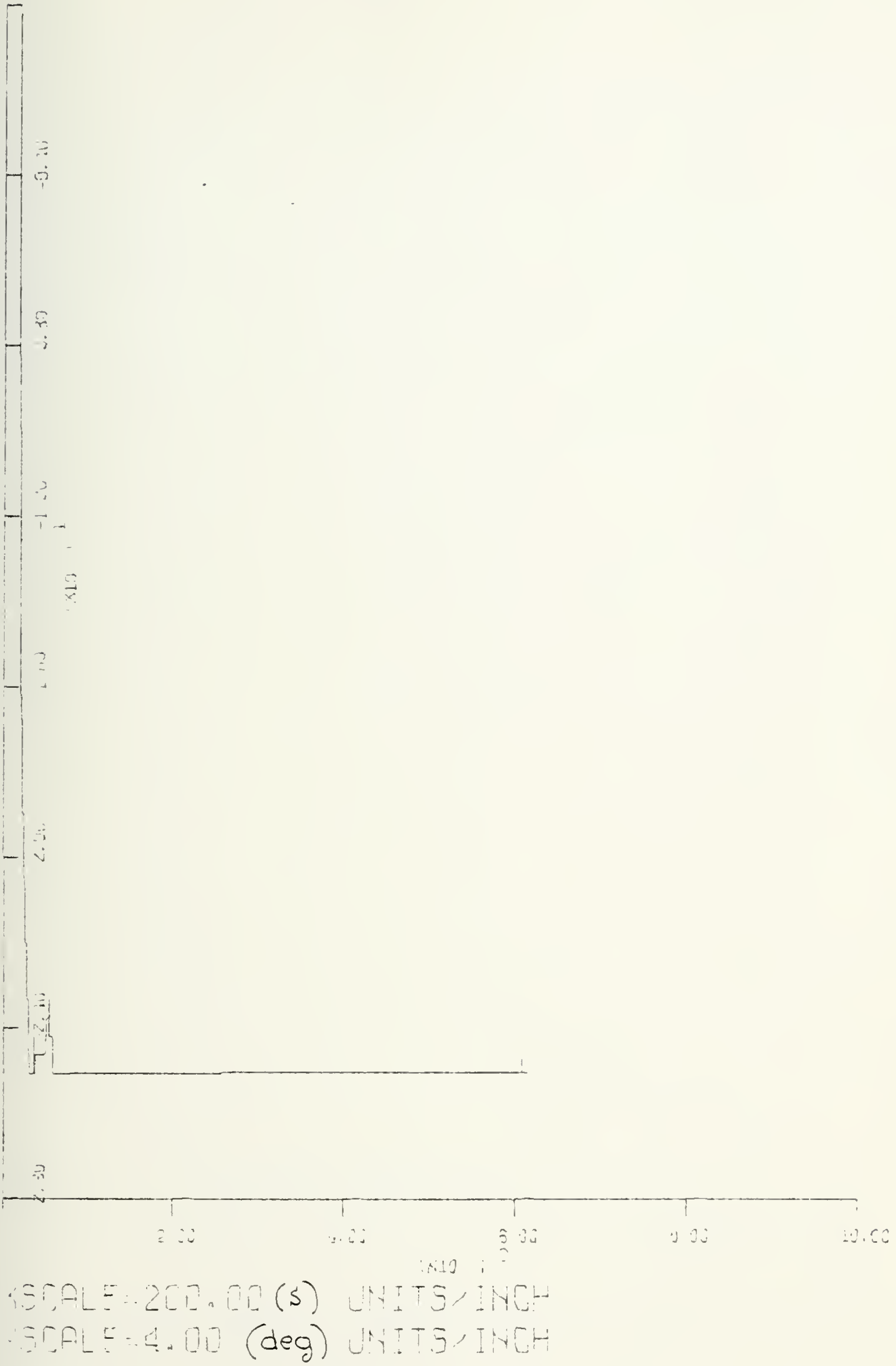


Fig. IV-7e. Fairwater Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=16

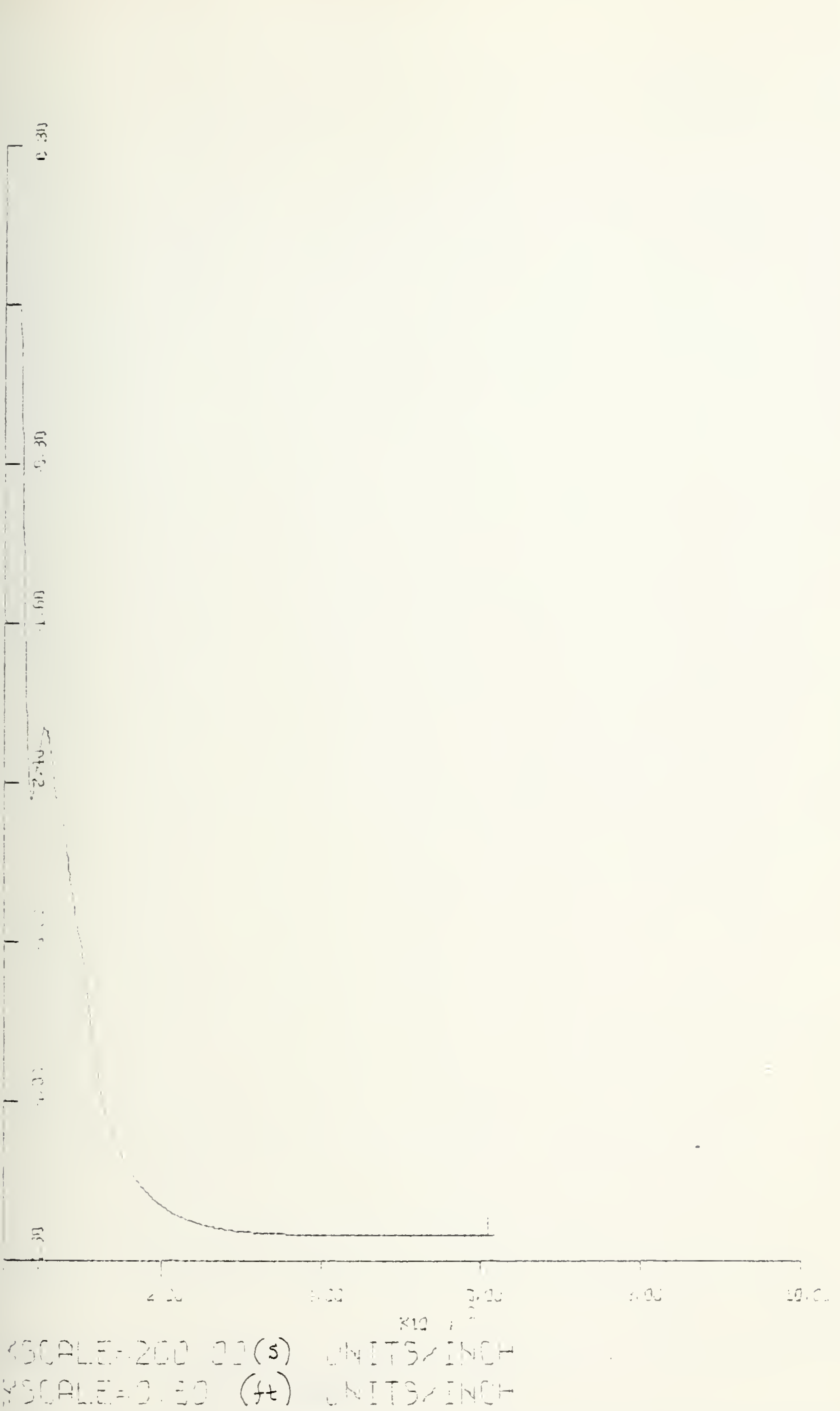


Fig. IV-8a. Depth vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=8

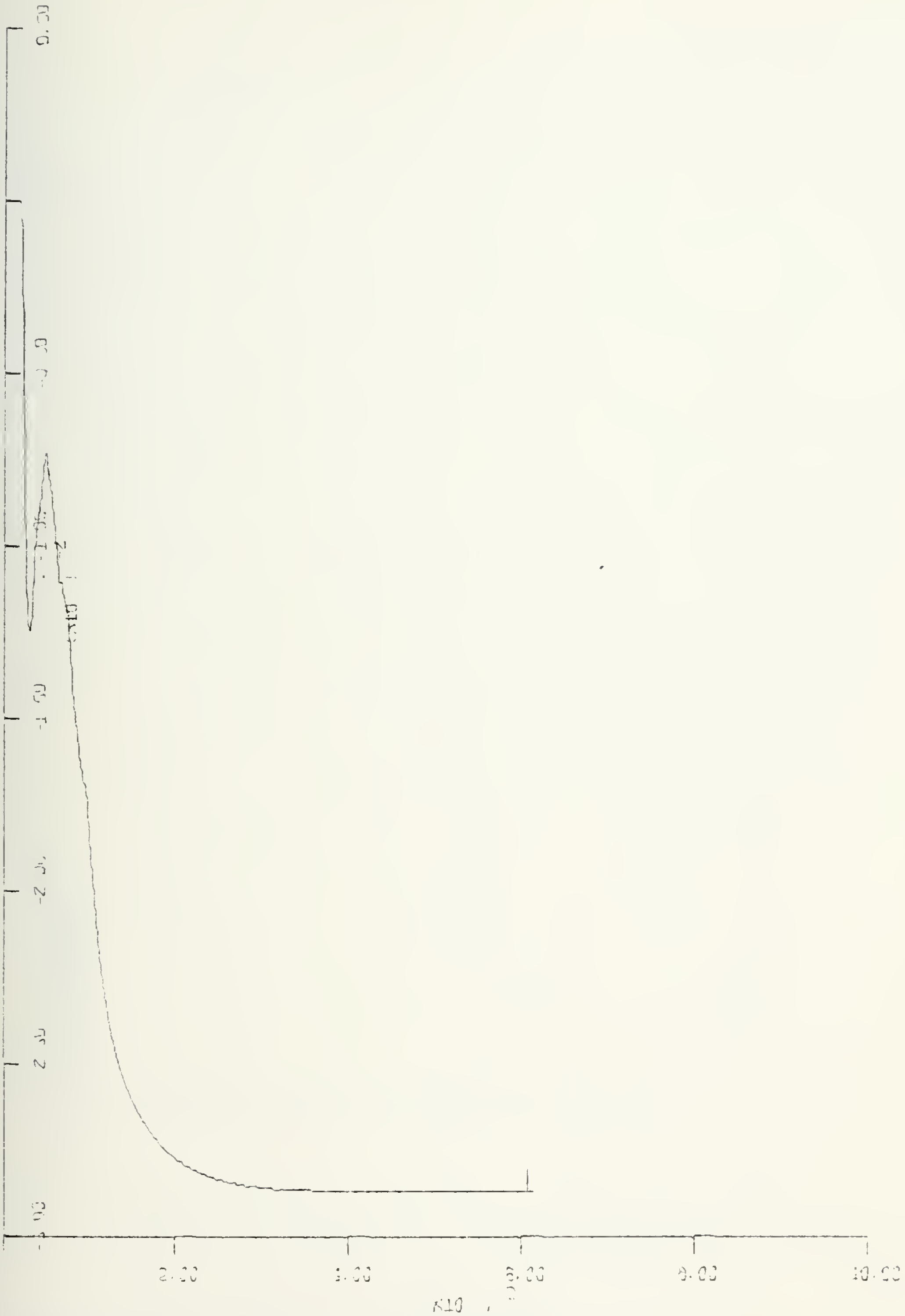


Fig. IV-8b. Pitch vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=8

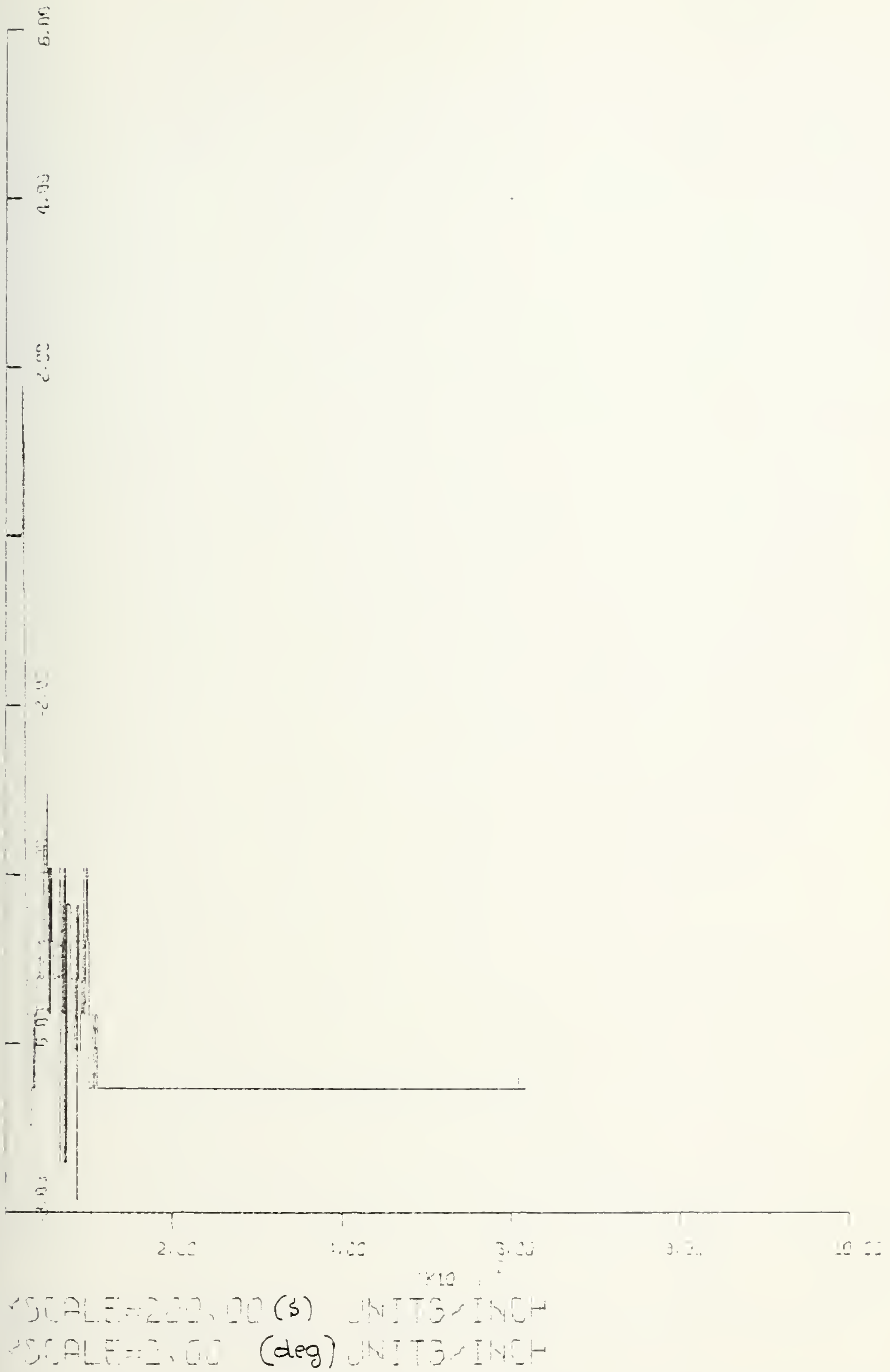
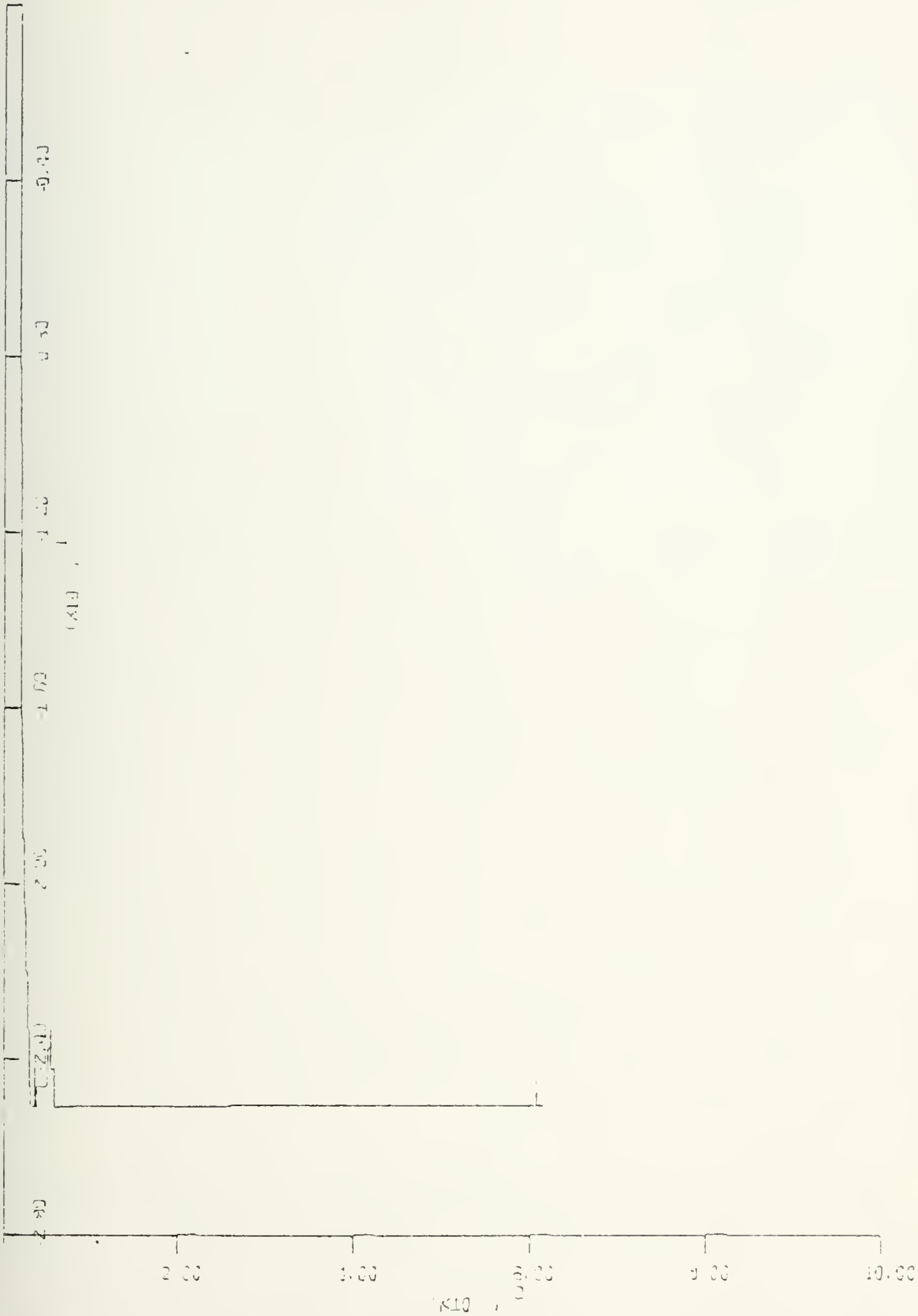


Fig. IV-8c. Stern Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=8



XSCALE=200.00(s) UNITS/INCH
YSCALE=4.00 (deg) UNITS/INCH

Fig. IV-8d. Fairwater Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=8

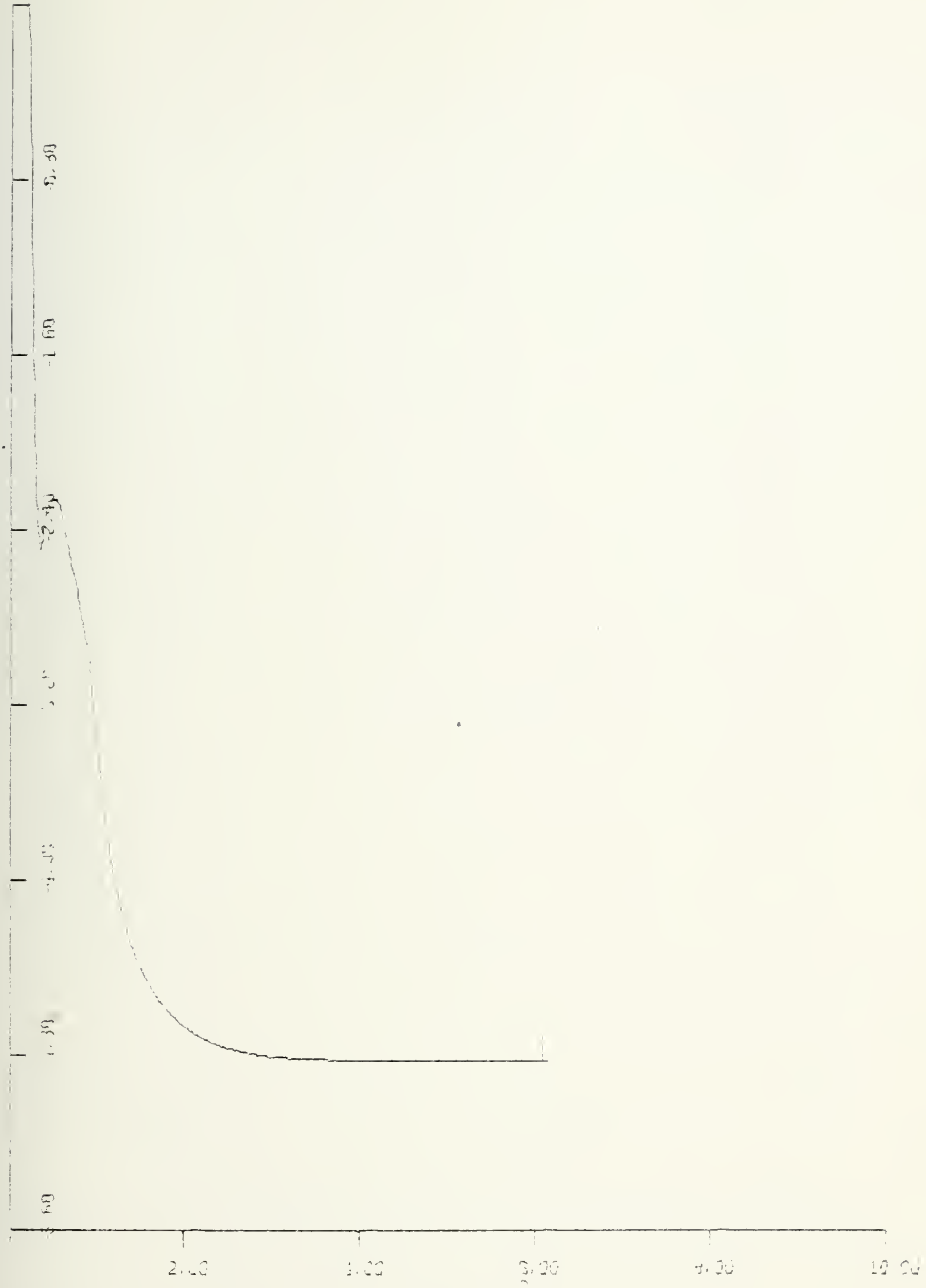


Fig. IV-9a. Depth vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=4

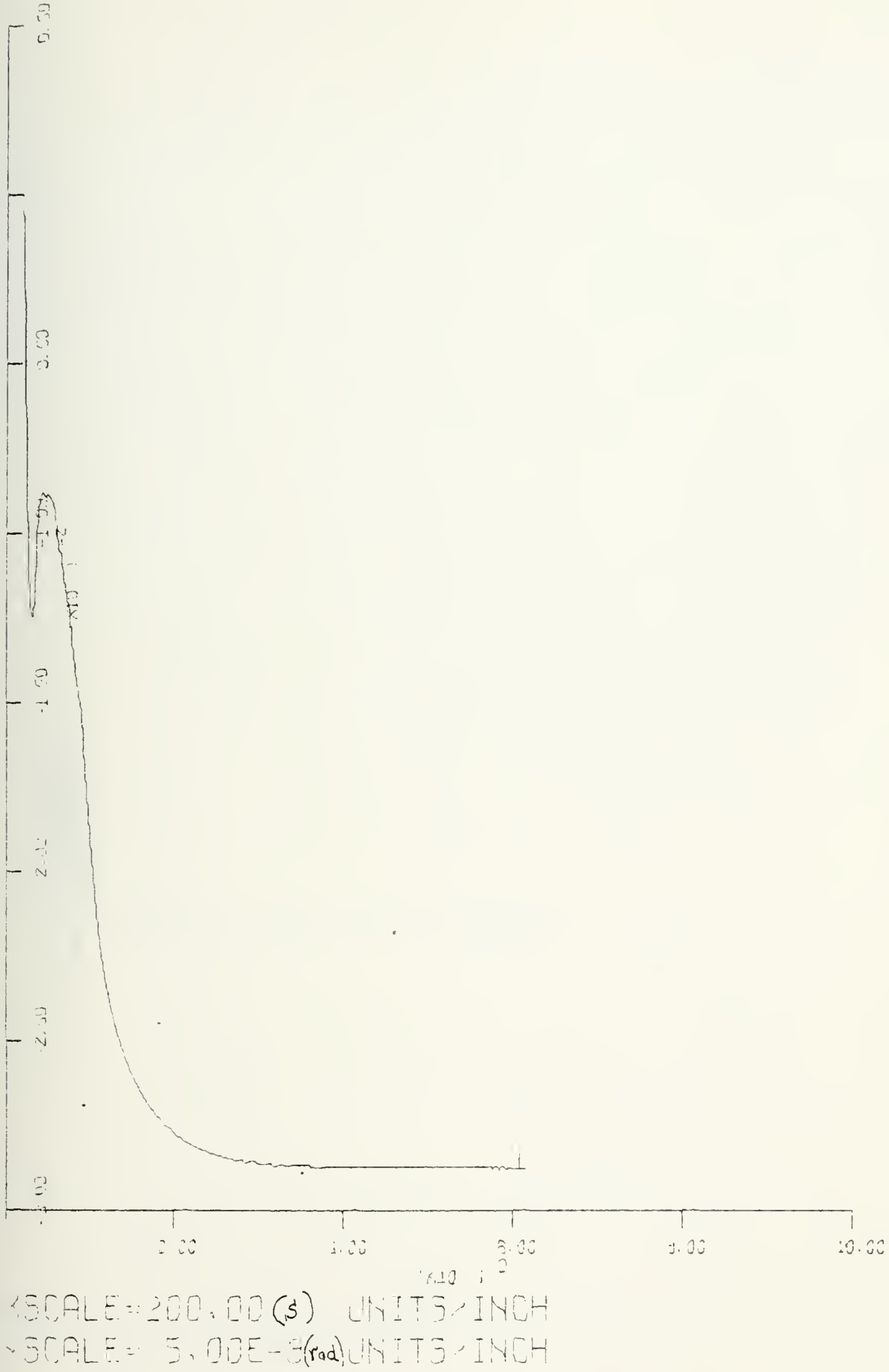


Fig. IV-9b. Pitch vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=4

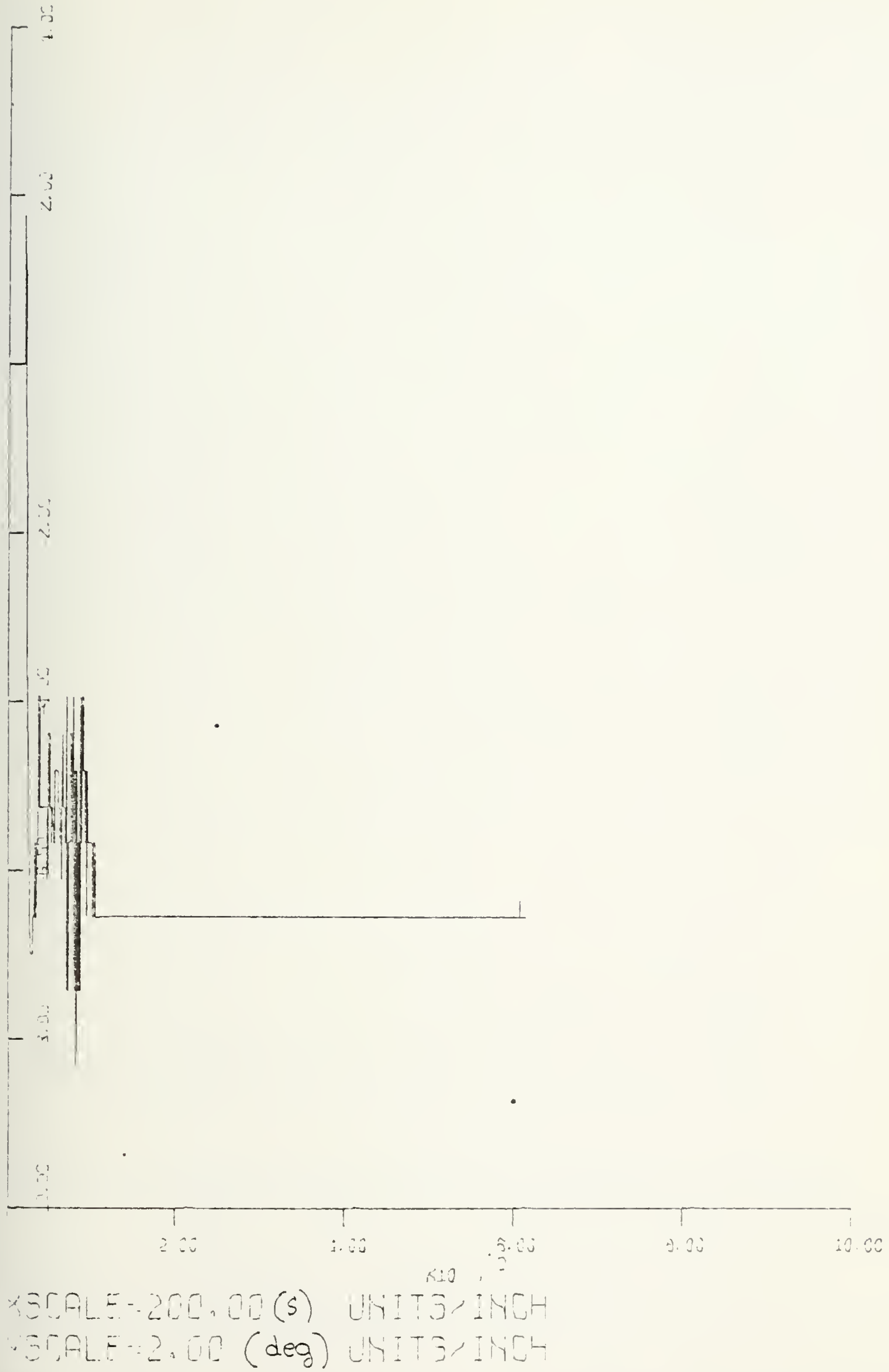


Fig. IV-9c. Stern Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=4

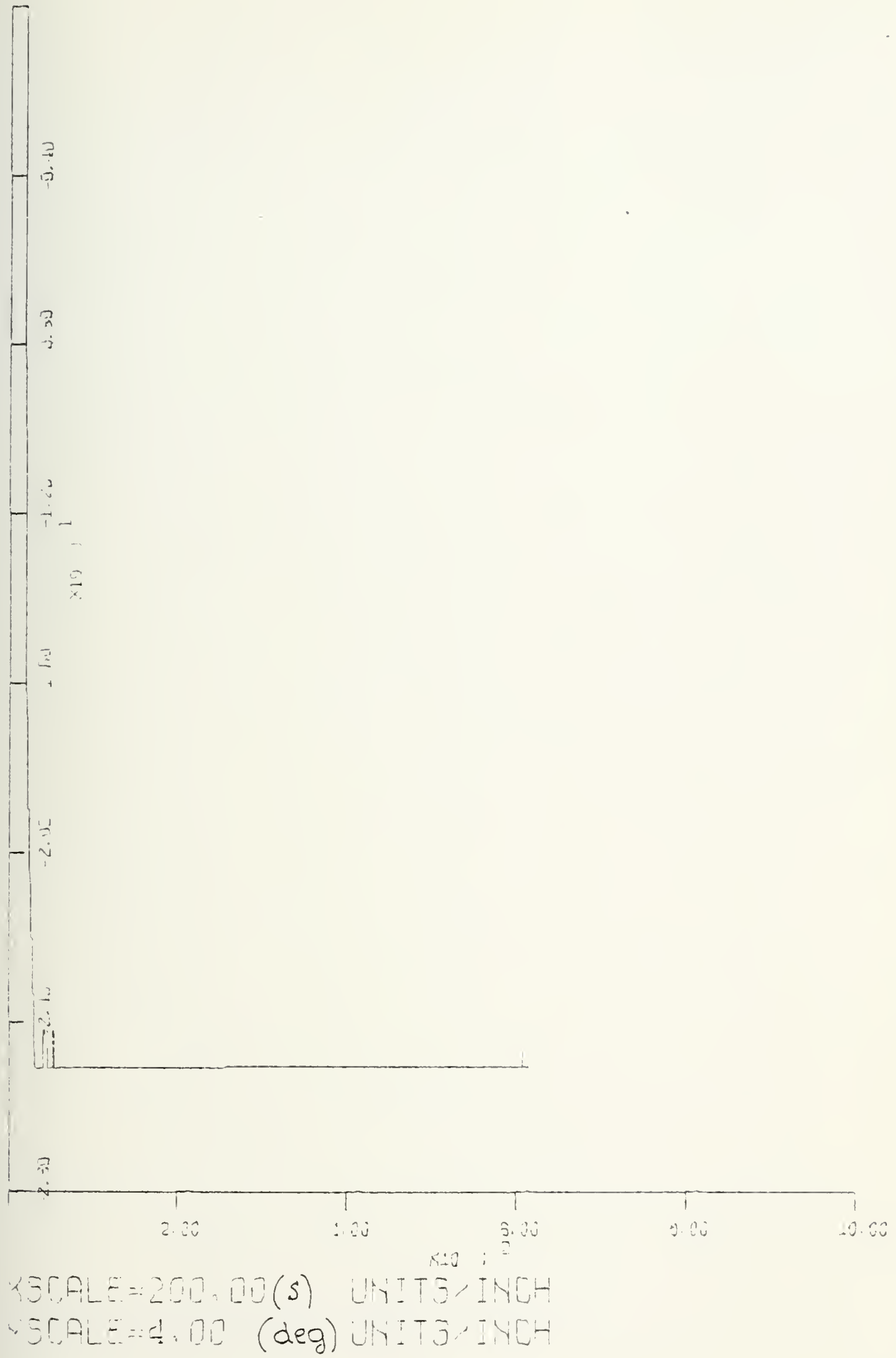


Fig. IV-9d. Fairwater Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=4

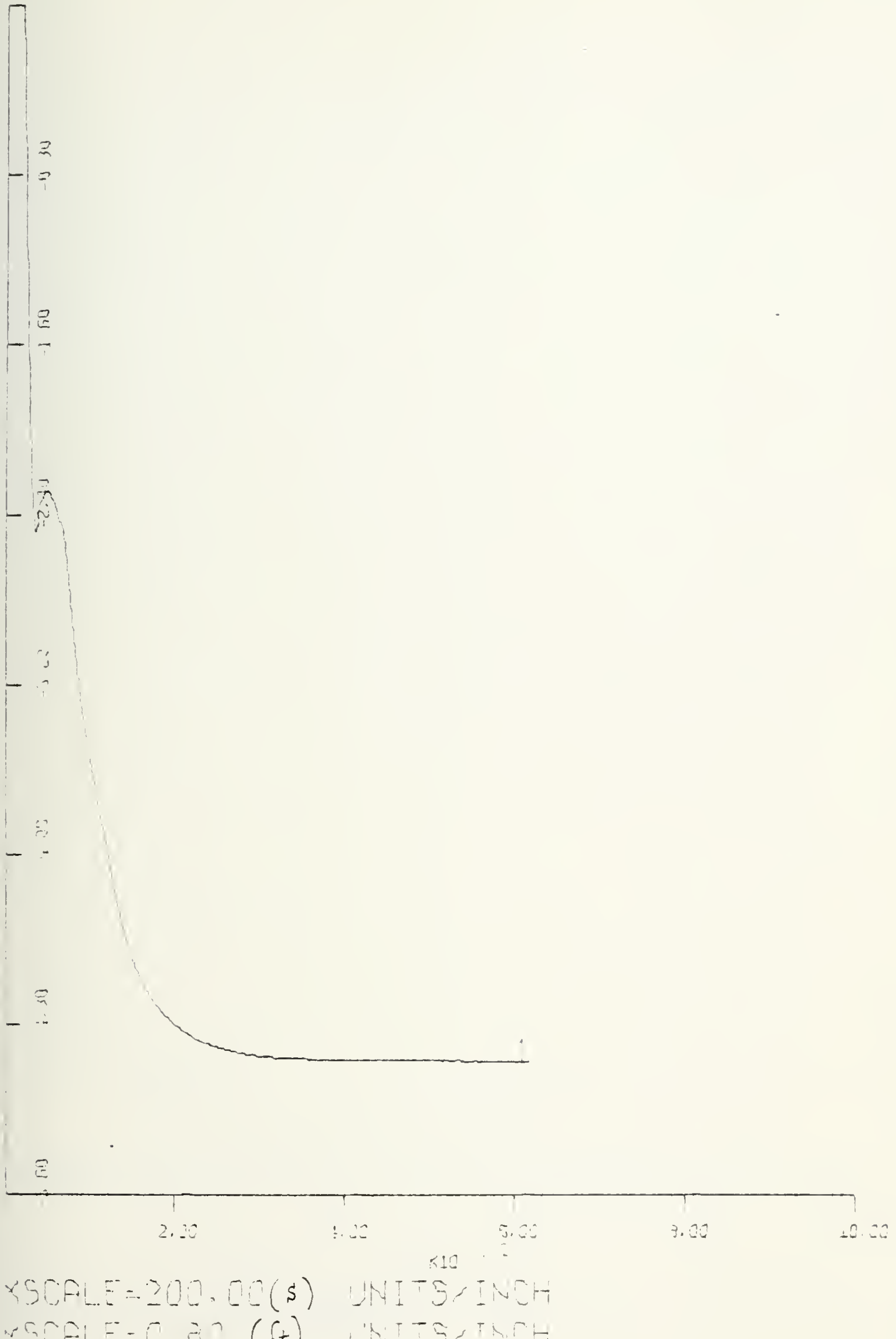


Fig. IV-10a. Depth vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=2.5

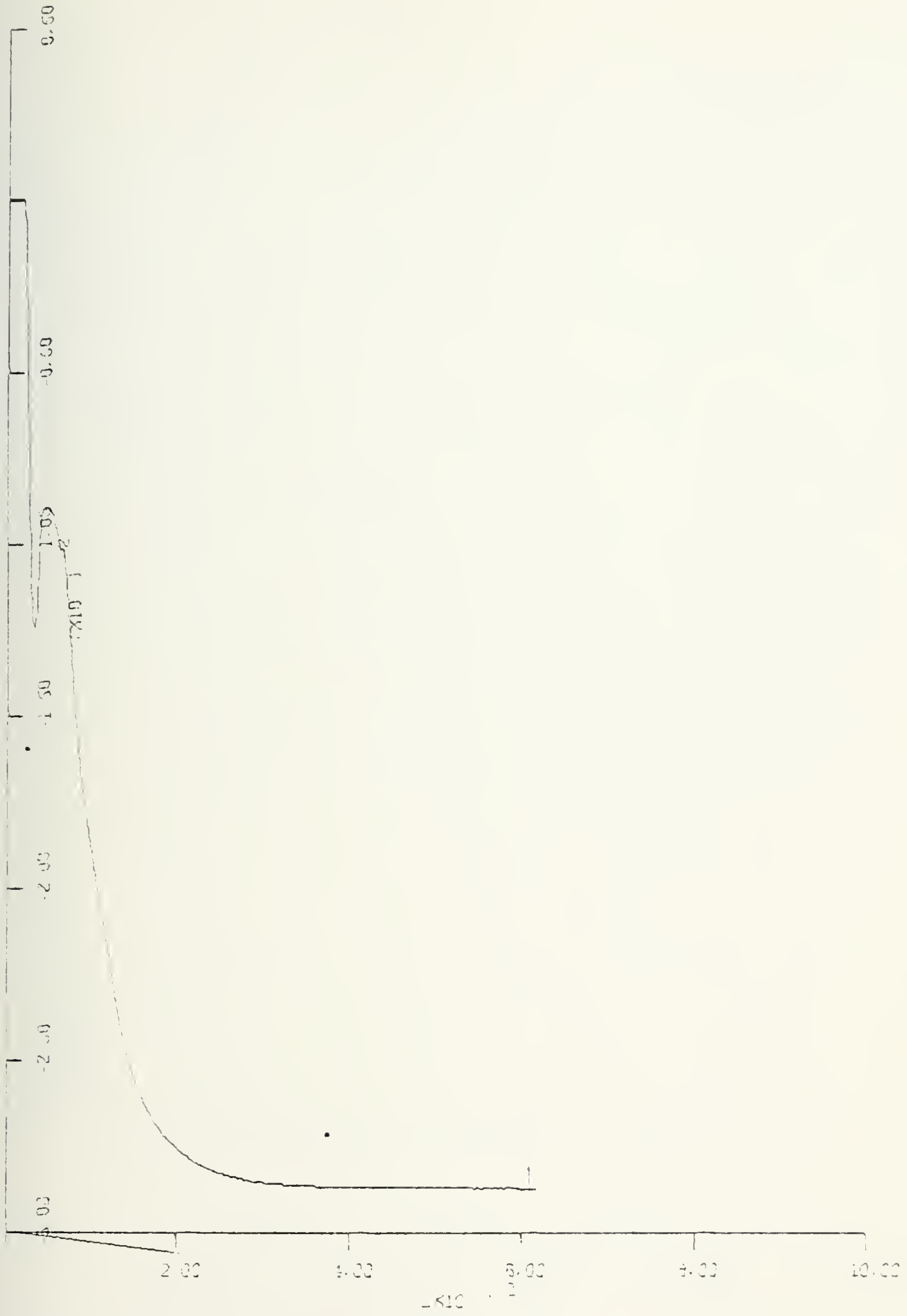


Fig. IV-10b. Pitch vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=2.5

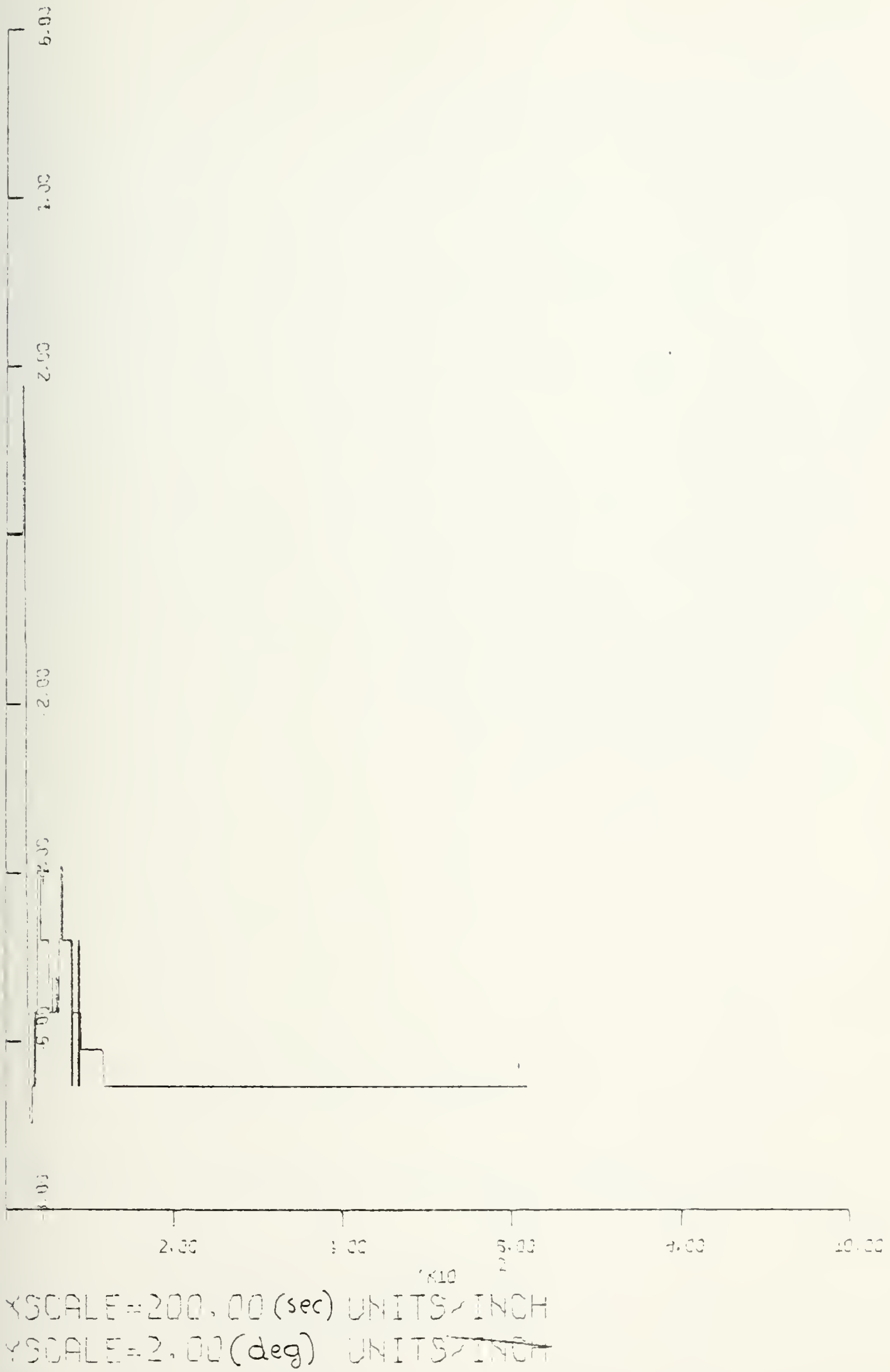


Fig. IV-10c. Stern Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=2.5

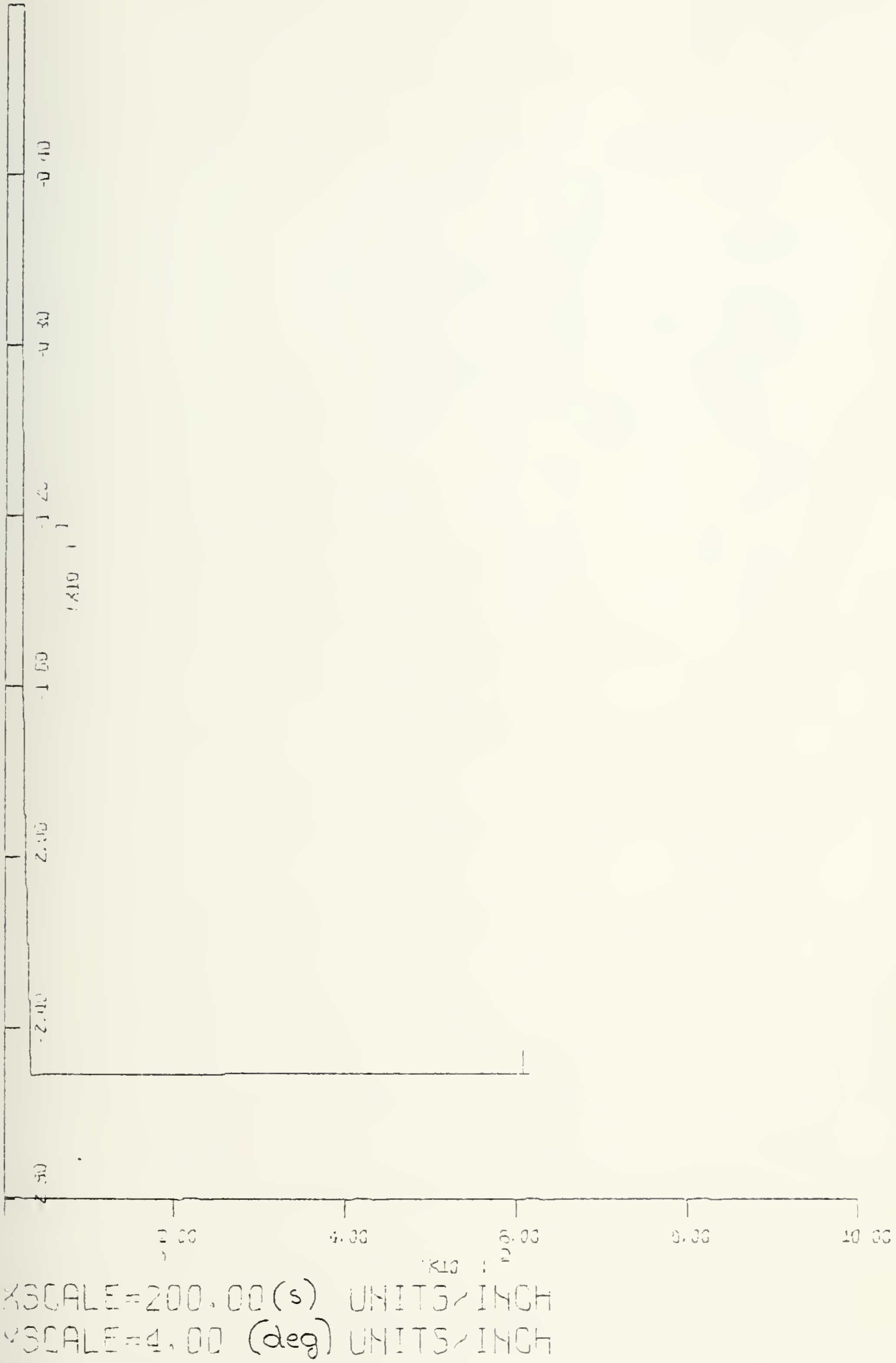


Fig. IV-10d. Fairwater Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=2.5

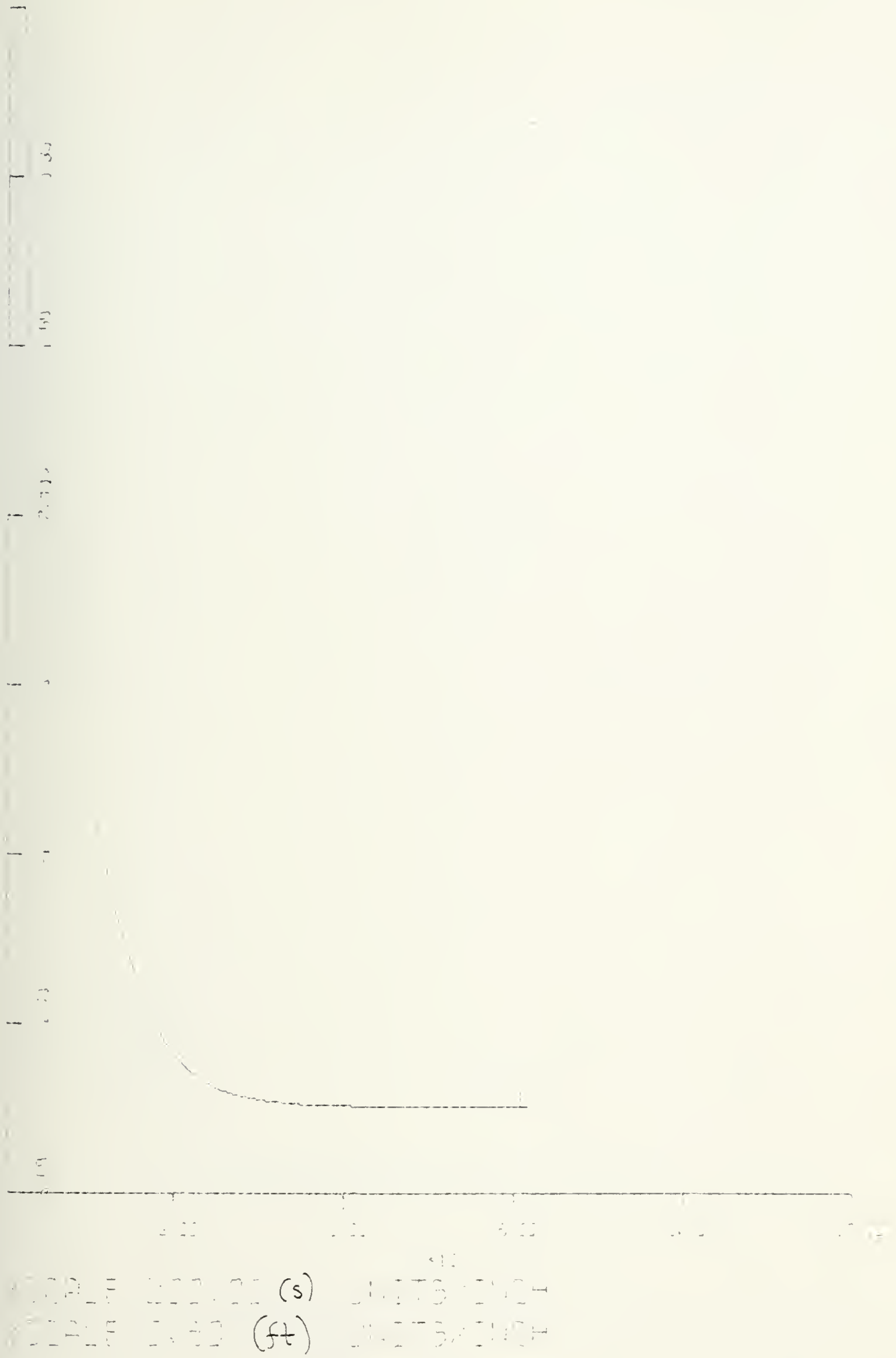


Fig. IV-11a. Depth vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=1.0

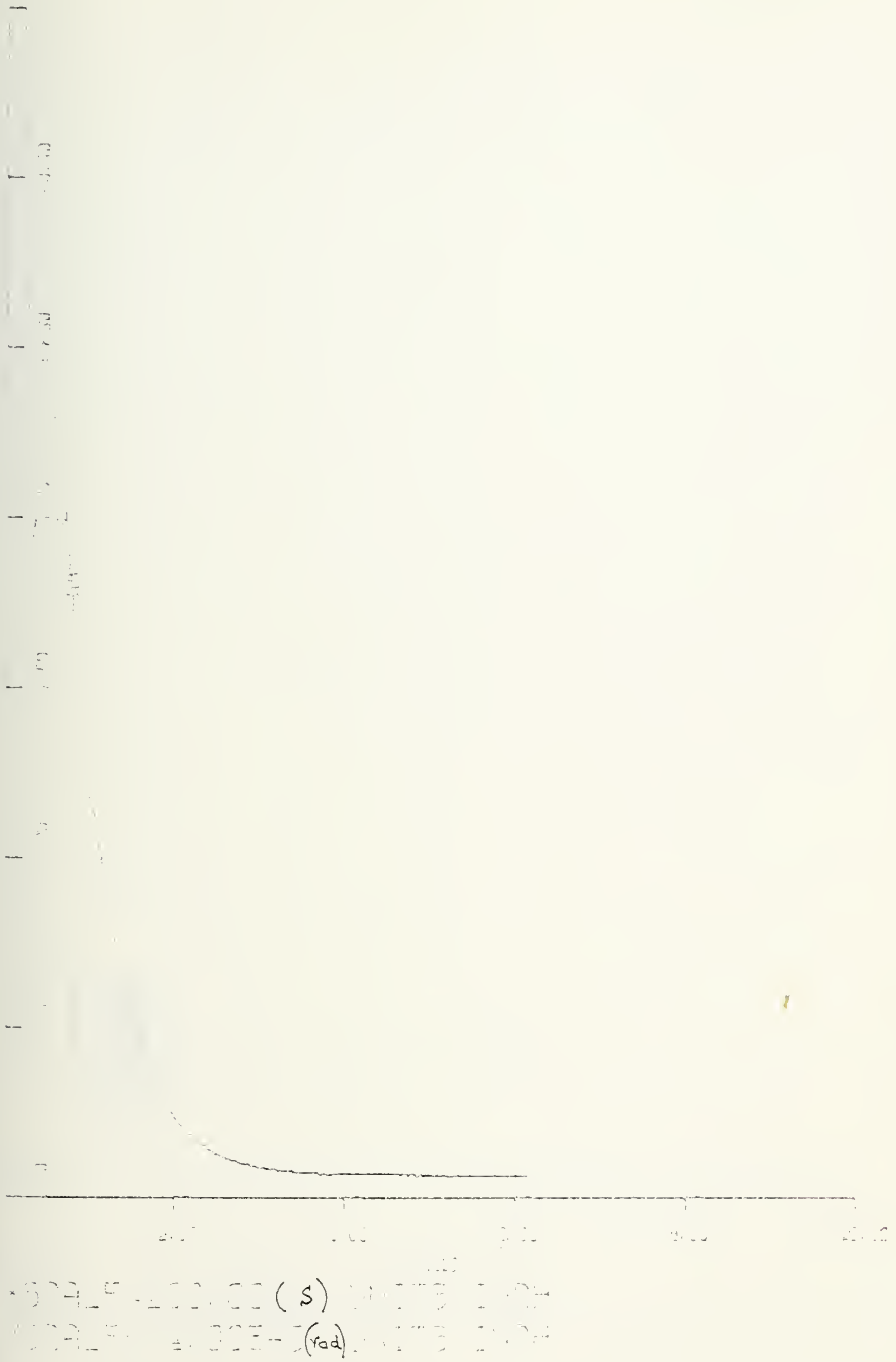


Fig. IV-11b. Pitch vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=1.0

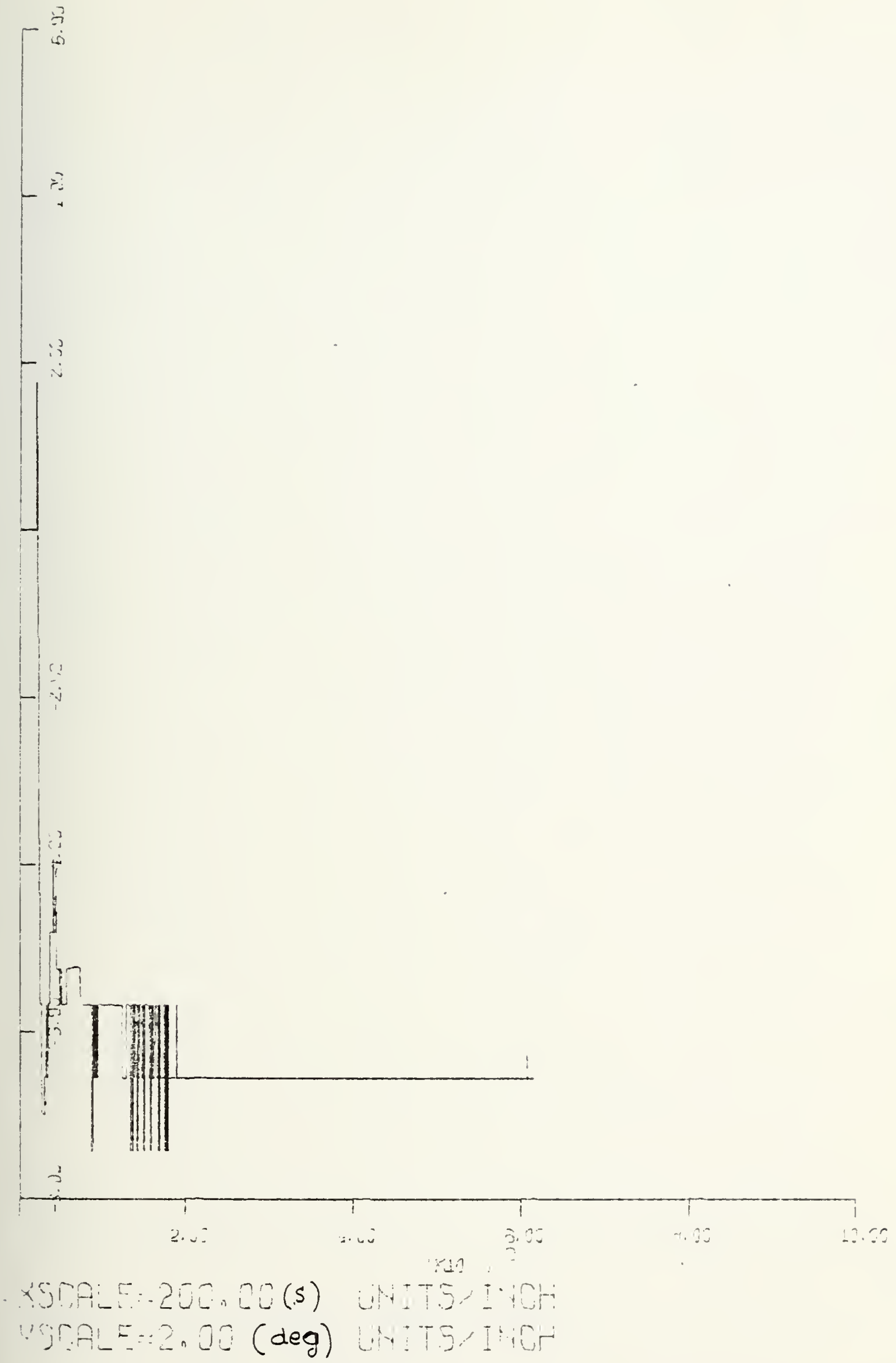


Fig. IV-11c. Stern Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=1.0

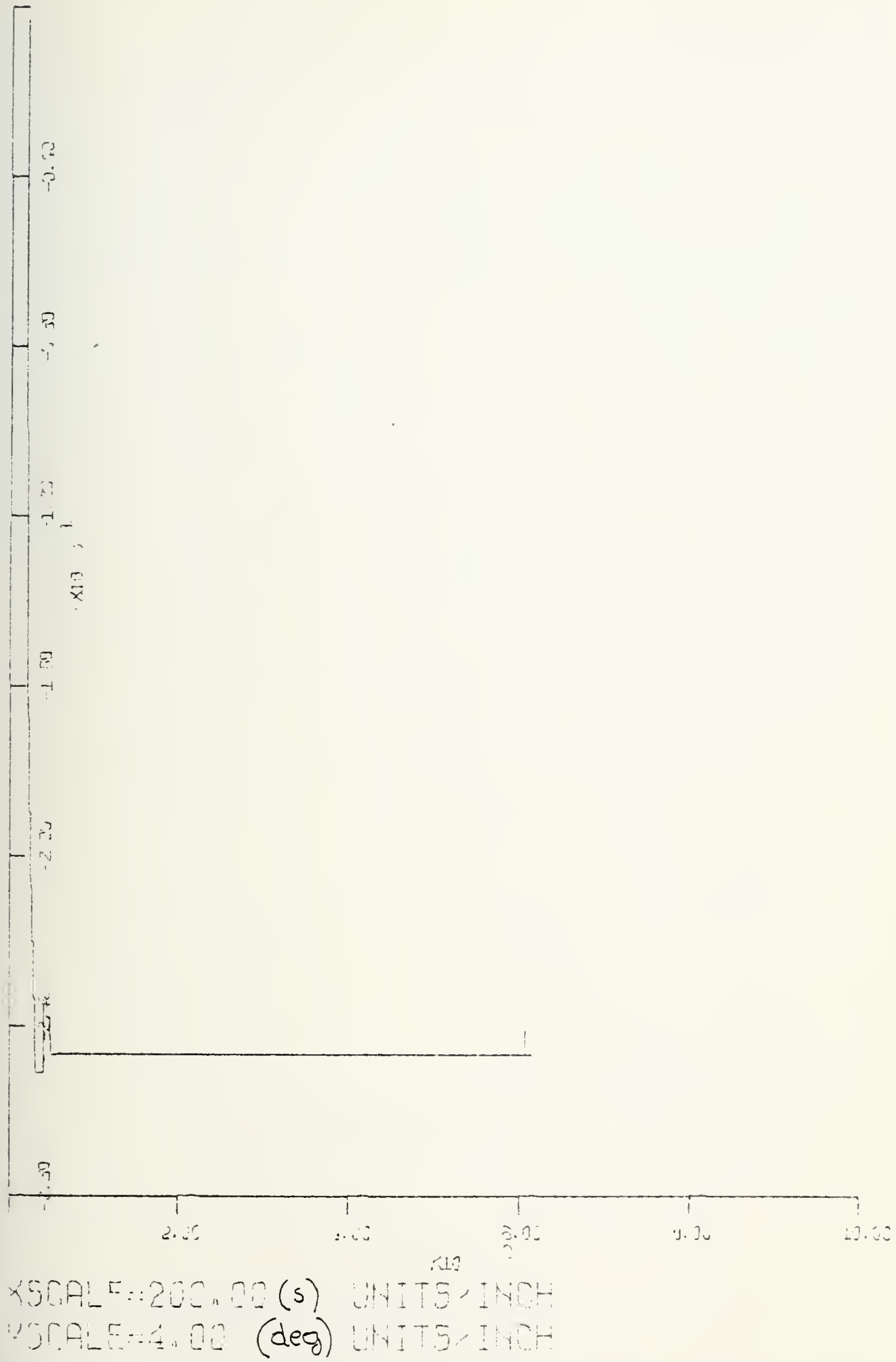


Fig. IV-11d. Fairwater Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=1.0

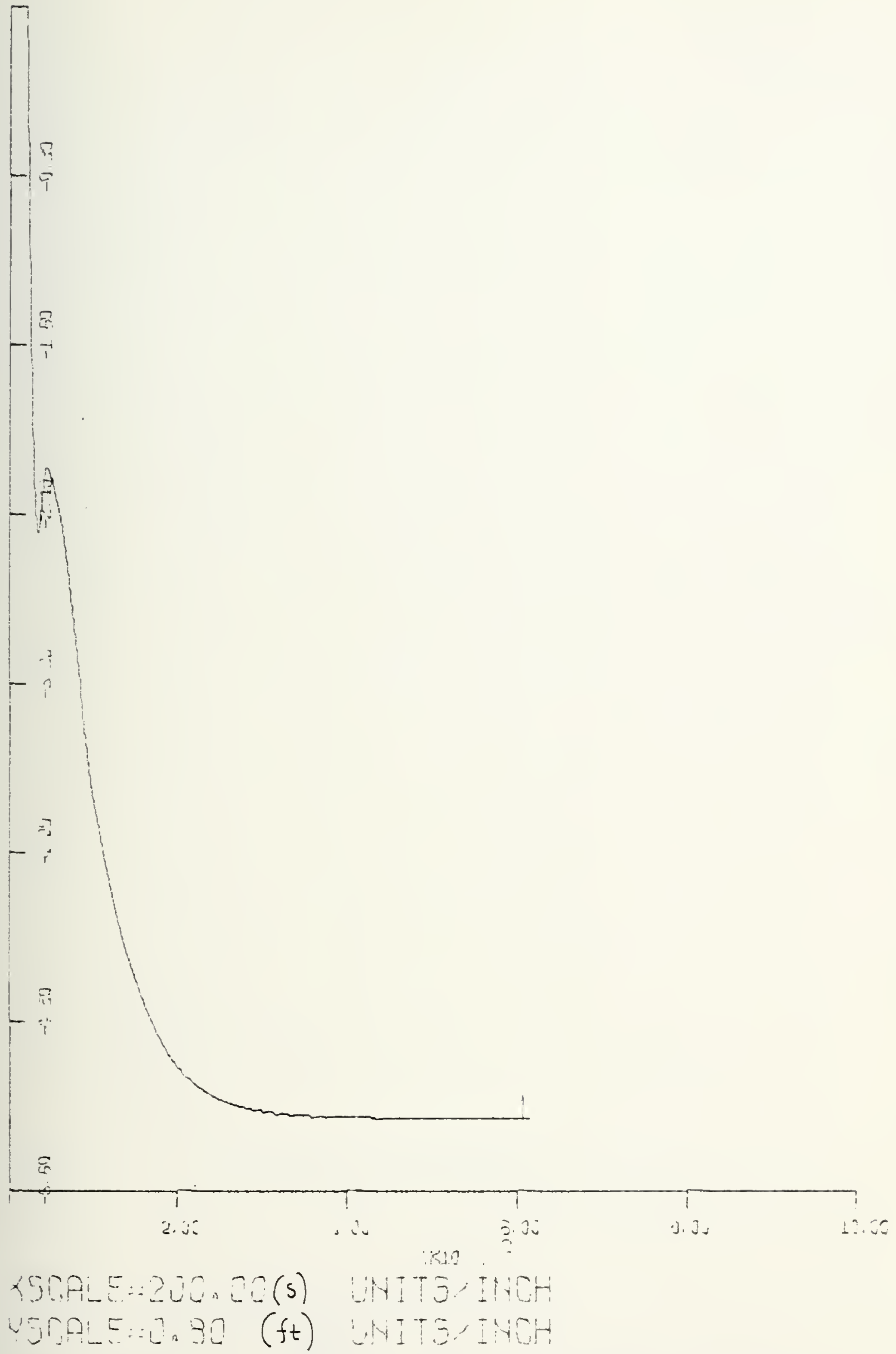


Fig. IV-12a. Depth vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.9

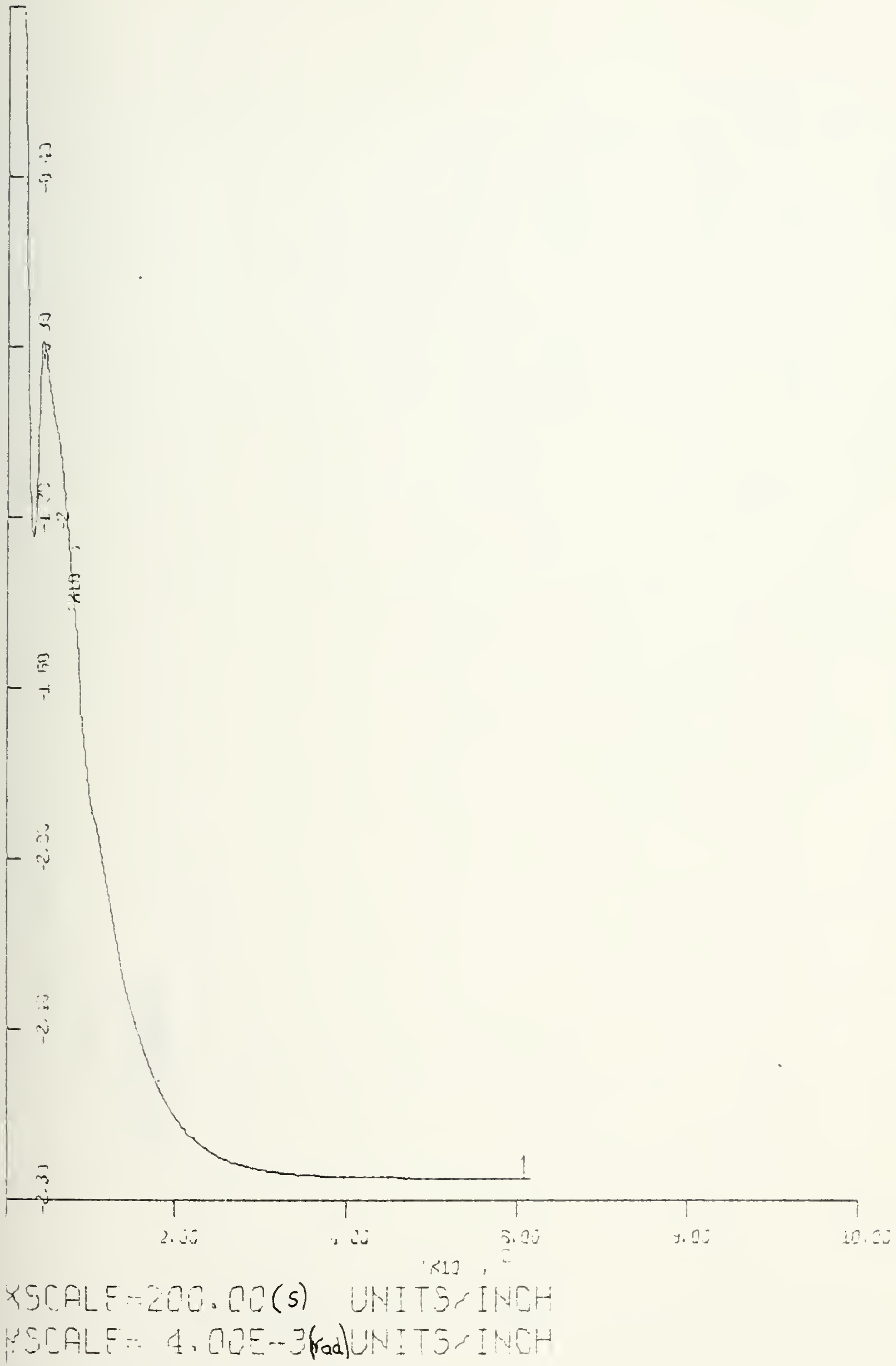


Fig. IV-12b. Pitch vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.9

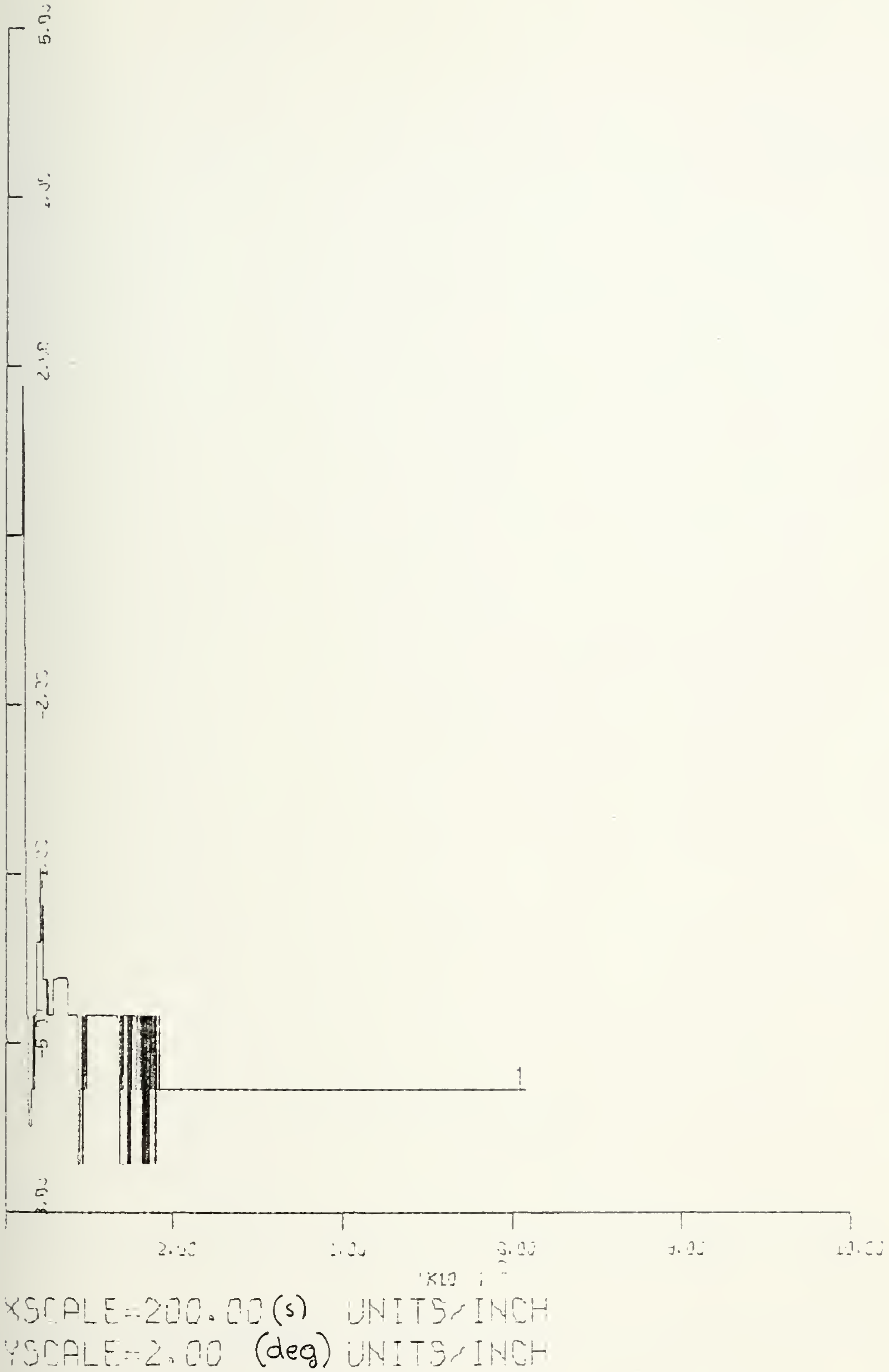


Fig. IV-12c. Stern Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.9

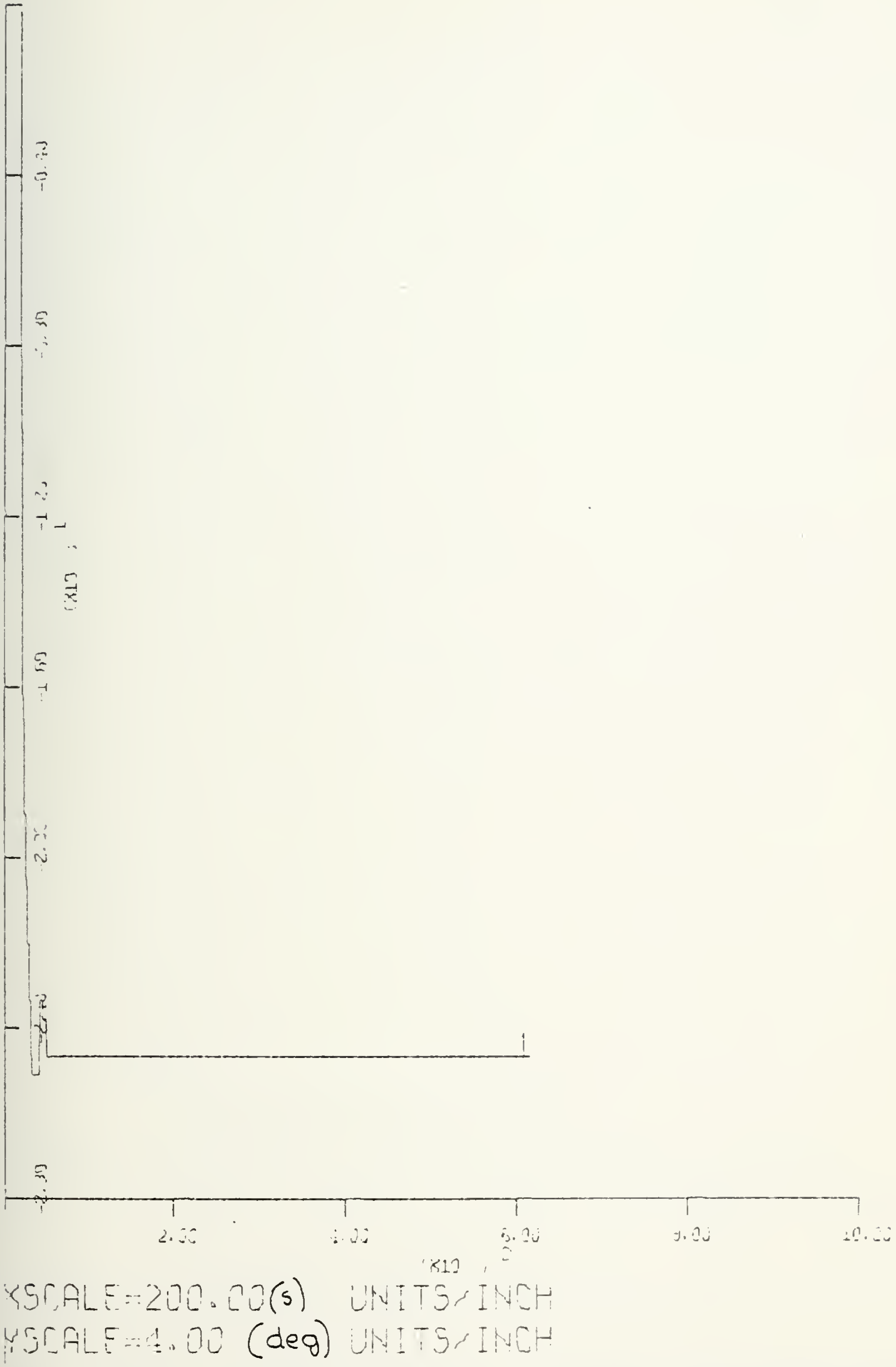


Fig. IV-12d. Fairwater Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.9

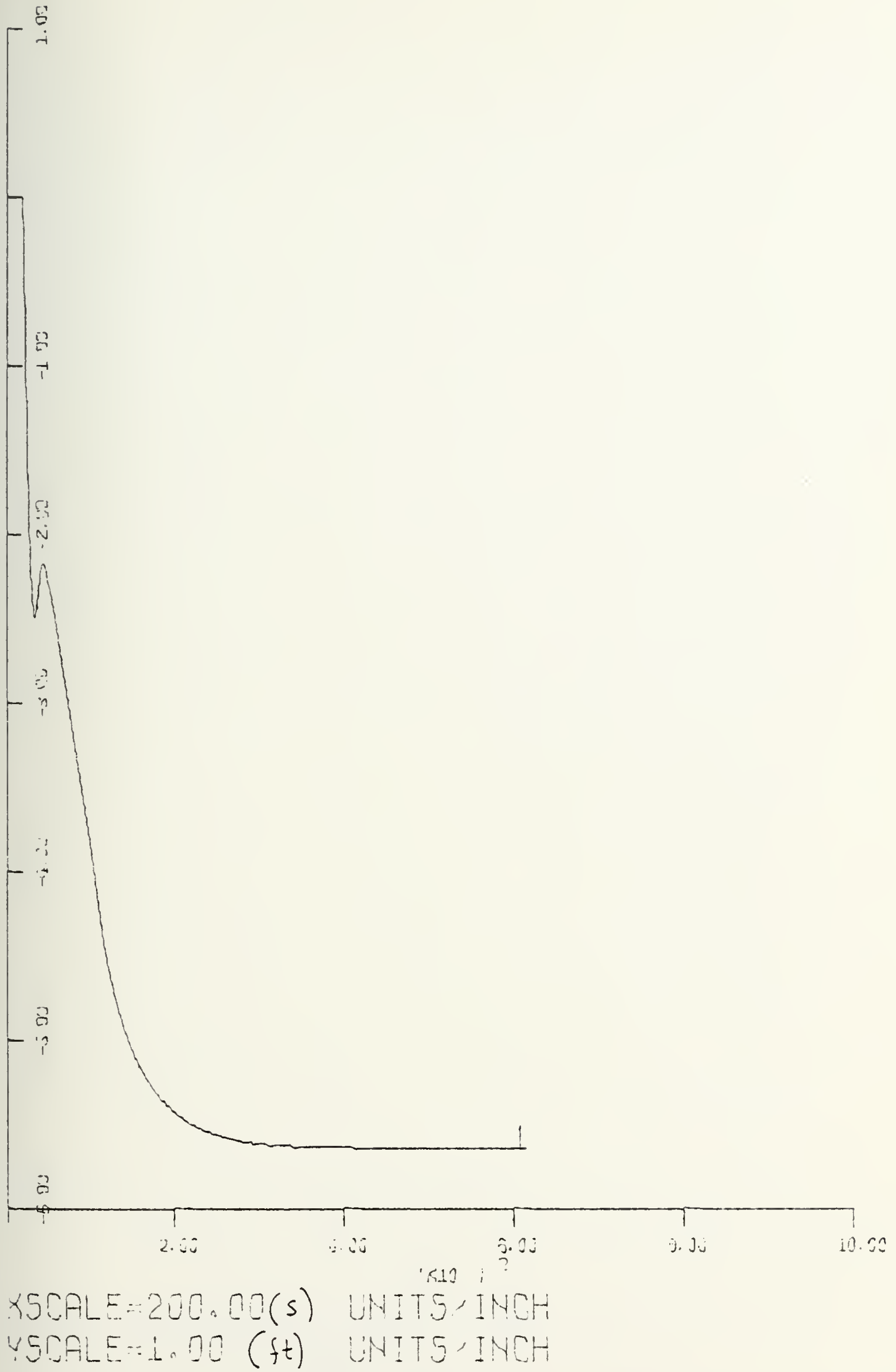
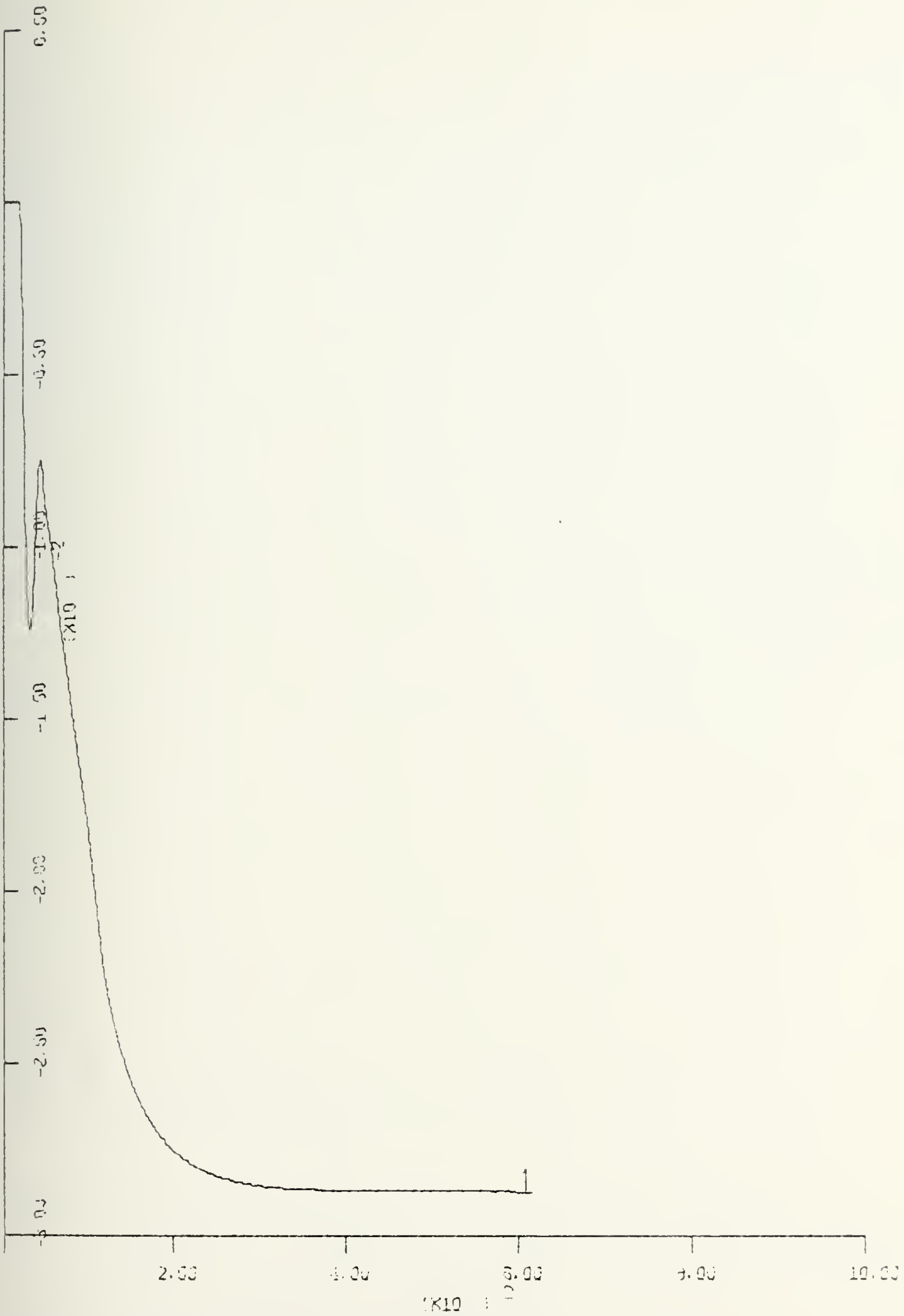


Fig. IV-13a. Depth vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.7



XSCALE=200.00(s) UNITS/INCH
YSCALE= 5.00E-3(rad)UNITS/INCH

Fig. IV-13b. Pitch vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.7

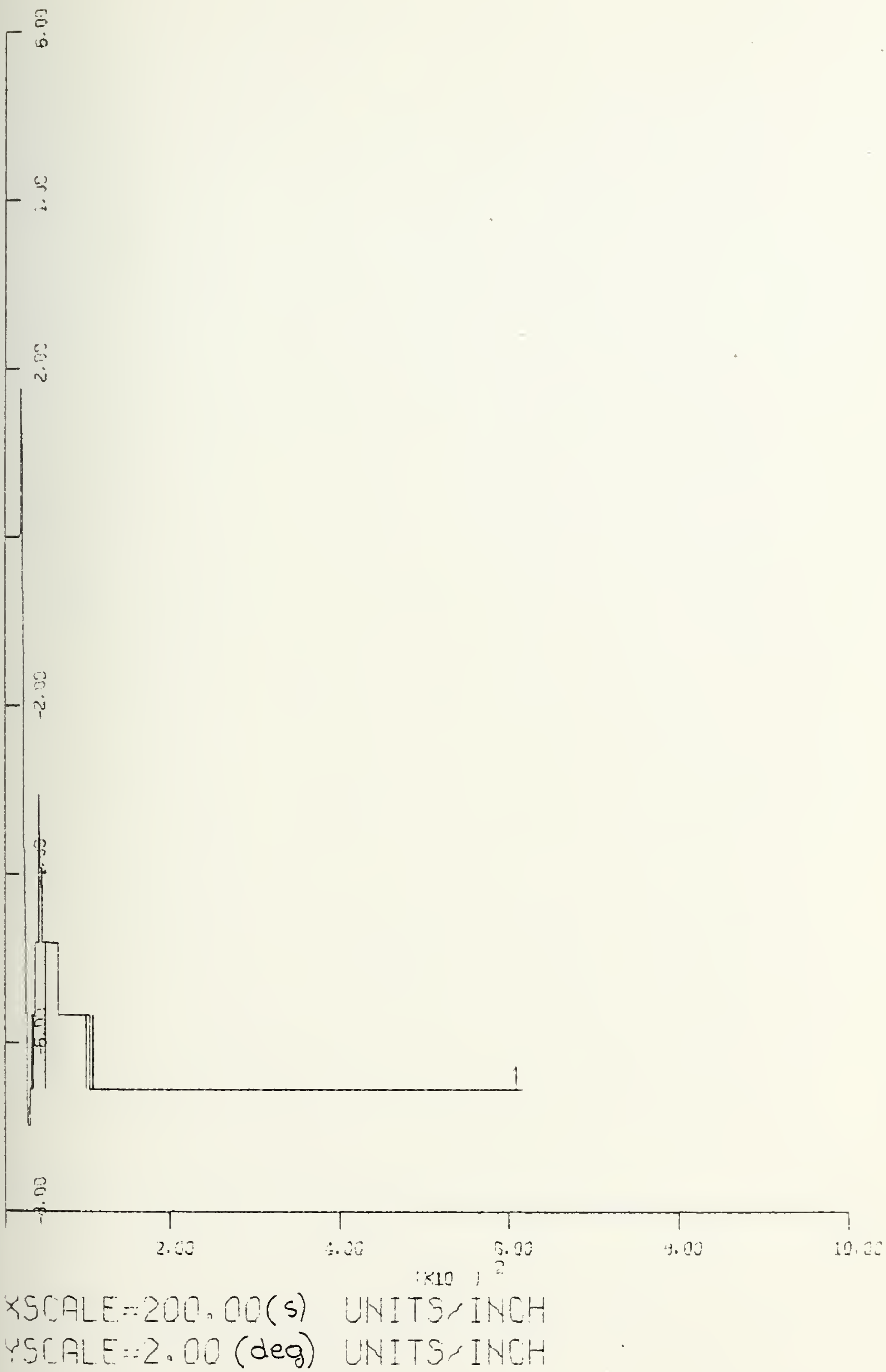


Fig. IV-13c. Stern Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.7

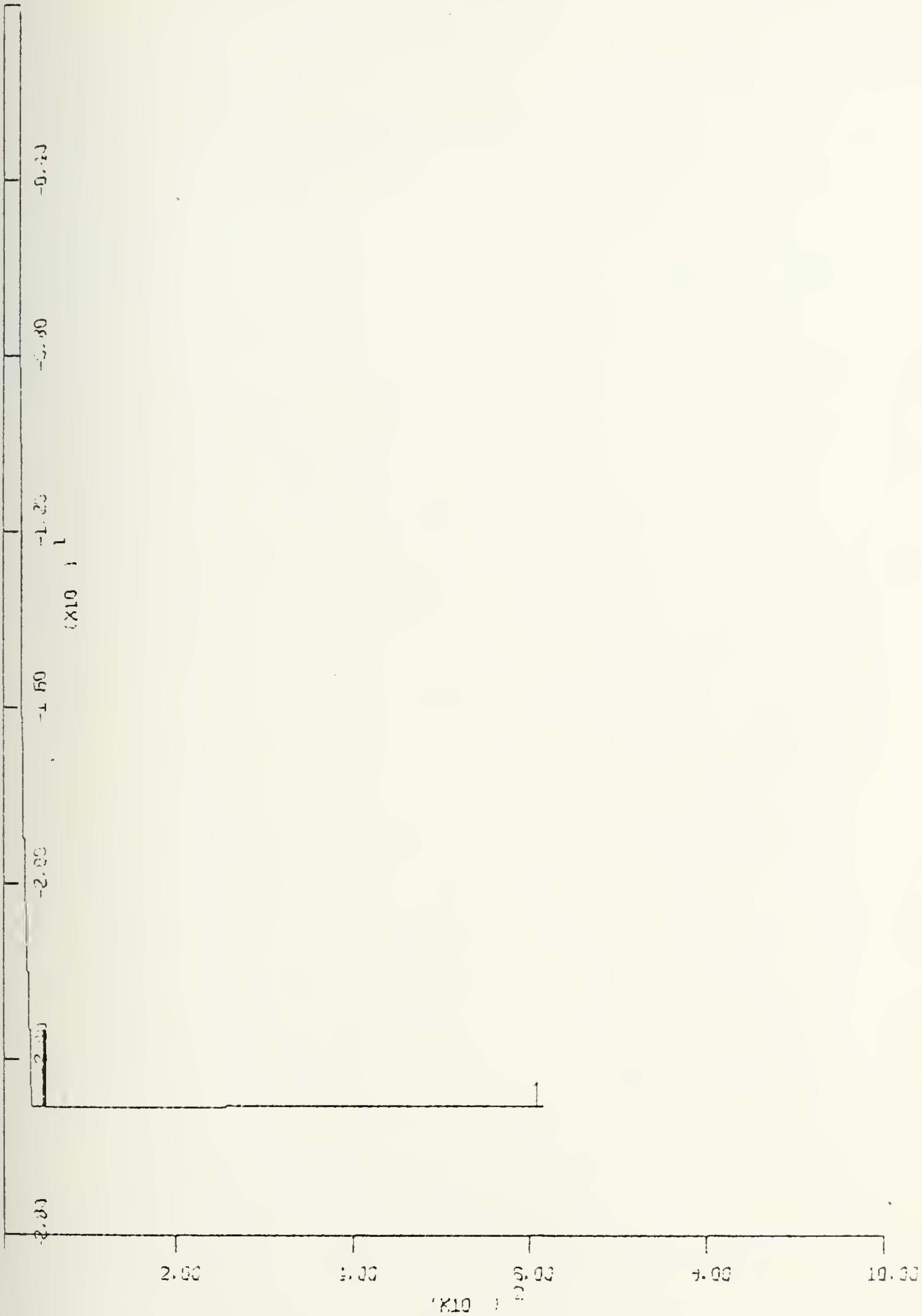


Fig. IV-13d. Fairwater Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.7

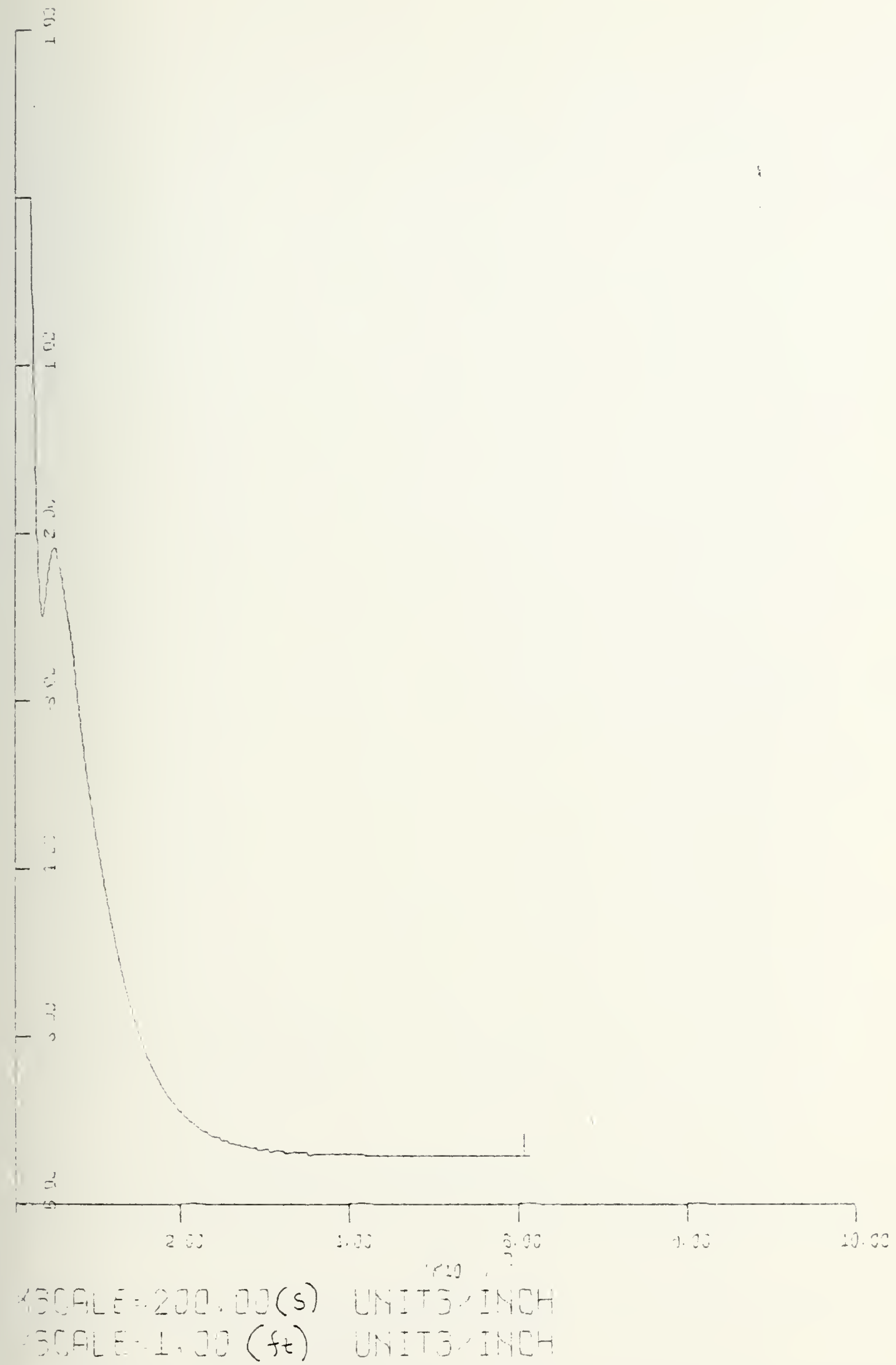


Fig. IV-14a. Depth vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.5

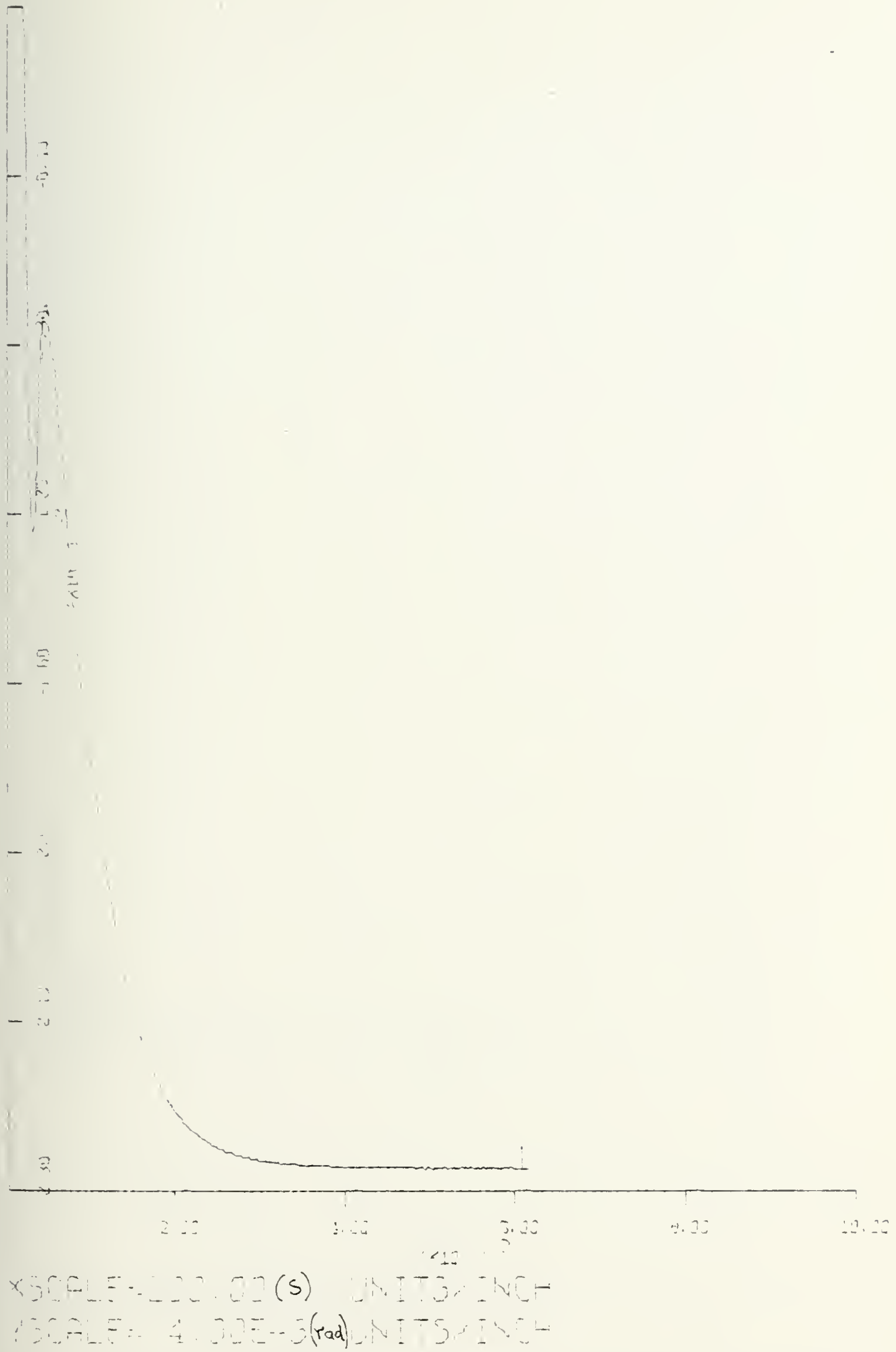


Fig. IV-14b. Pitch vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.5

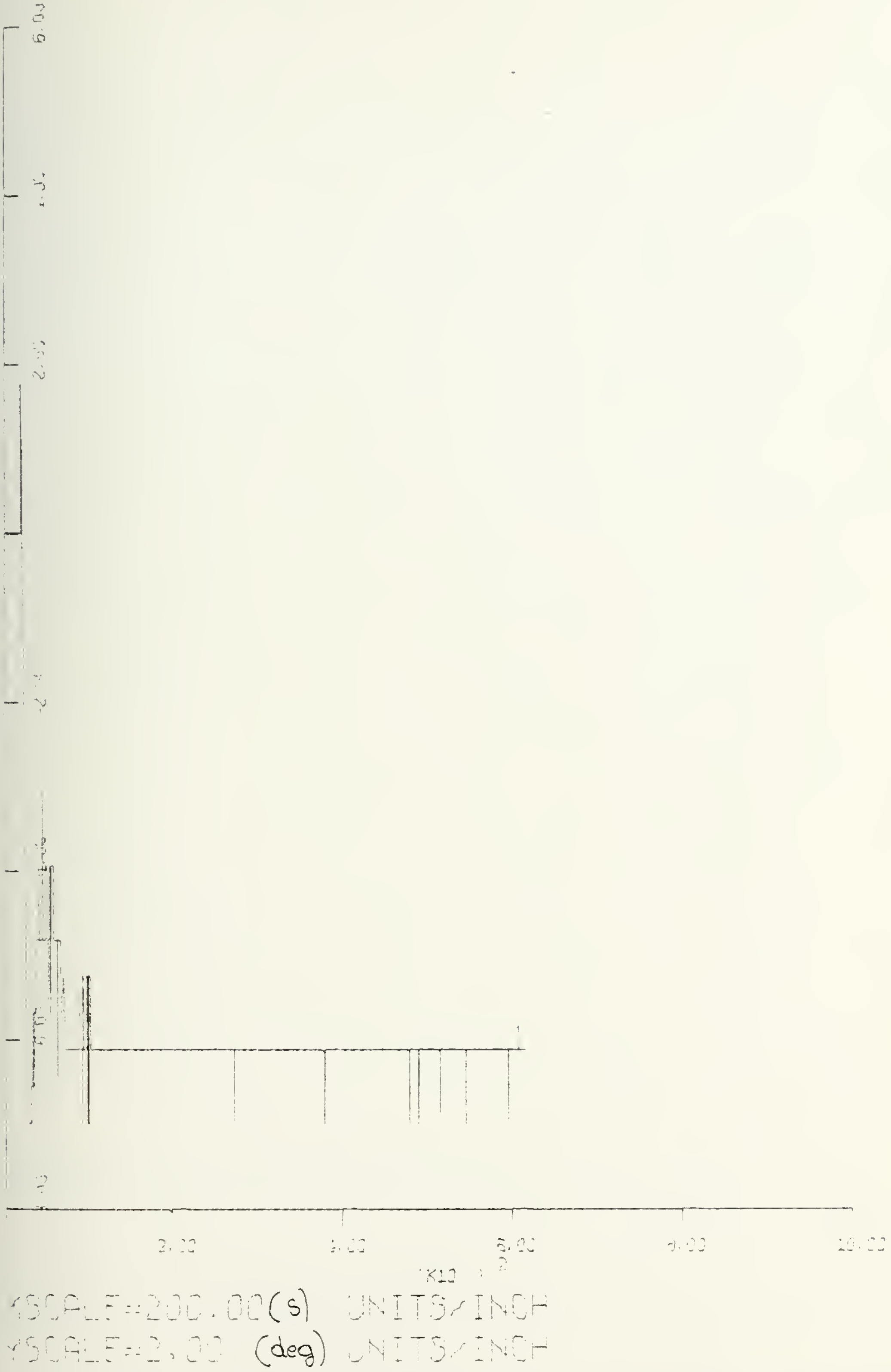


Fig. IV-14c. Stern Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.5

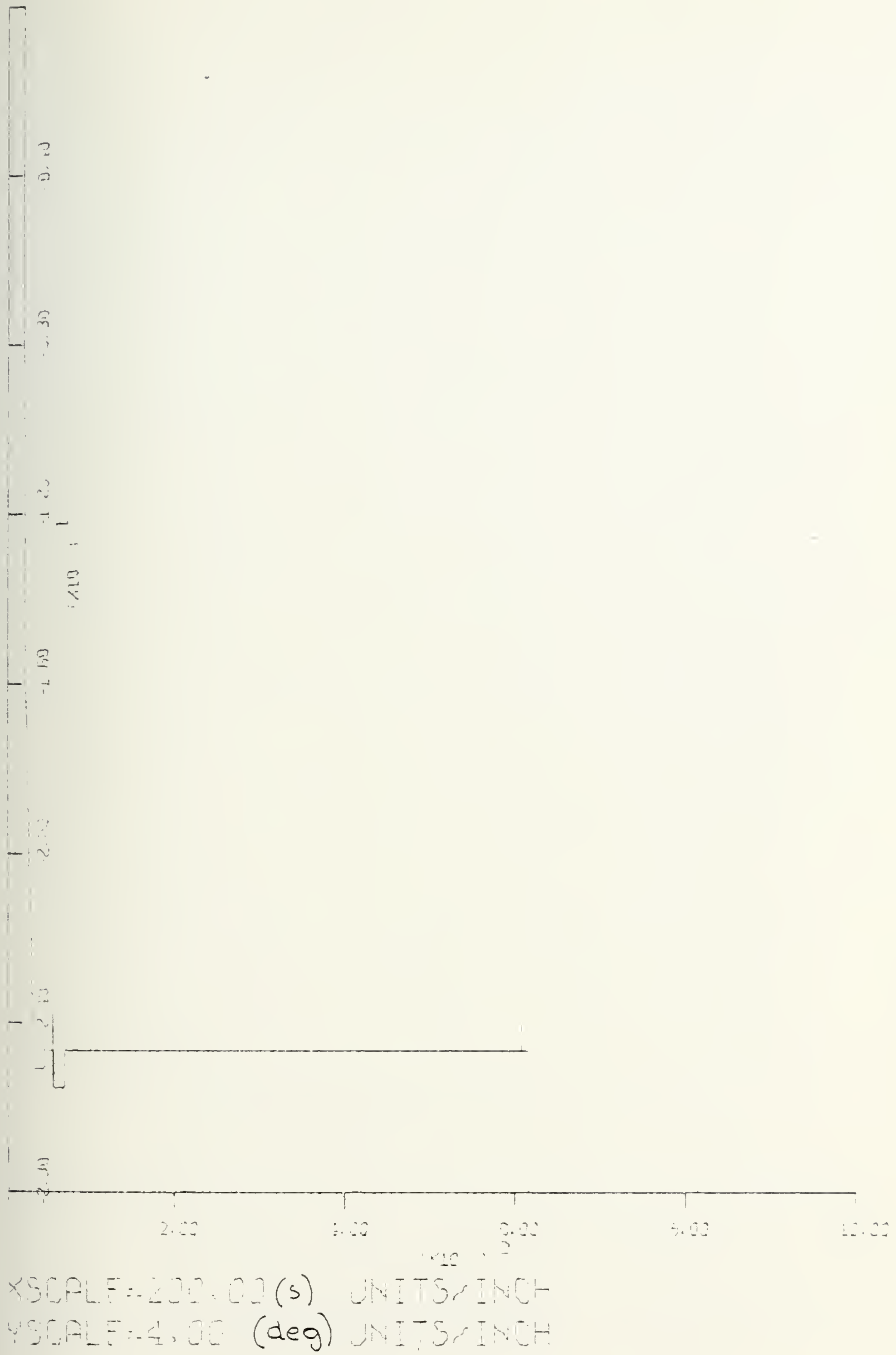


Fig. IV-14d. Fairwater Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.5

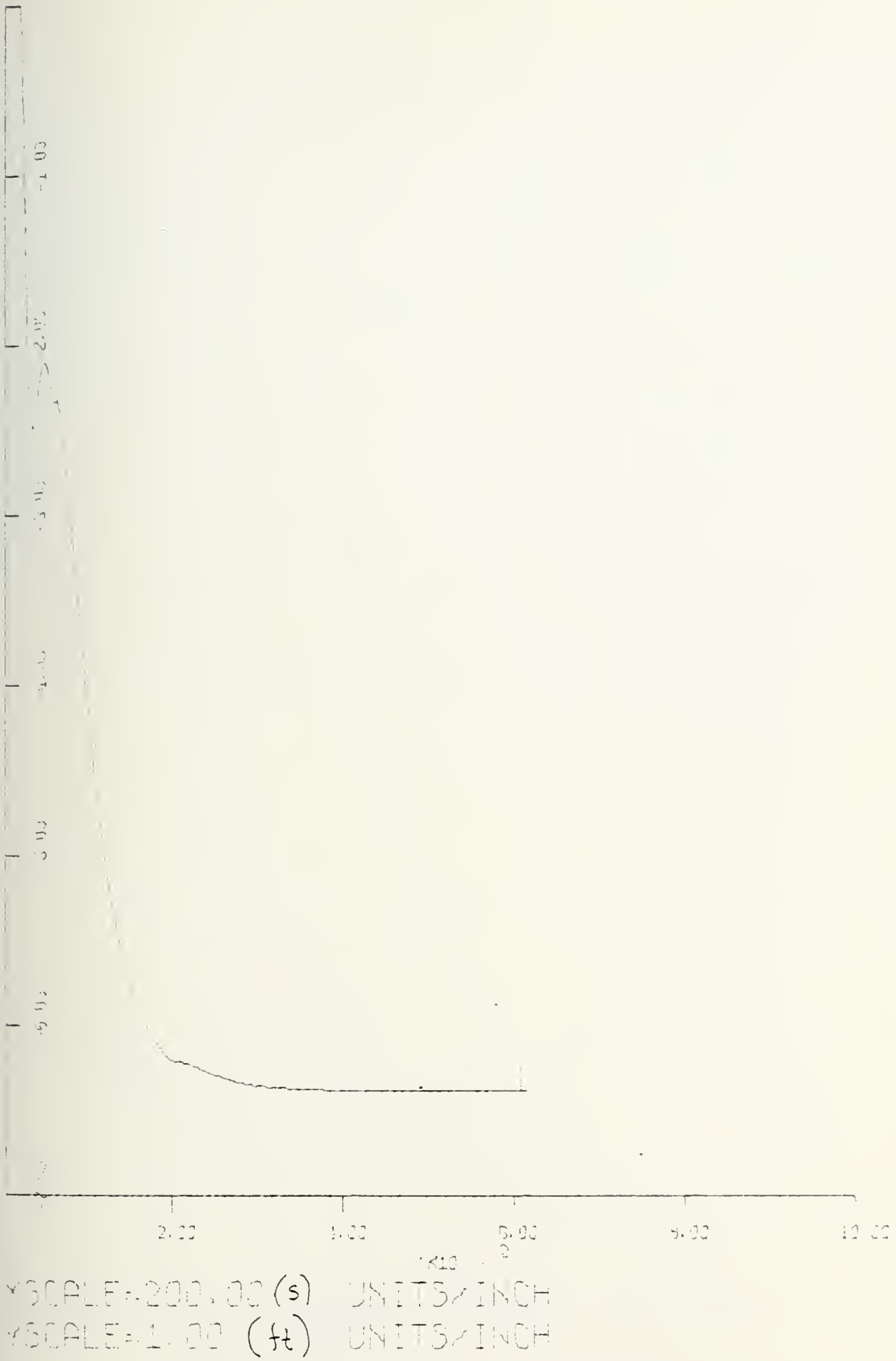


Fig. IV-15a. Depth vs. Time. Response to a step force at at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.3

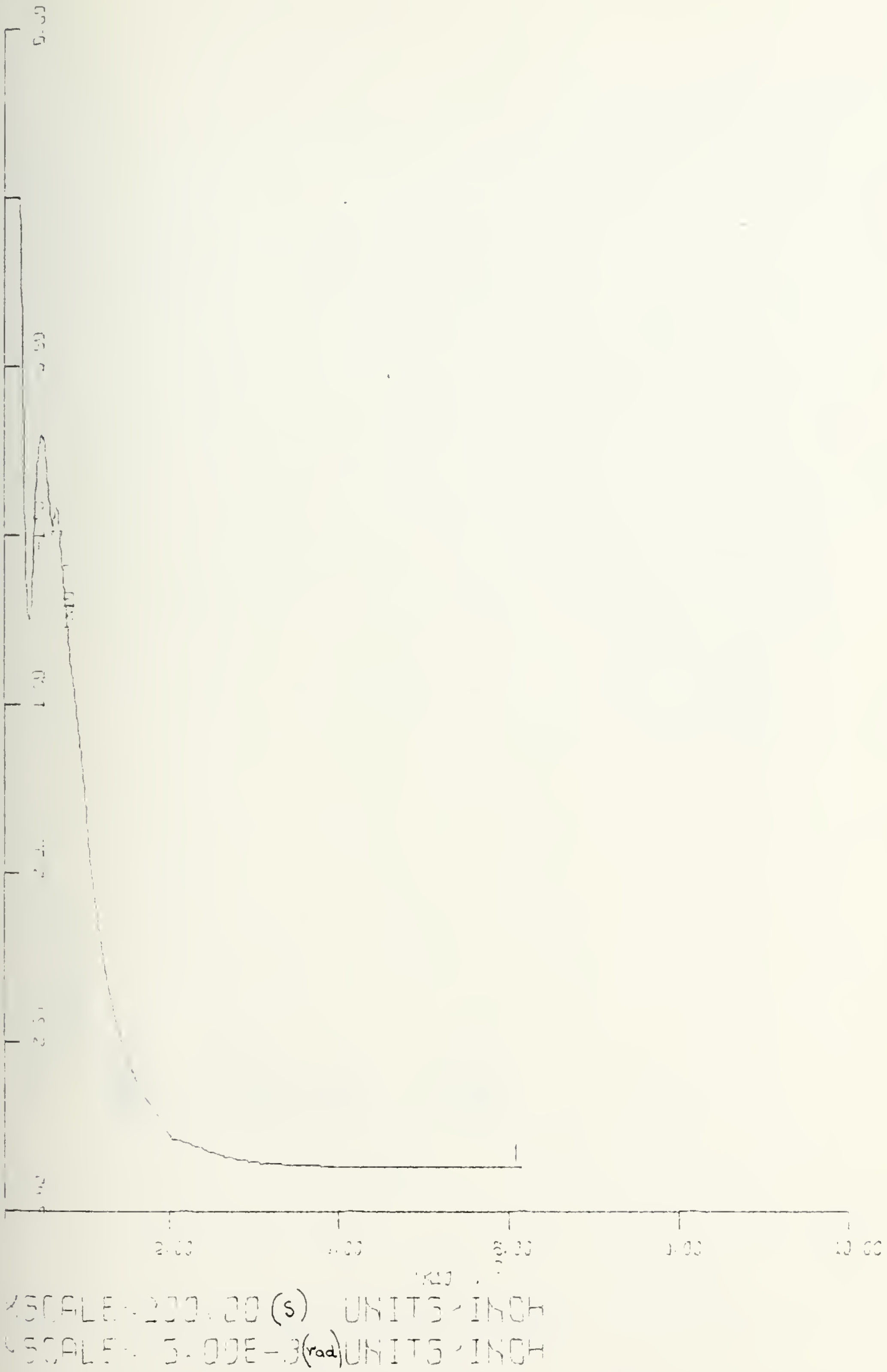


Fig. IV-15b. Pitch vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.3

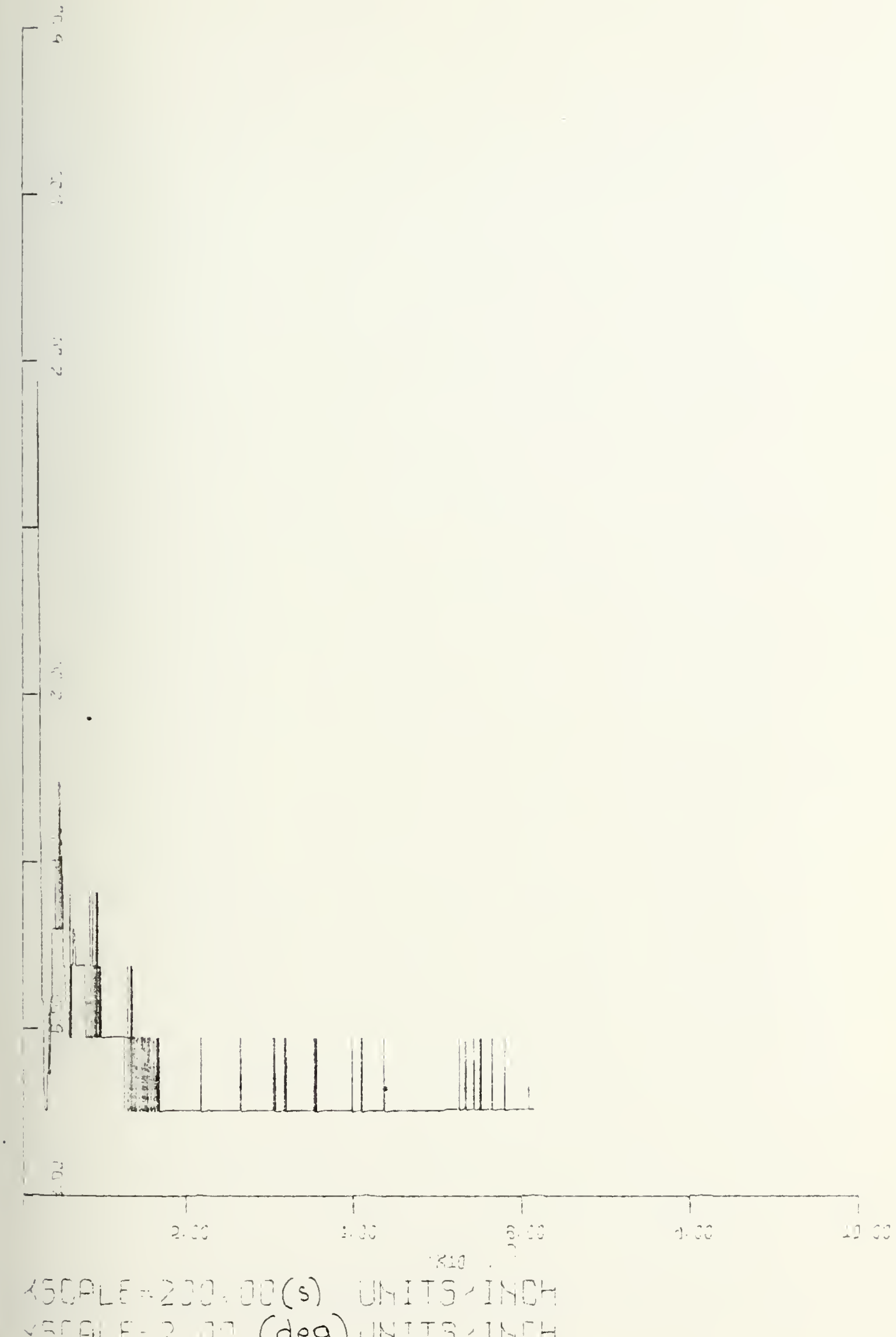


Fig. IV-15c. Stern Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.3

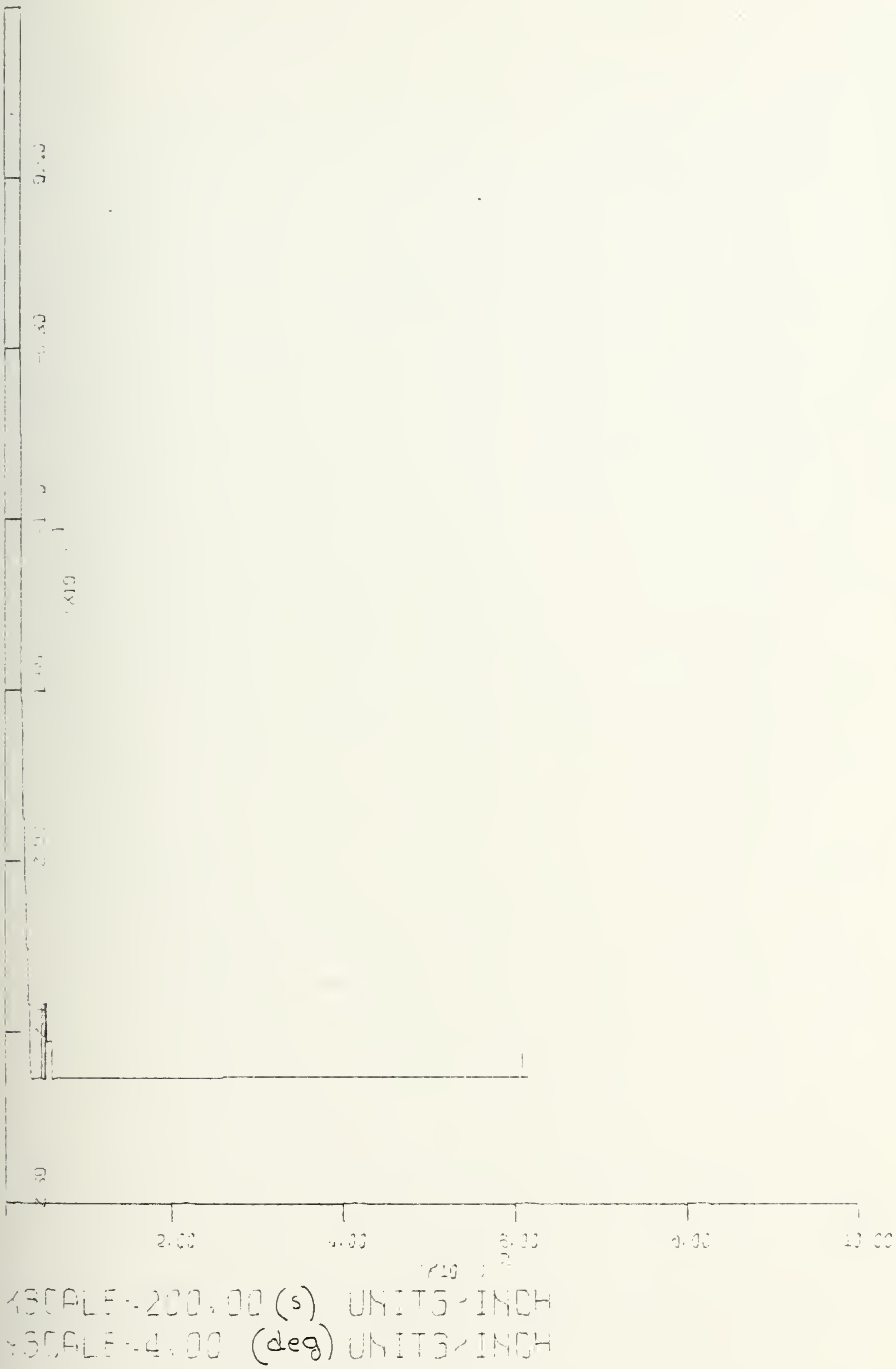


Fig. IV-15d. Fairwater Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.3

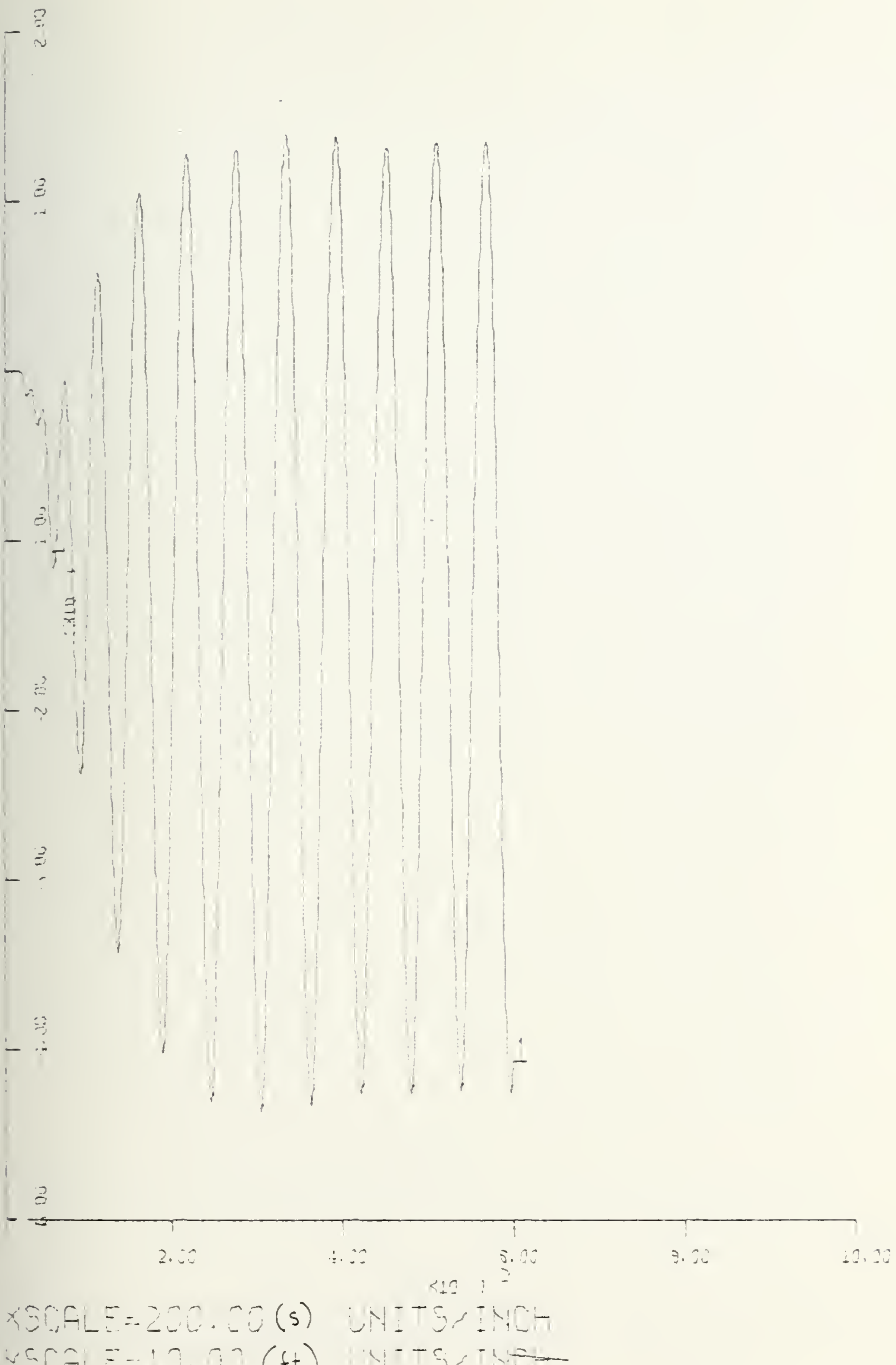
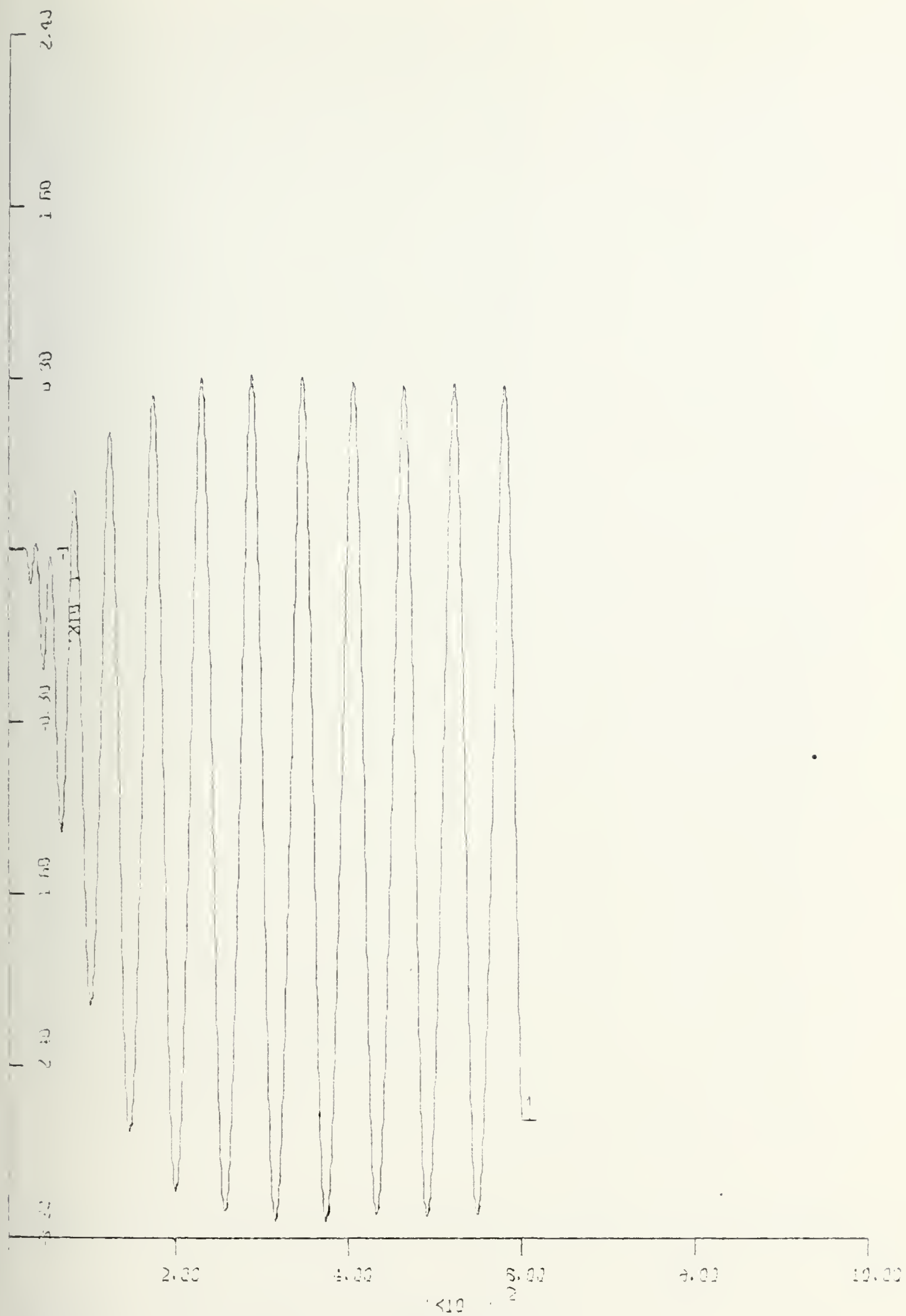


Fig. IV-16a. Depth vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=164, C=0.001, E=0.001. Parameter X=6



XSCALE=200.00(s) UNITS/INCH
YSCALE=0.08 (rad) UNITS/INCH

Fig. IV-16b. Pitch vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=164, C=0.001, E=0.001. Parameter X=6

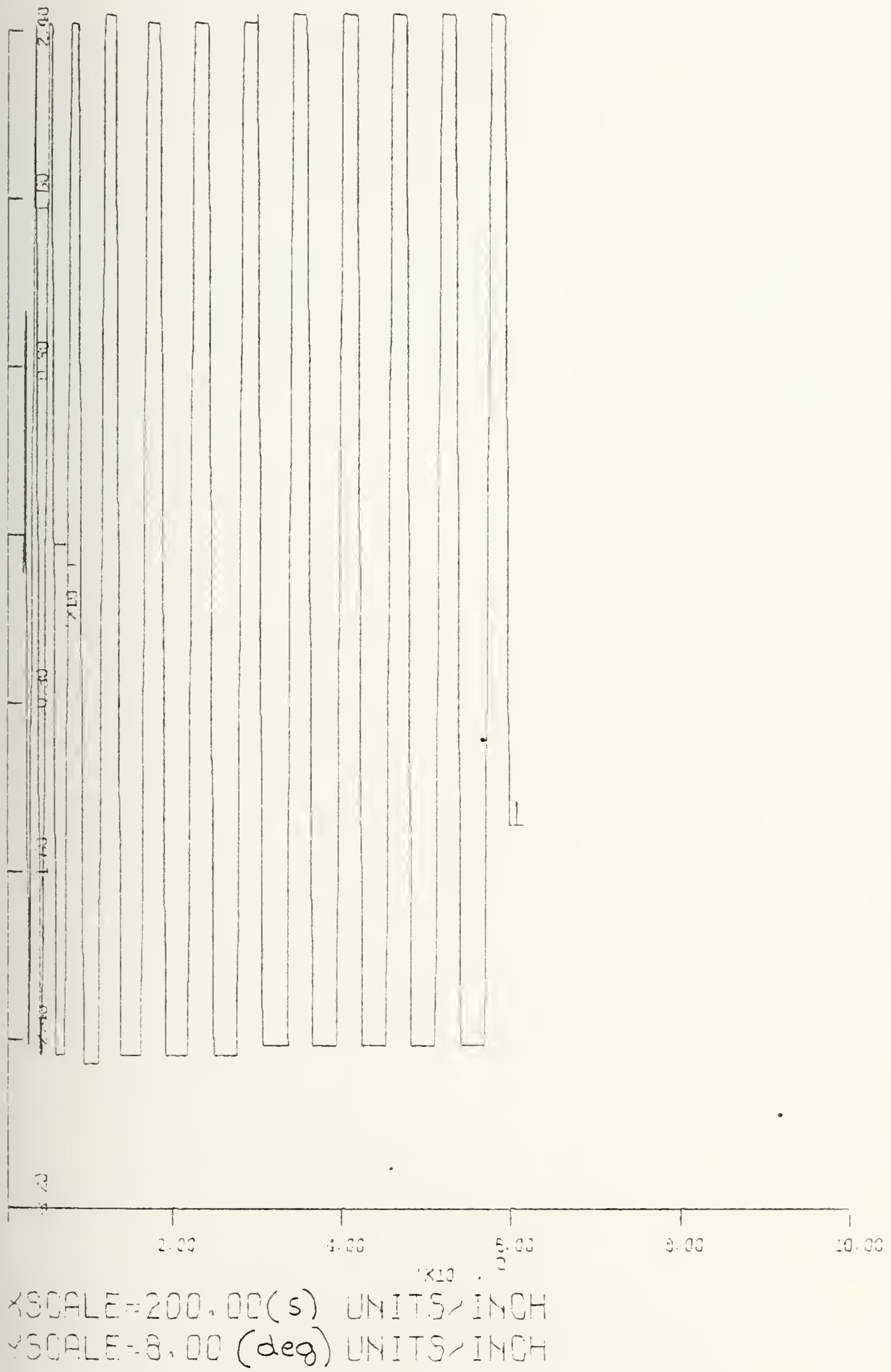


Fig. IV-16c. Stern Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=164, C=0.001, E=0.001. Parameter X=6

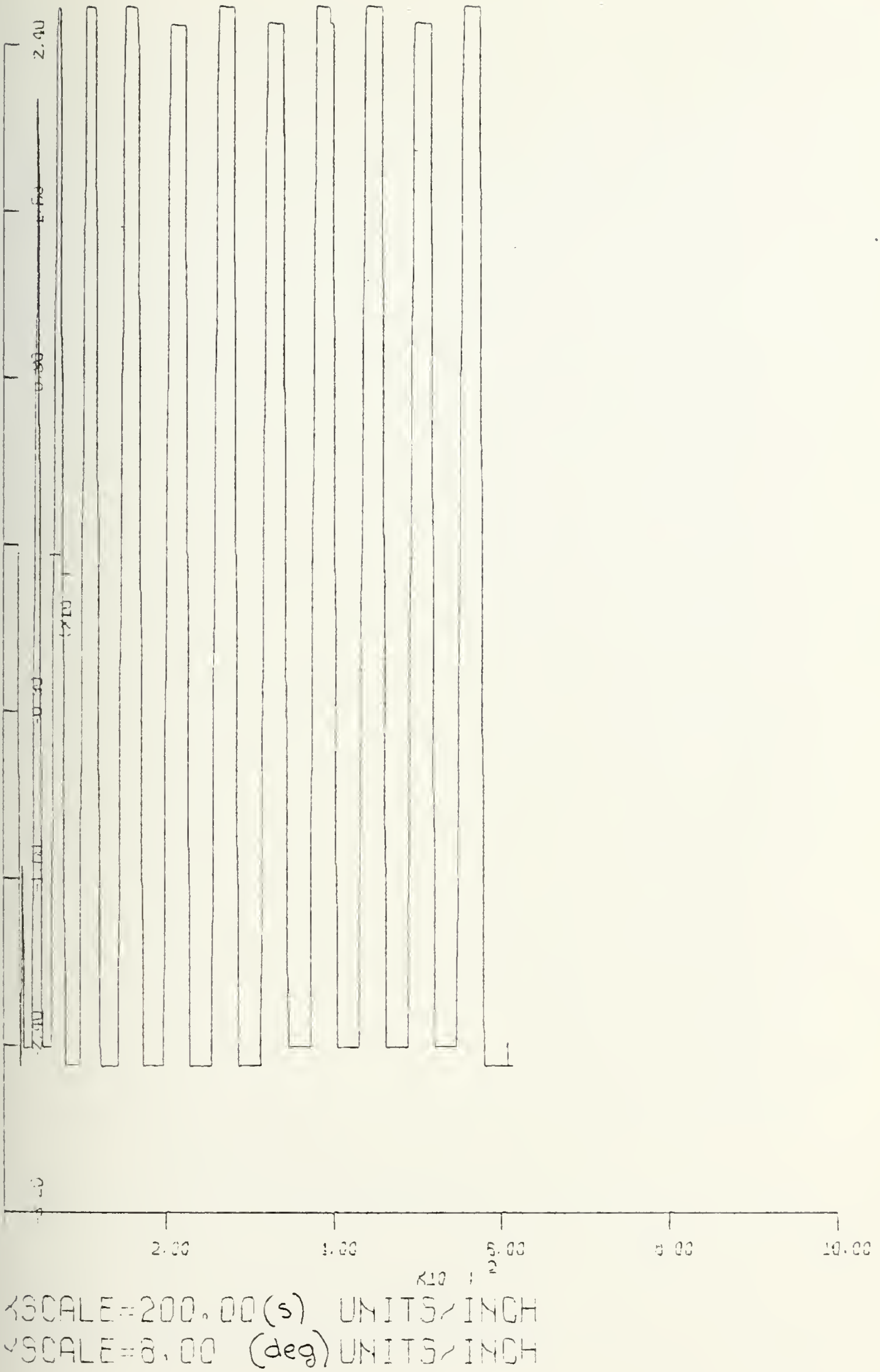


Fig. IV-16d. Fairwater Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=164, C=0.001, E=0.001. Parameter X=6

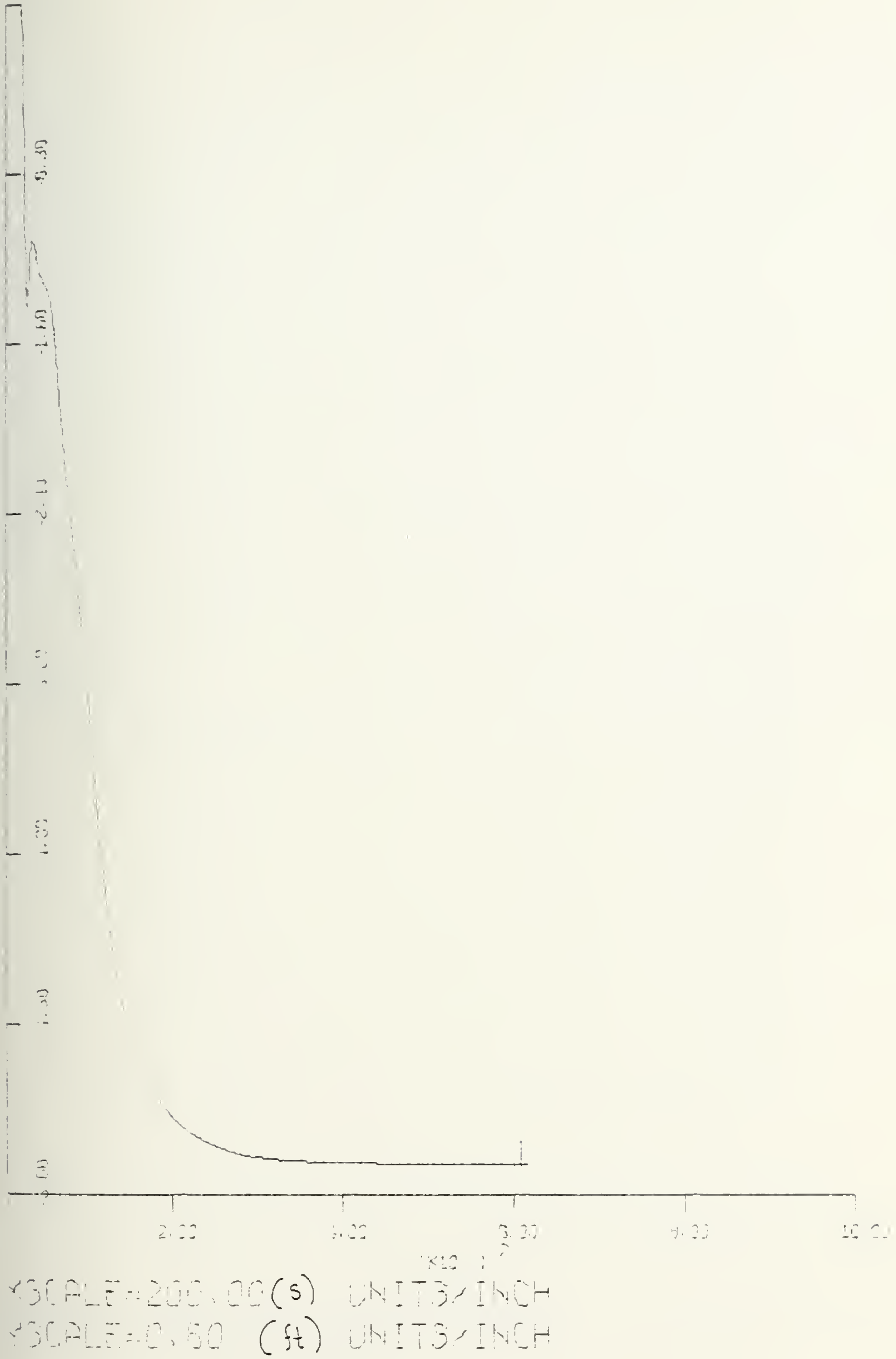


Fig. IV-17a. Depth vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=164, C=0.001, E=0.001. Parameter X=3.5

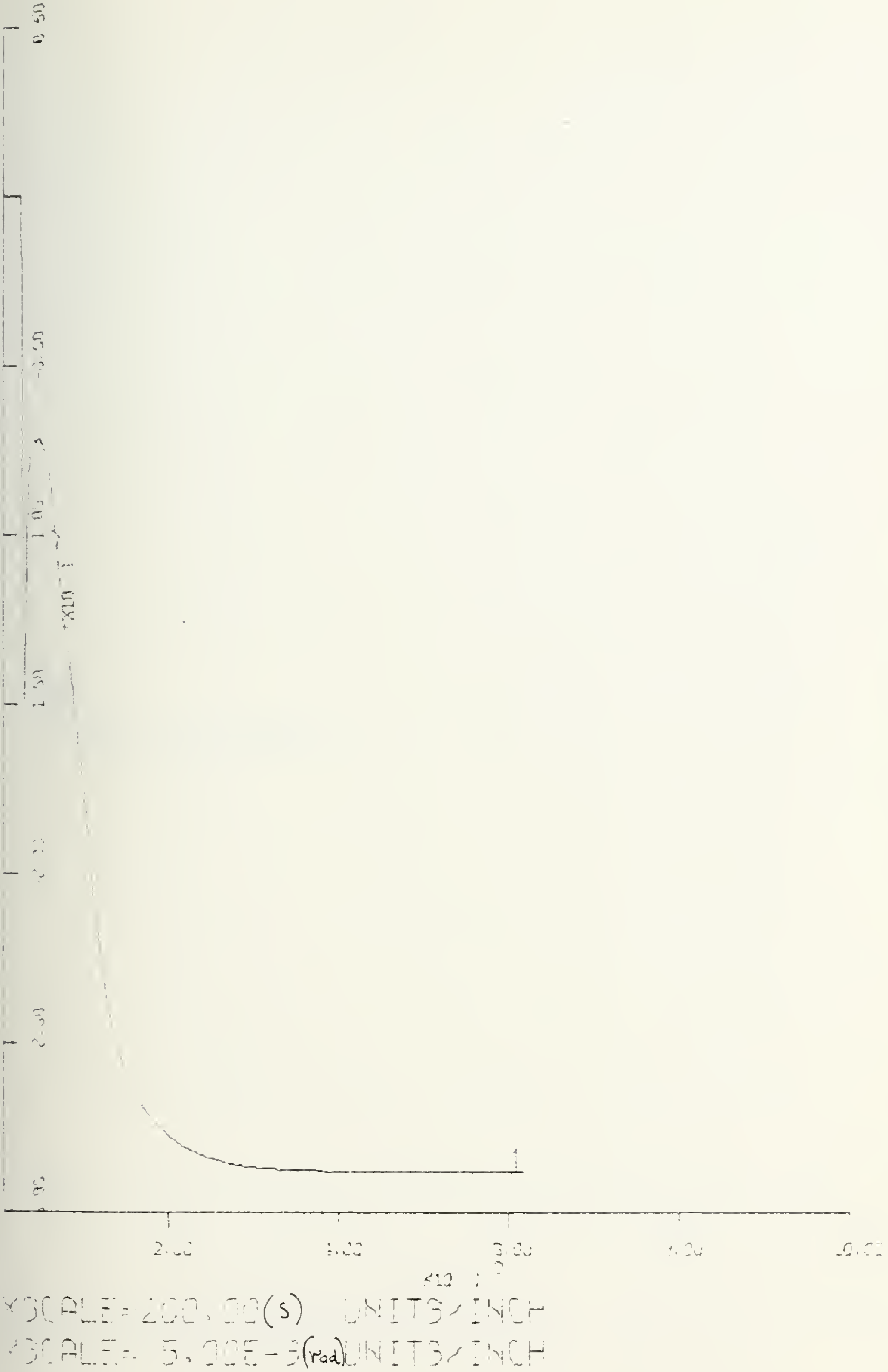


Fig. IV-17b. Pitch vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=164, C=0.001, E=0.001. Parameter X=3.5

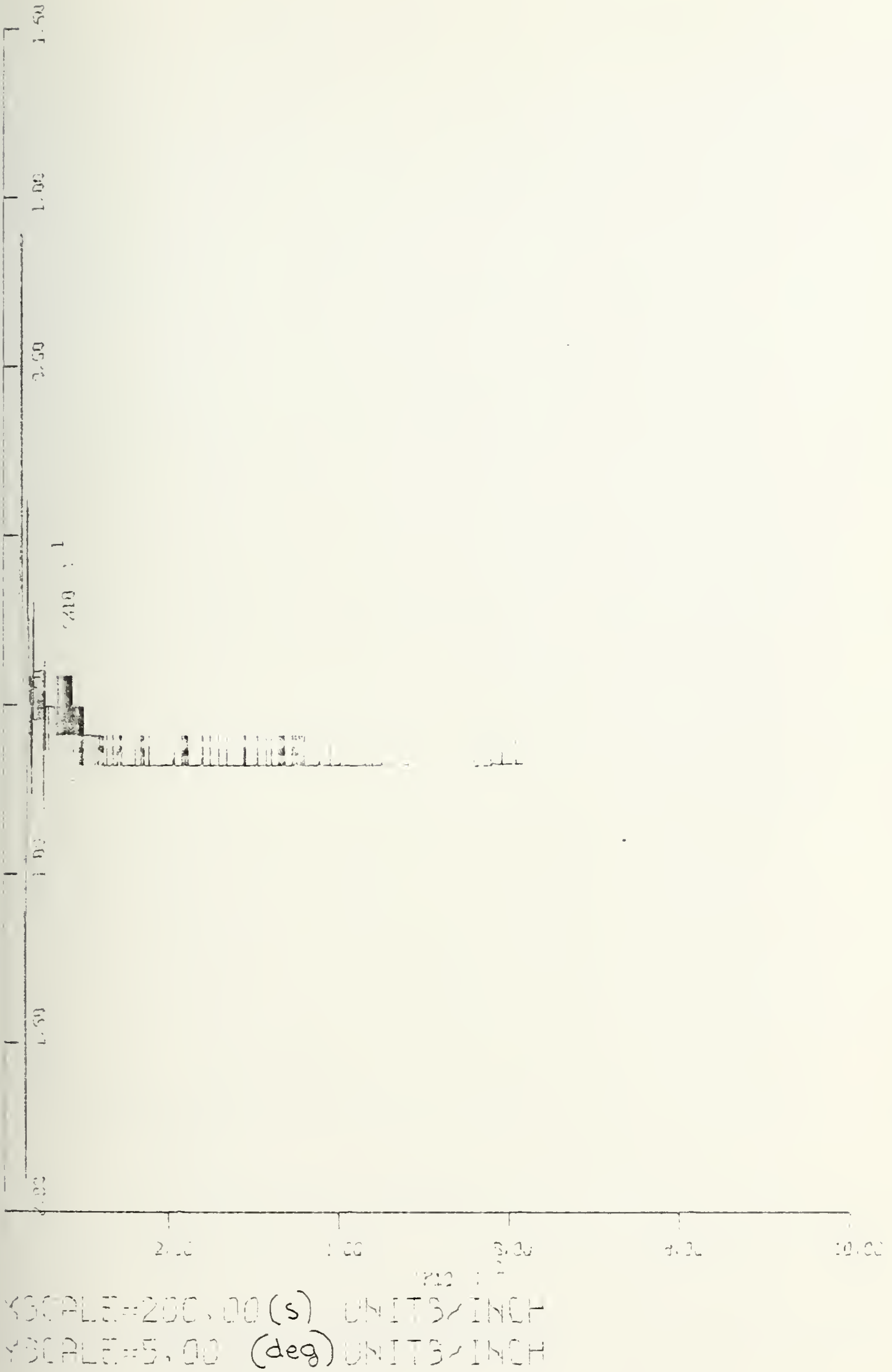


Fig. IV-17c. Stern Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=164, C=0.001, E=0.001. Parameter X=3.5

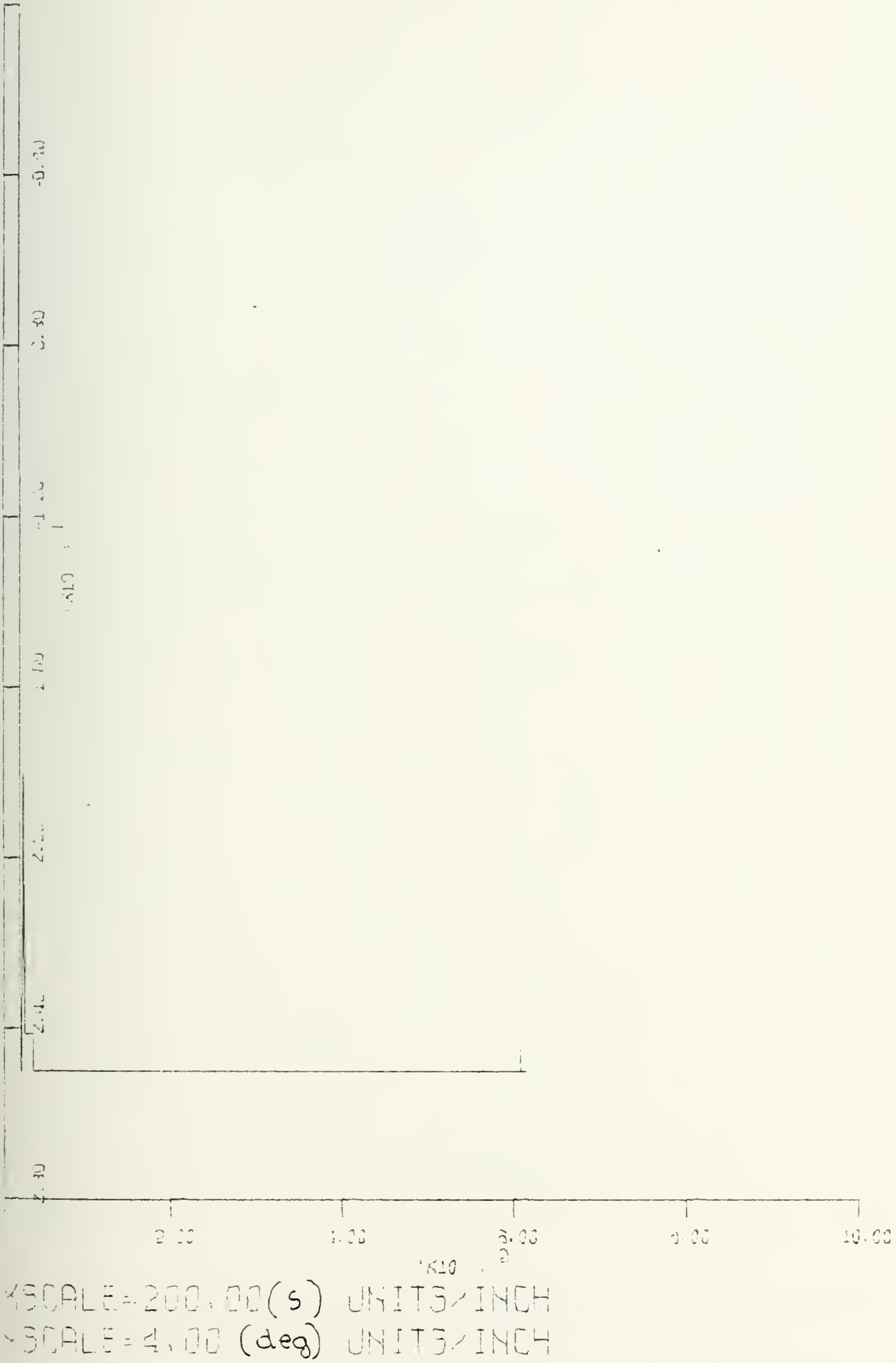


Fig. IV-17d. Fairwater Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=164, C=0.001, E=0.001. Parameter X=3.5

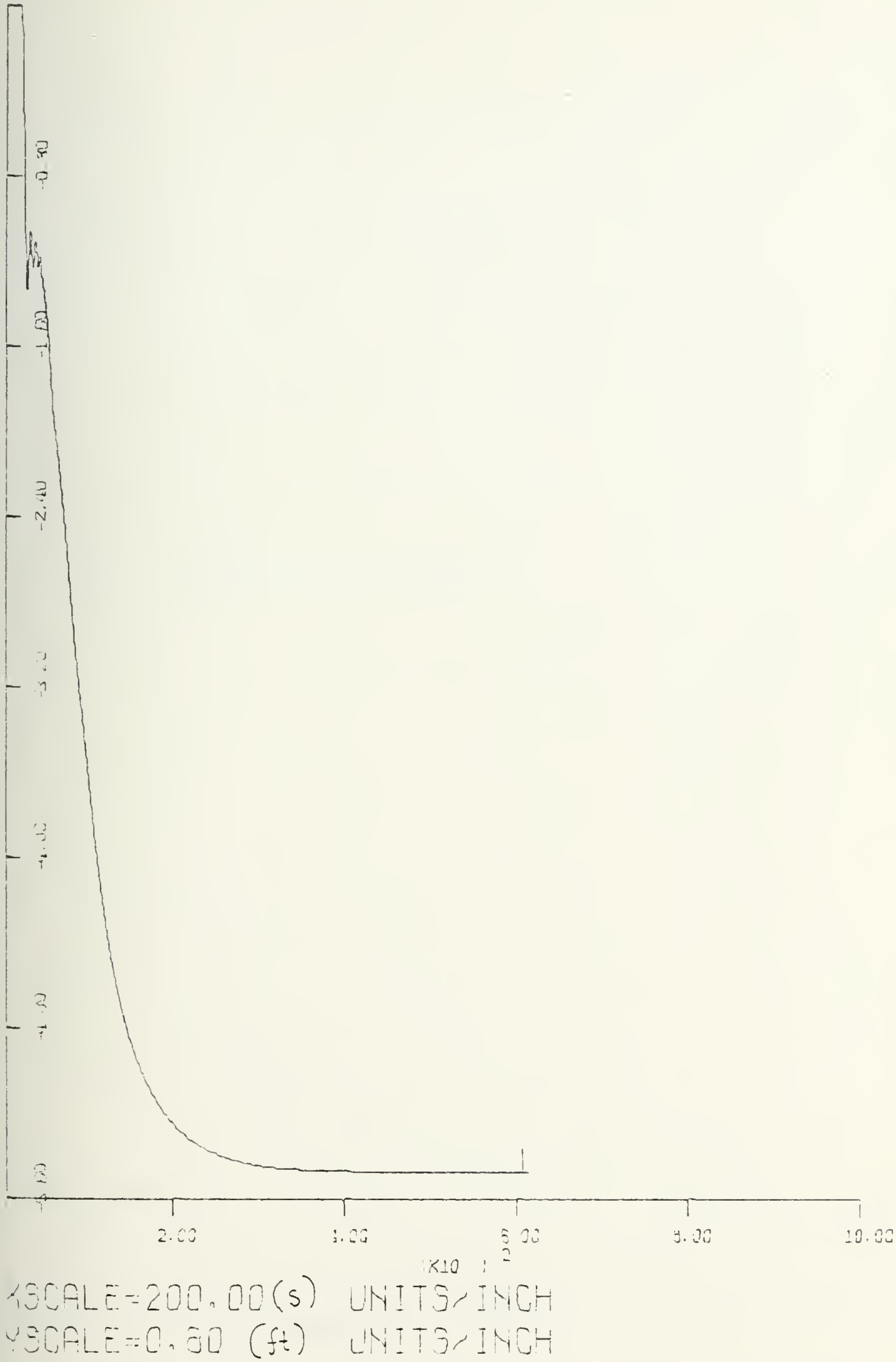


Fig. IV-18a. Depth vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=164, C=0.001, E=0.001. Parameter X=2.

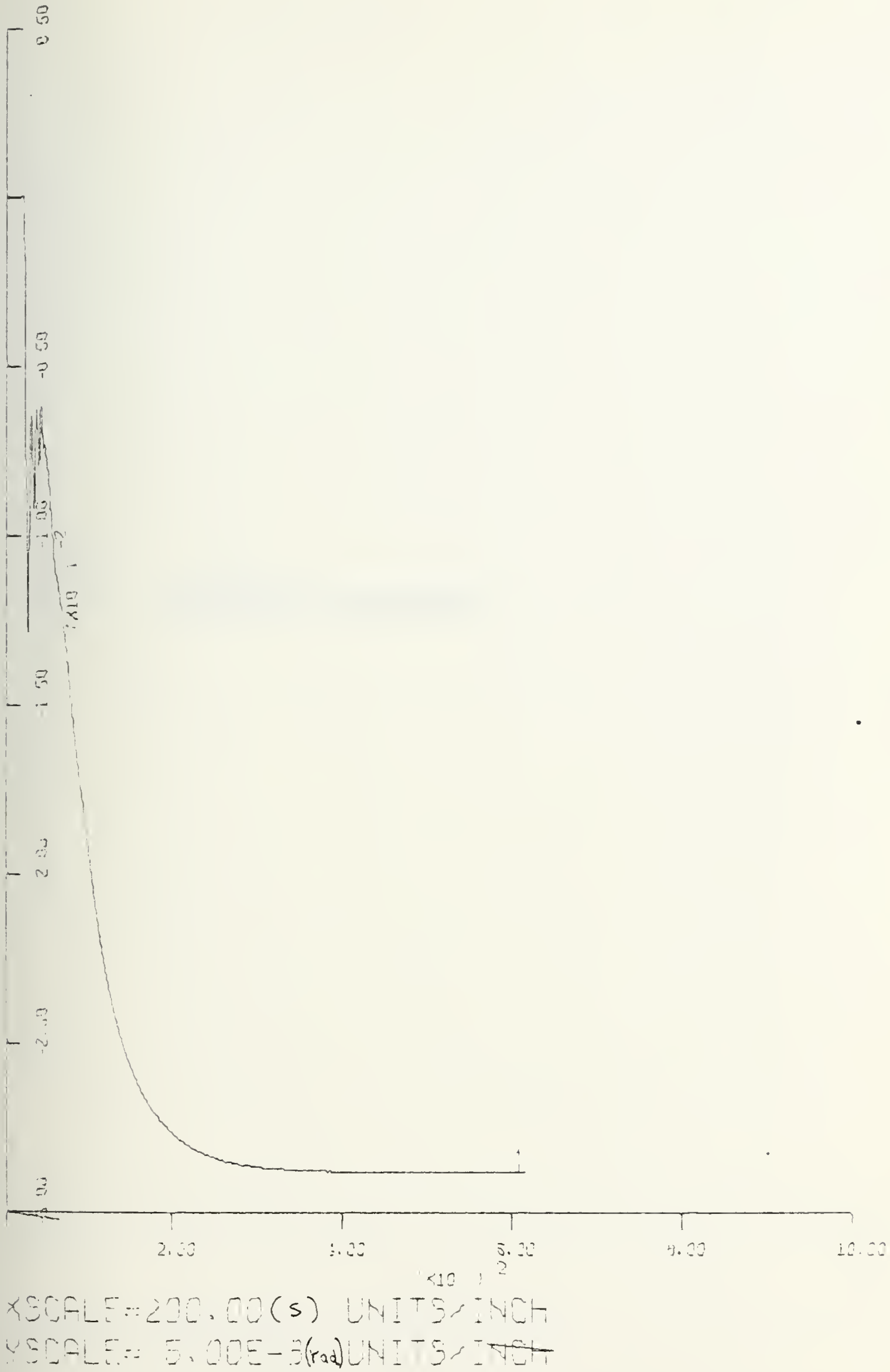


Fig. IV-18b. Pitch vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=164, C=0.001, E=0.001. Parameter X=2

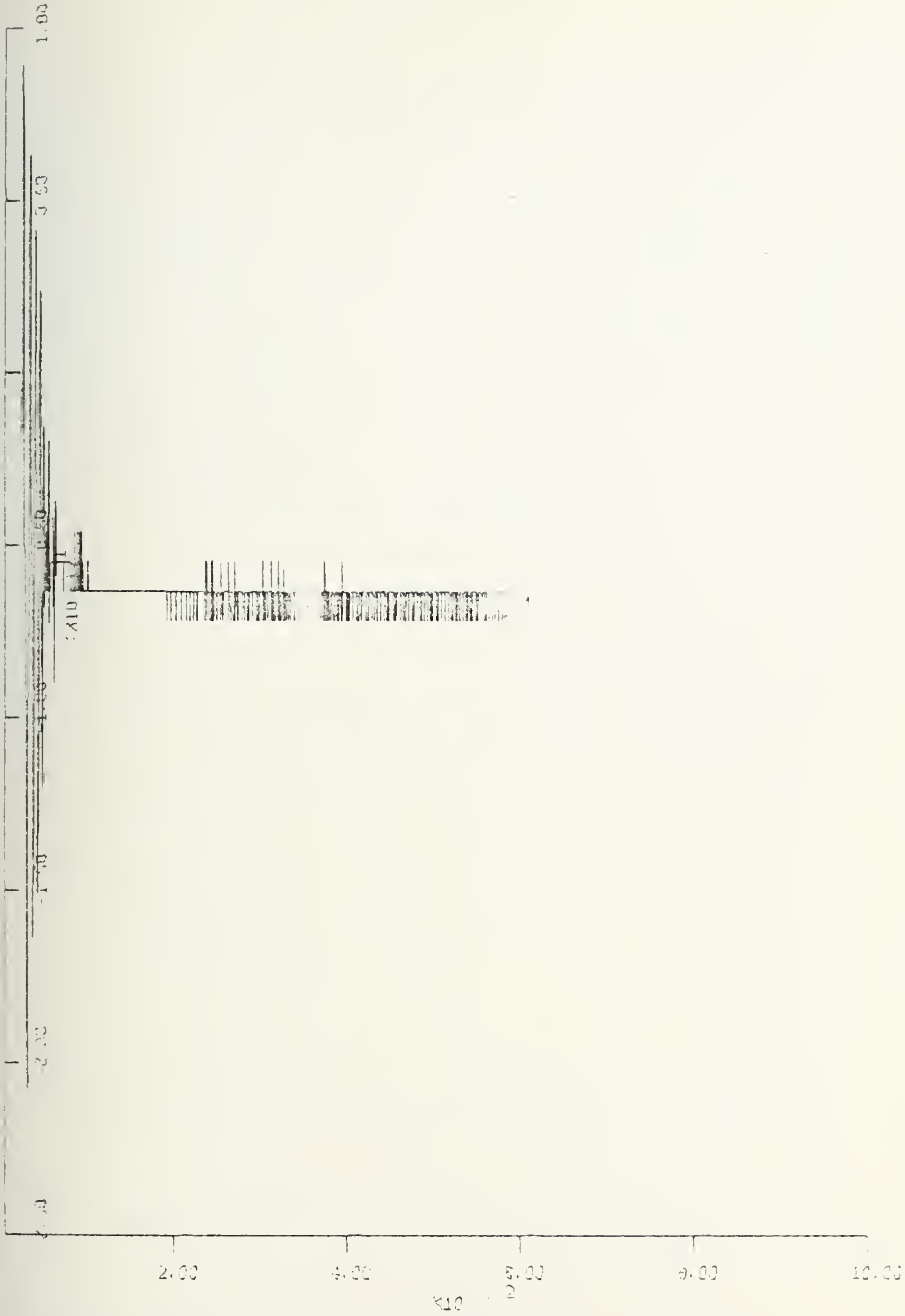


Fig. IV-18c. Stern Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=164, C=0.001, E=0.001. Parameter X=2

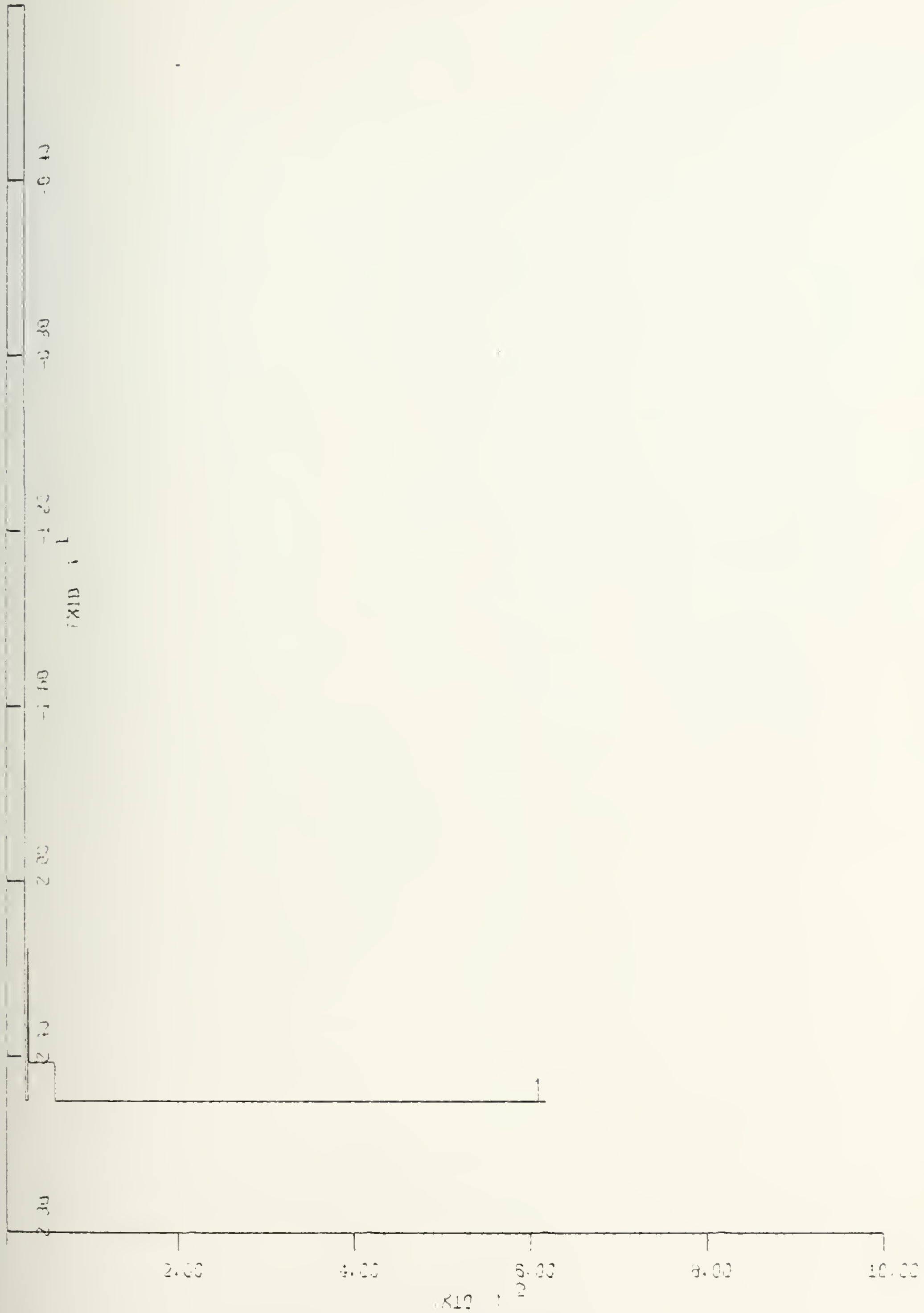


Fig. IV-18d. Fairwater Plane Angle vs. Time. Response to a step force at AU. CMC uses optimal design in combined controllers with B=164, C=0.001, E=0.001. Parameter X=2

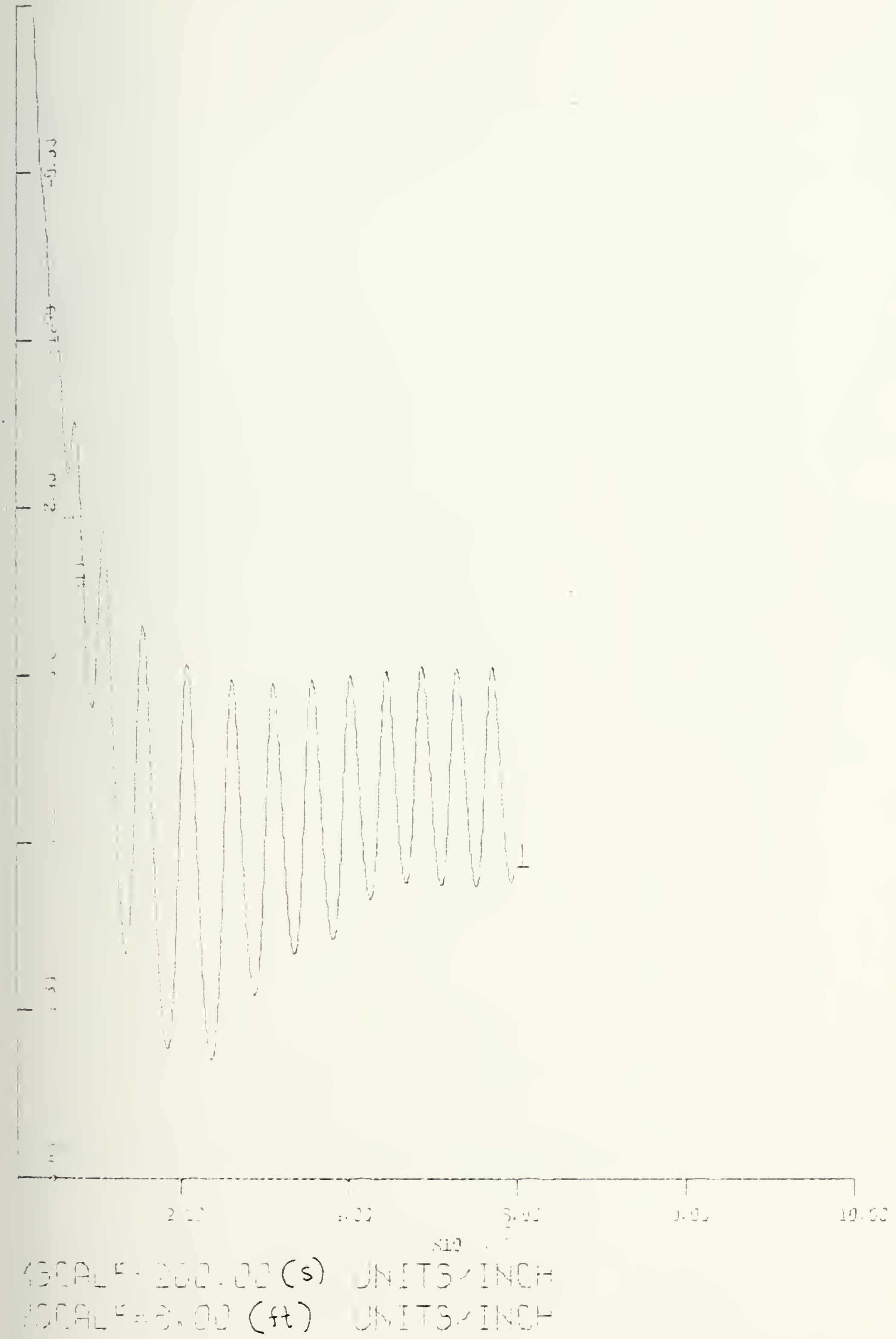


Fig. IV-19a. Depth vs. Time. Response to a step input at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=1.0

Fig. IV-19b. Pitch vs. Time. Response to a step input at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=1.0

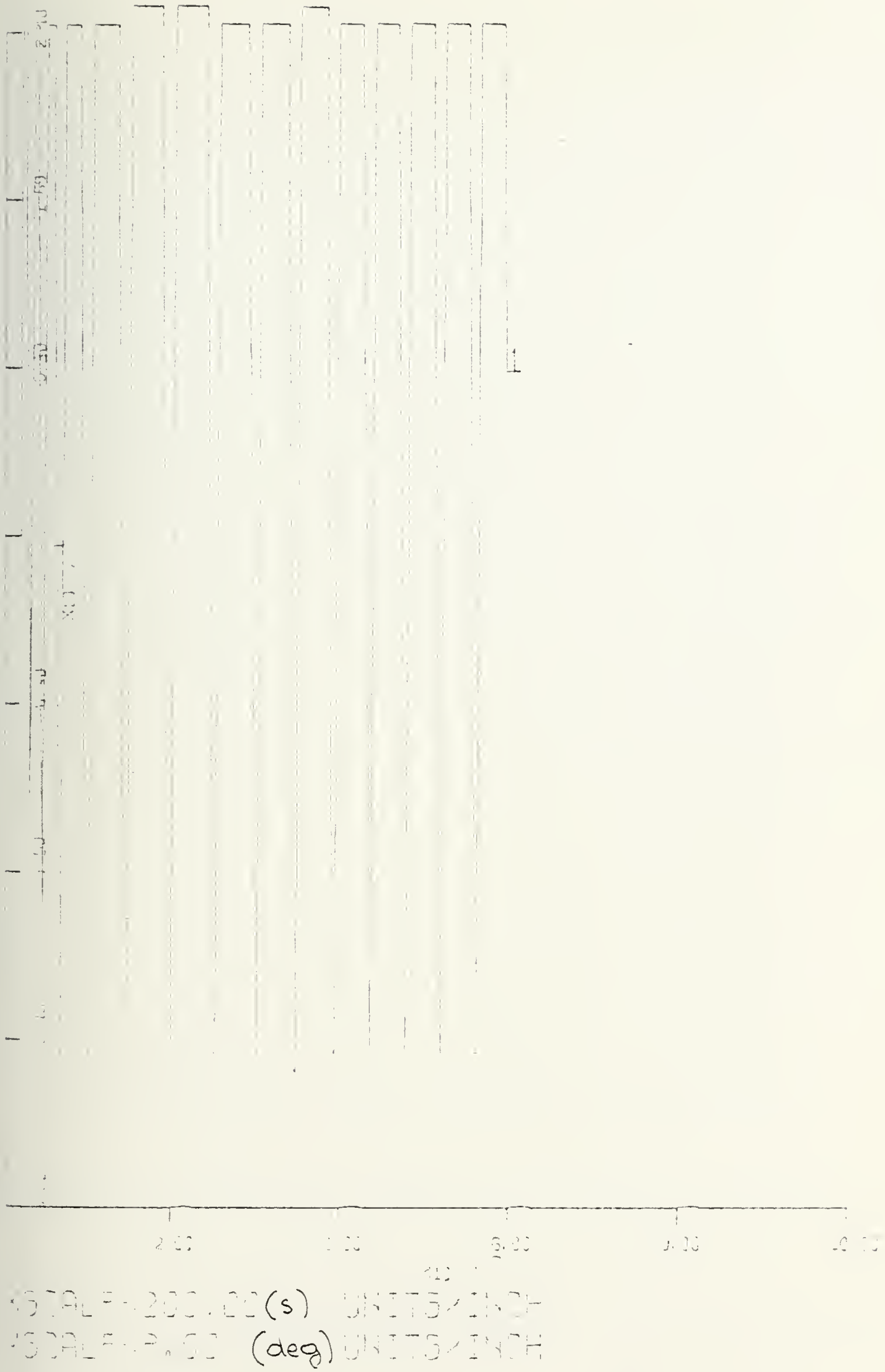


Fig. IV-19c. Stern Plane Angle vs. Time. Response to a step input at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=1.0

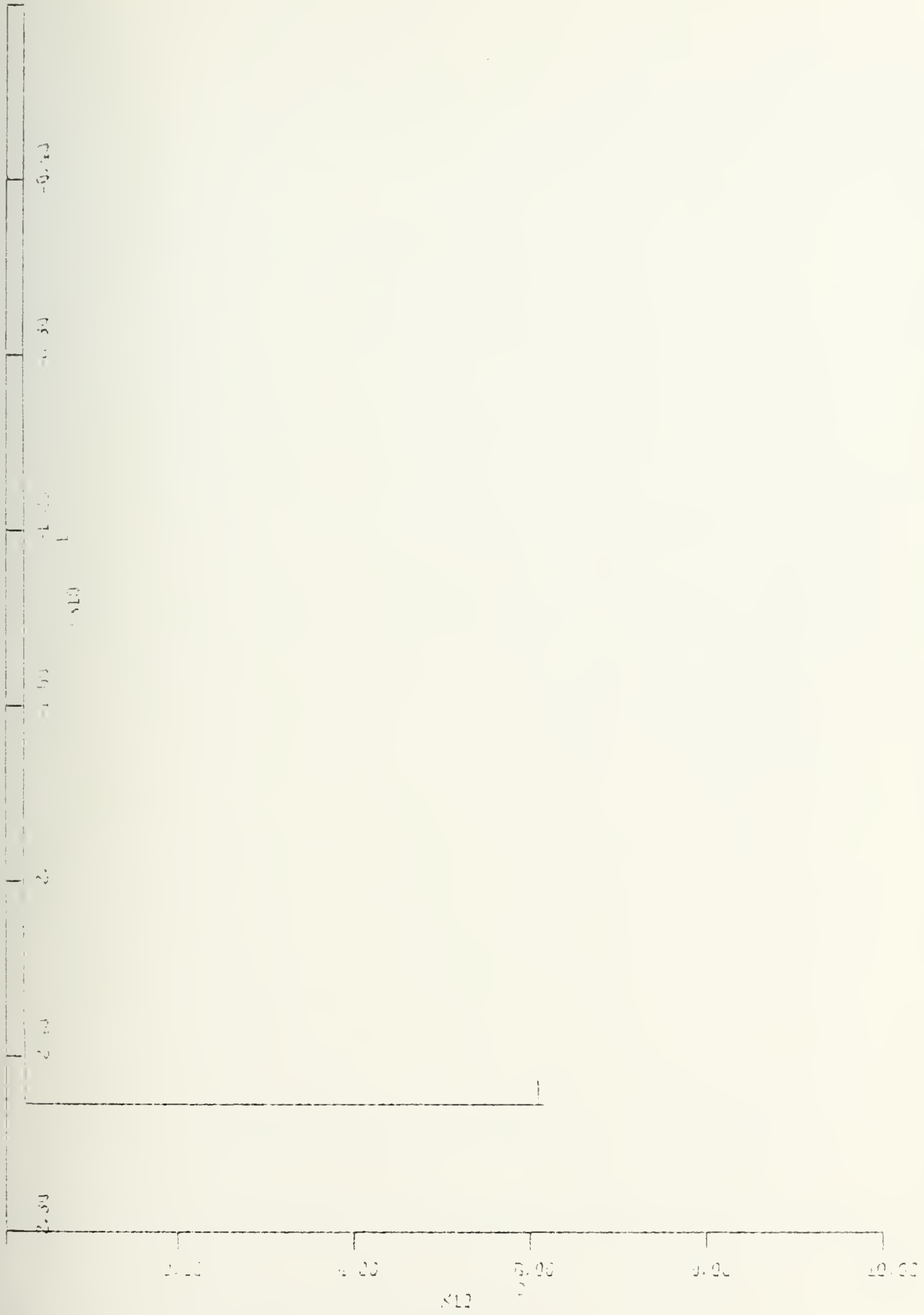


Fig. IV-19d. Fairwater Plane Angle vs. Time. Response to a step input at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=1.0

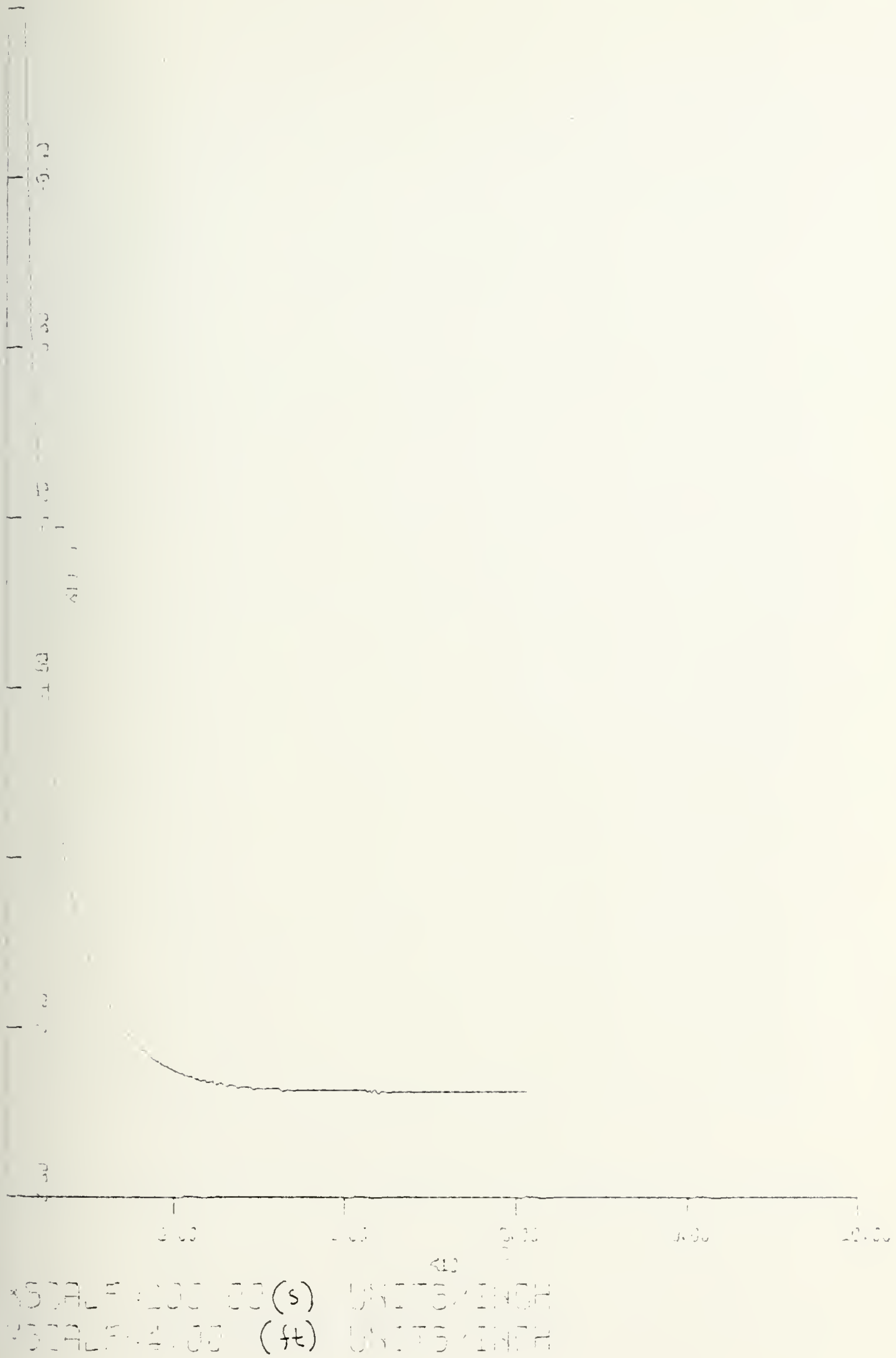
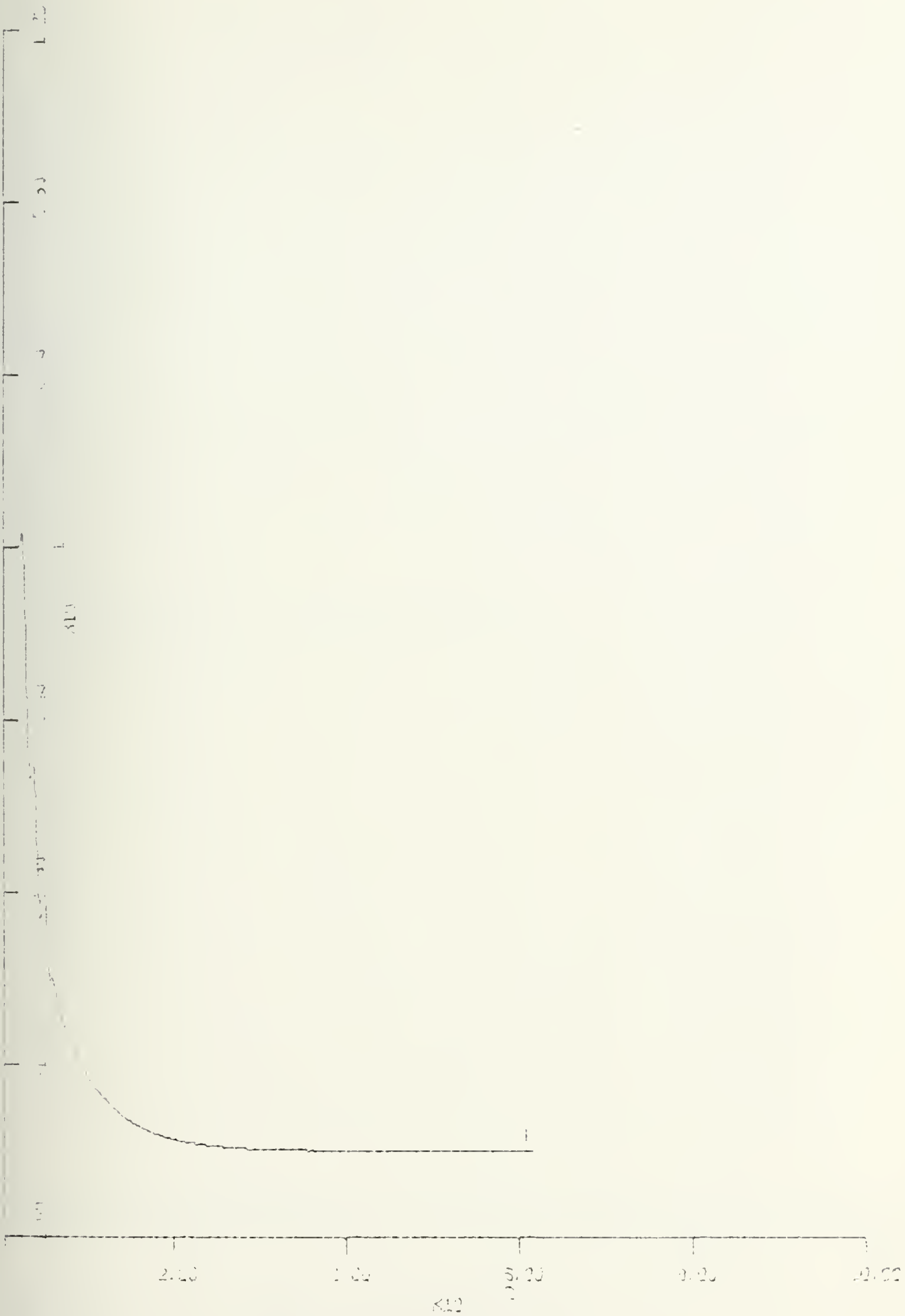


Fig. IV-20a. Depth vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.9



XSCALE=2.00E 00 (s) UNITS/INCH
YSCALE=0.04 (rad) UNITS/INCH

Fig. IV-20b. Pitch vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.9

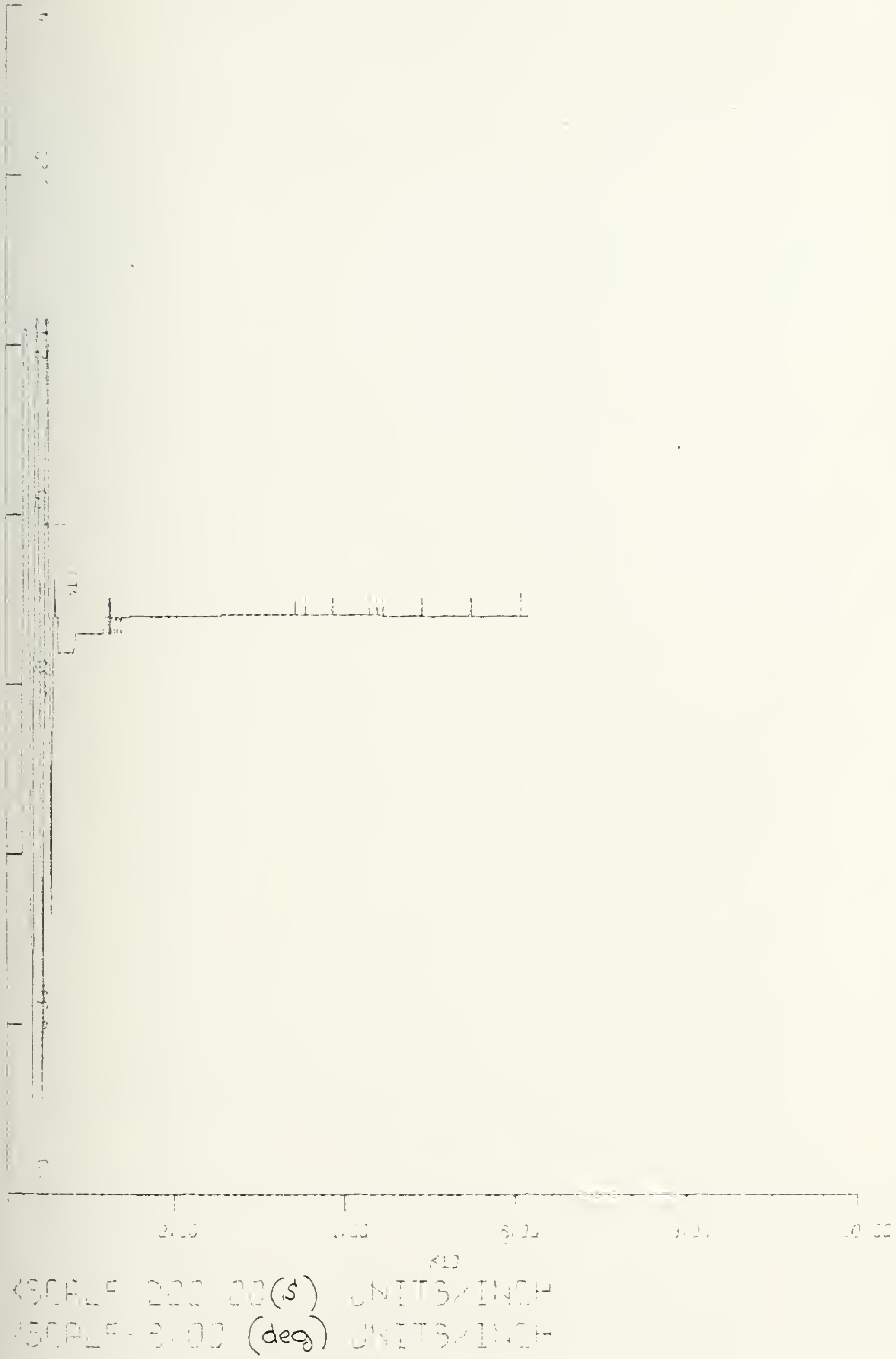


Fig. IV-20c. Stern Plane Angle vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.9

Fig. IV-20d. Fairwater Plane Angle vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.9

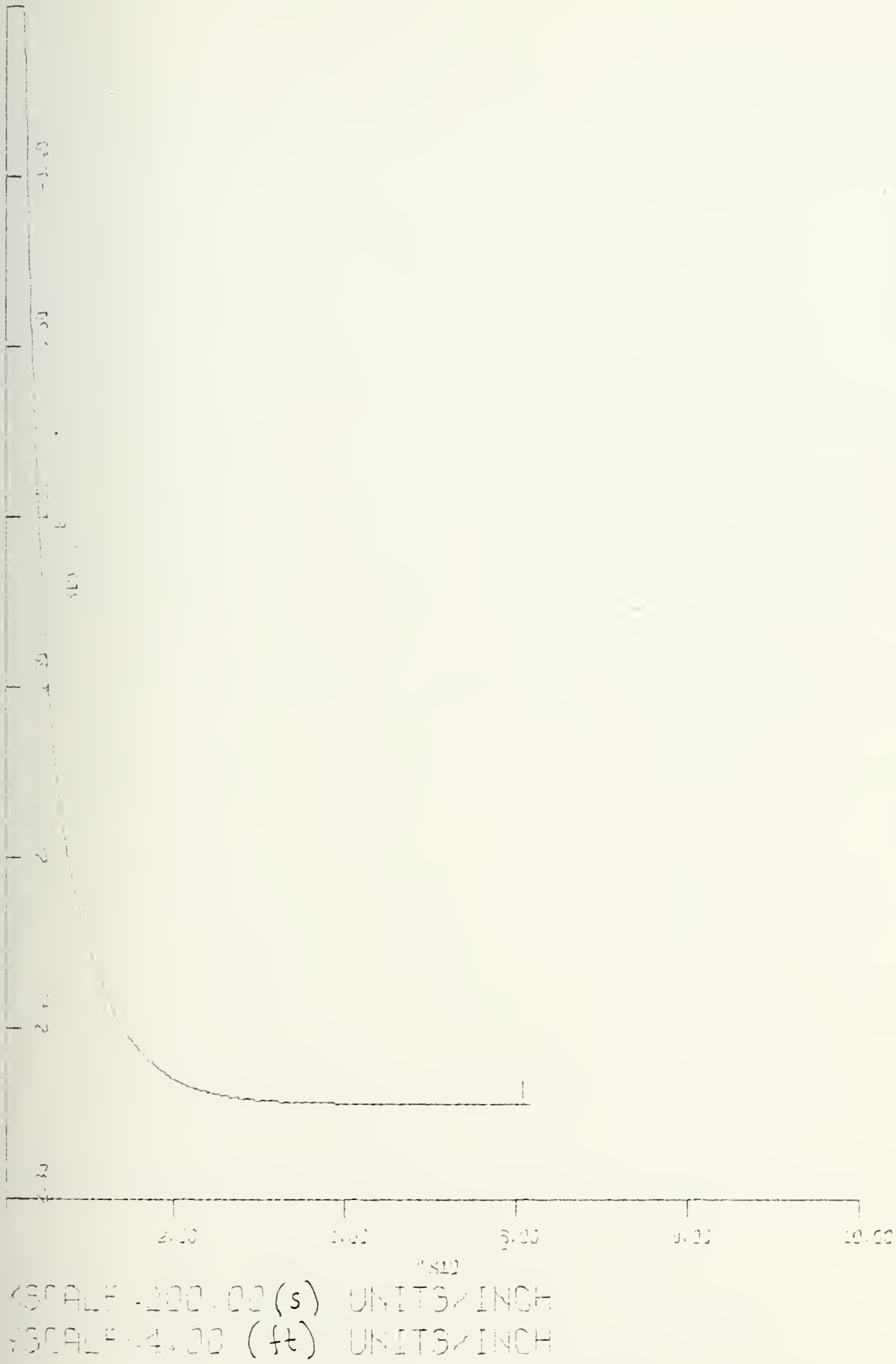


Fig. IV-21a. Depth vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.8

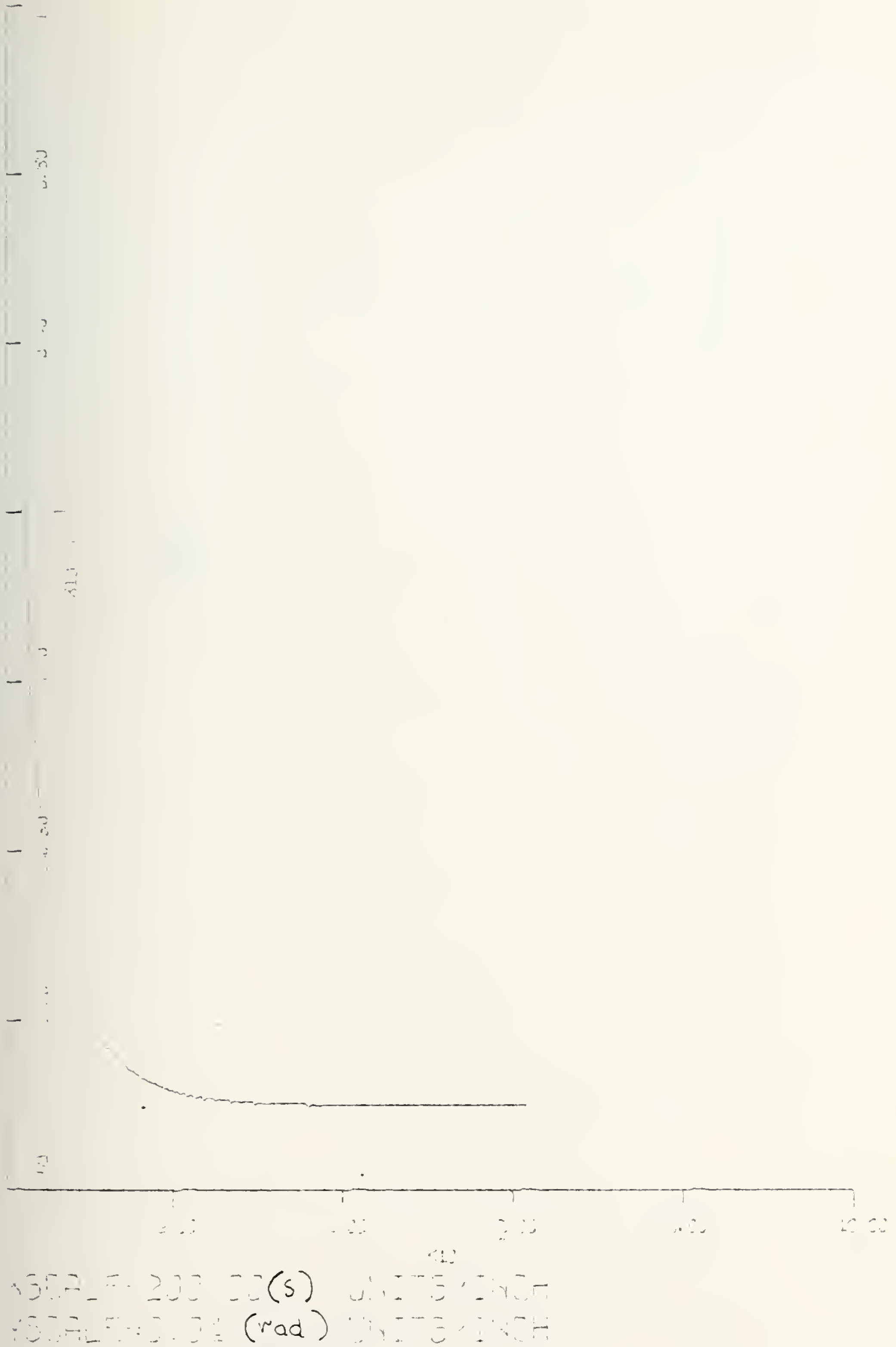


Fig. IV-21b. Pitch vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.8

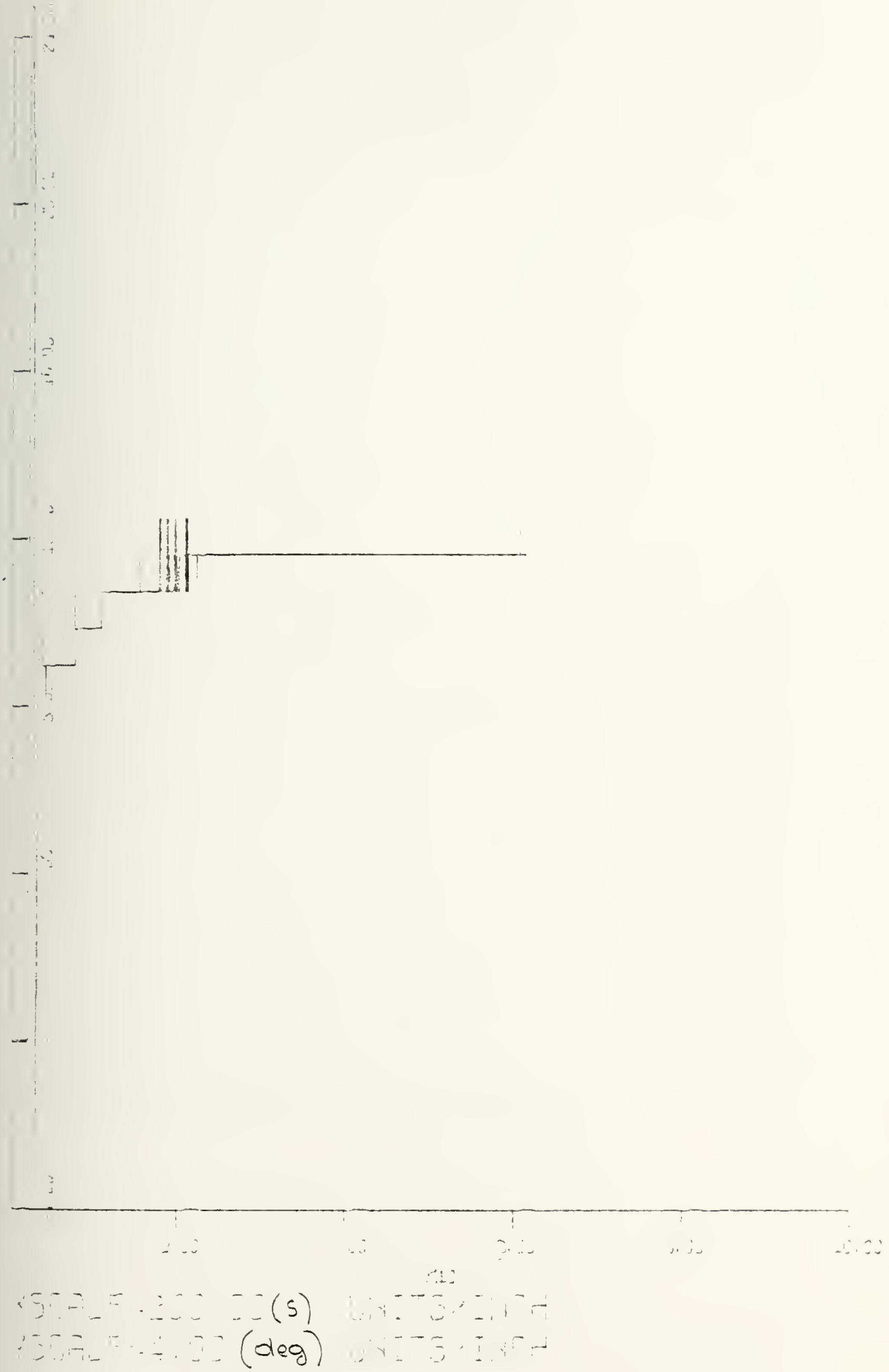


Fig. IV-21c. Stern Plane Angle vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.8

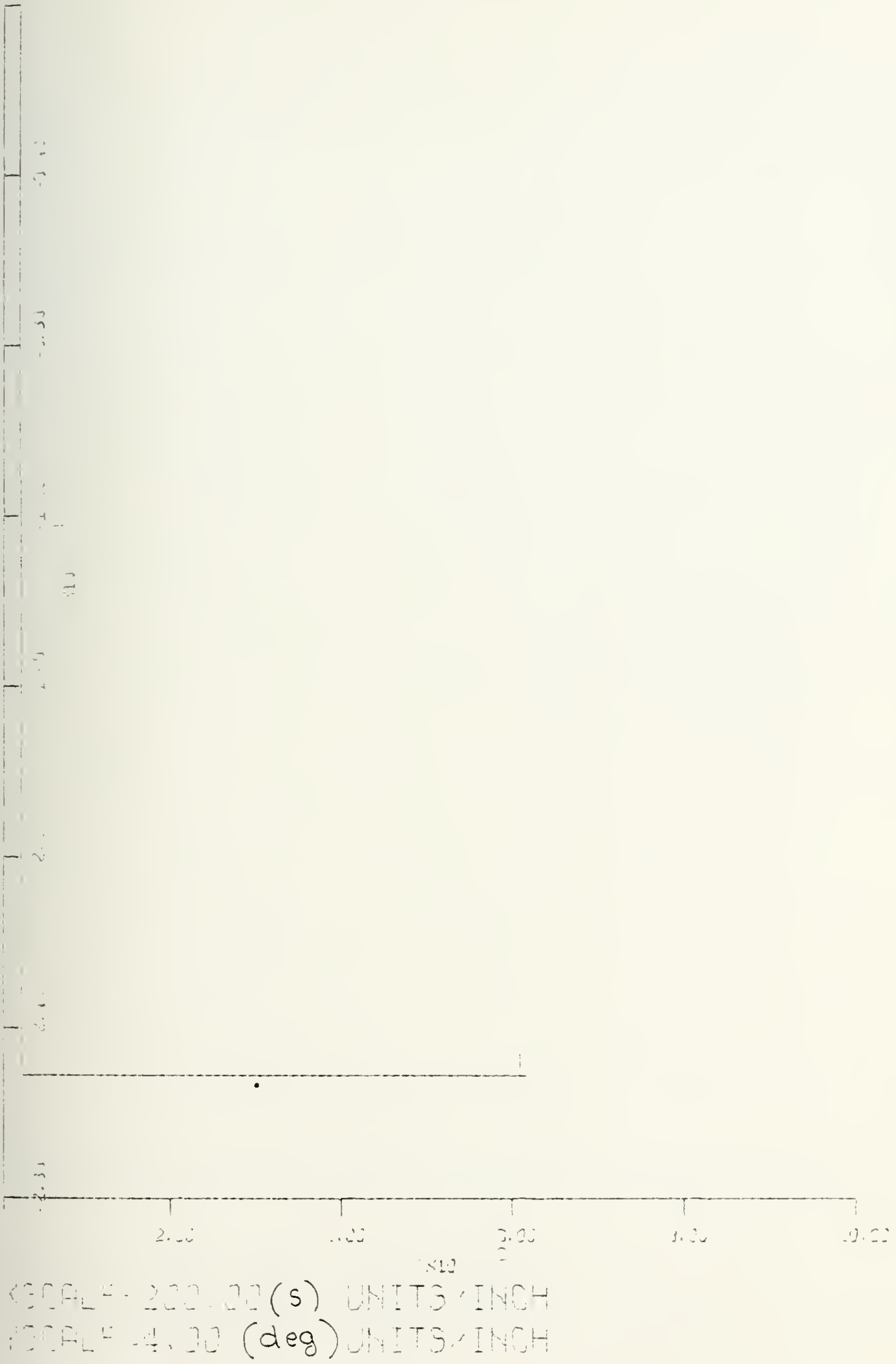


Fig. IV-21d. Fairwater Plane Angle vs Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.8

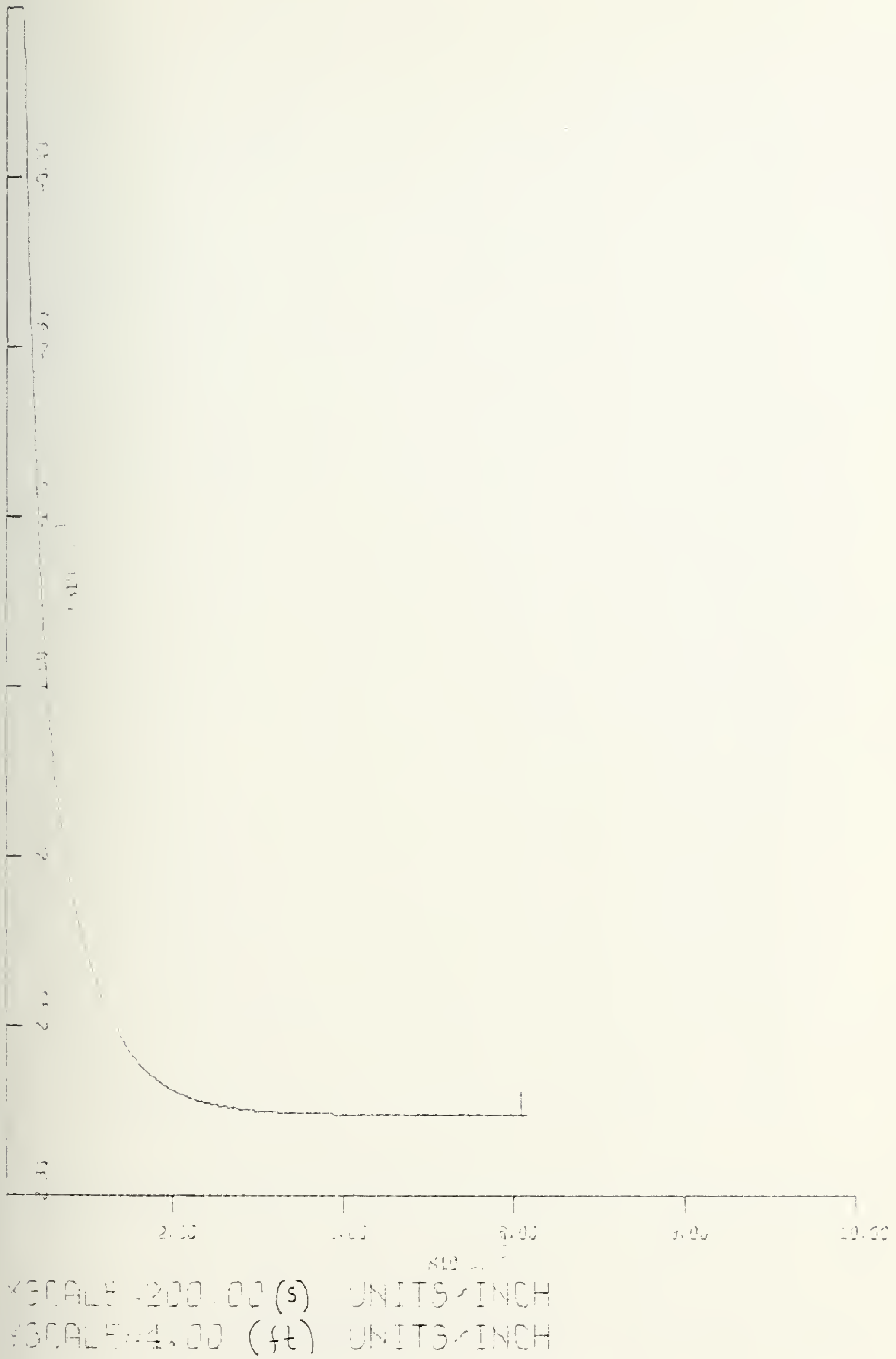


Fig. IV-22a. Depth vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.7

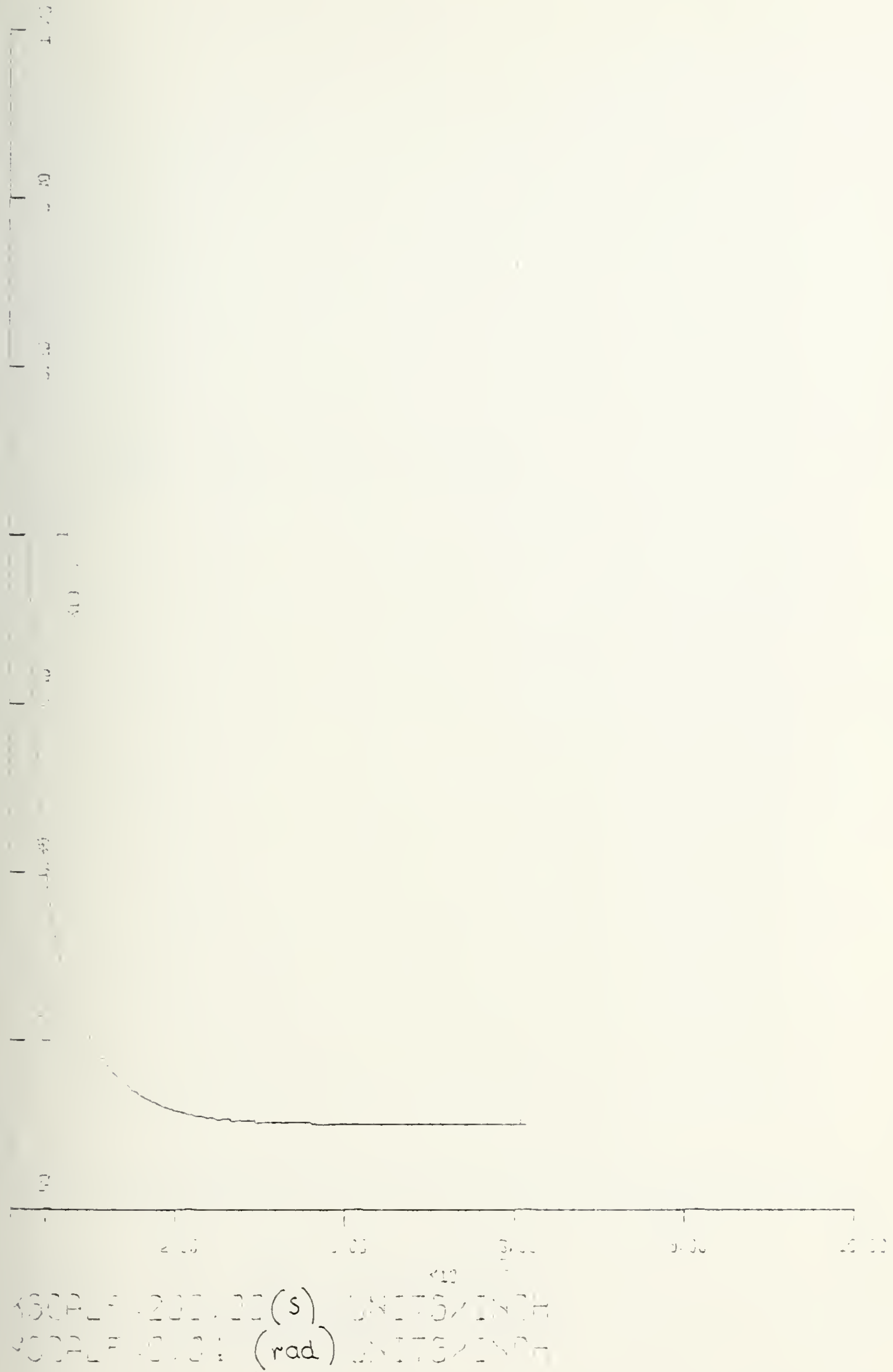


Fig. IV-22b. Pitch vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=800, C-10, E=1. Parameter X=0.7

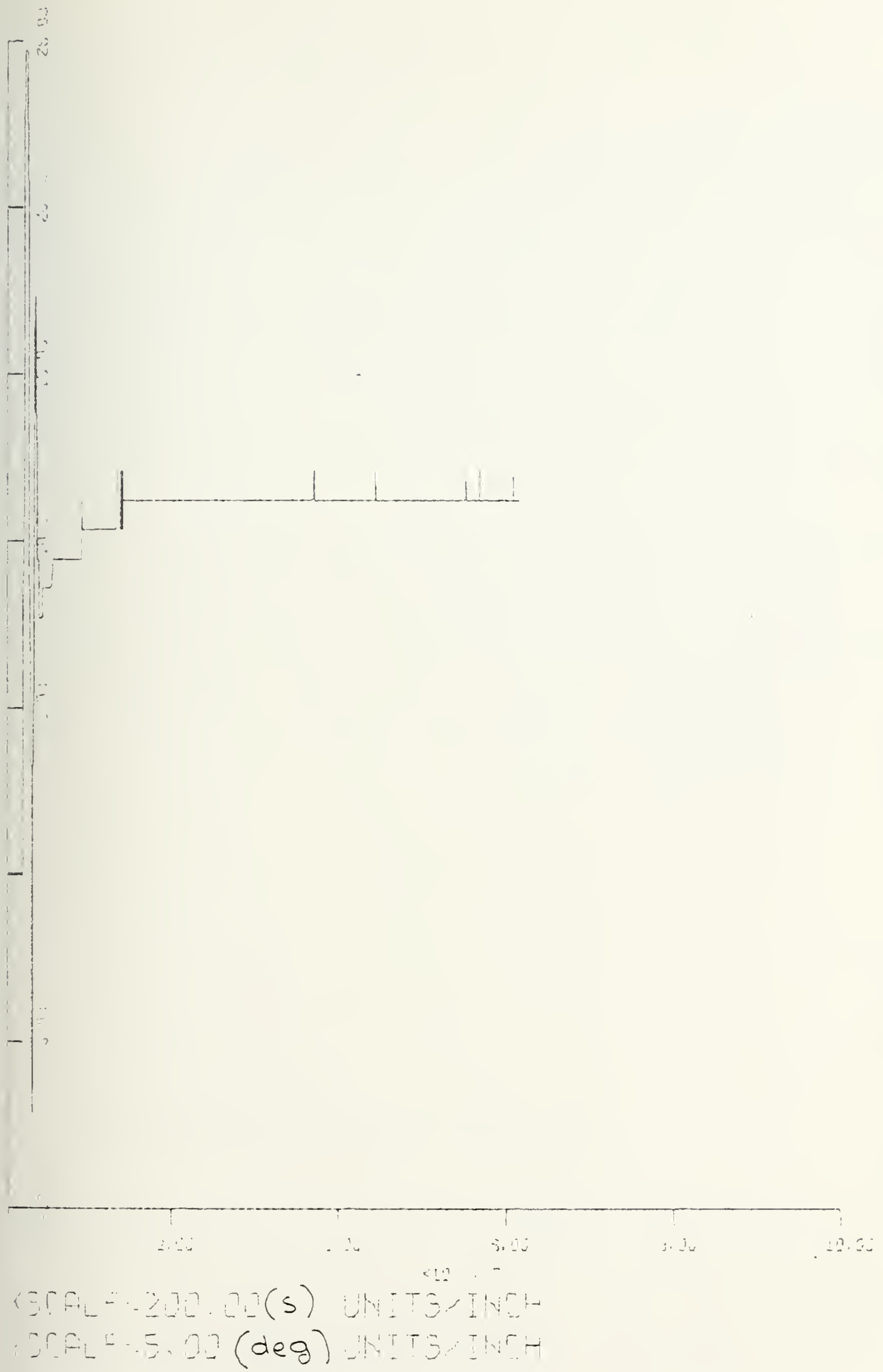


Fig. IV-22c. Stern Plane Angle vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.7

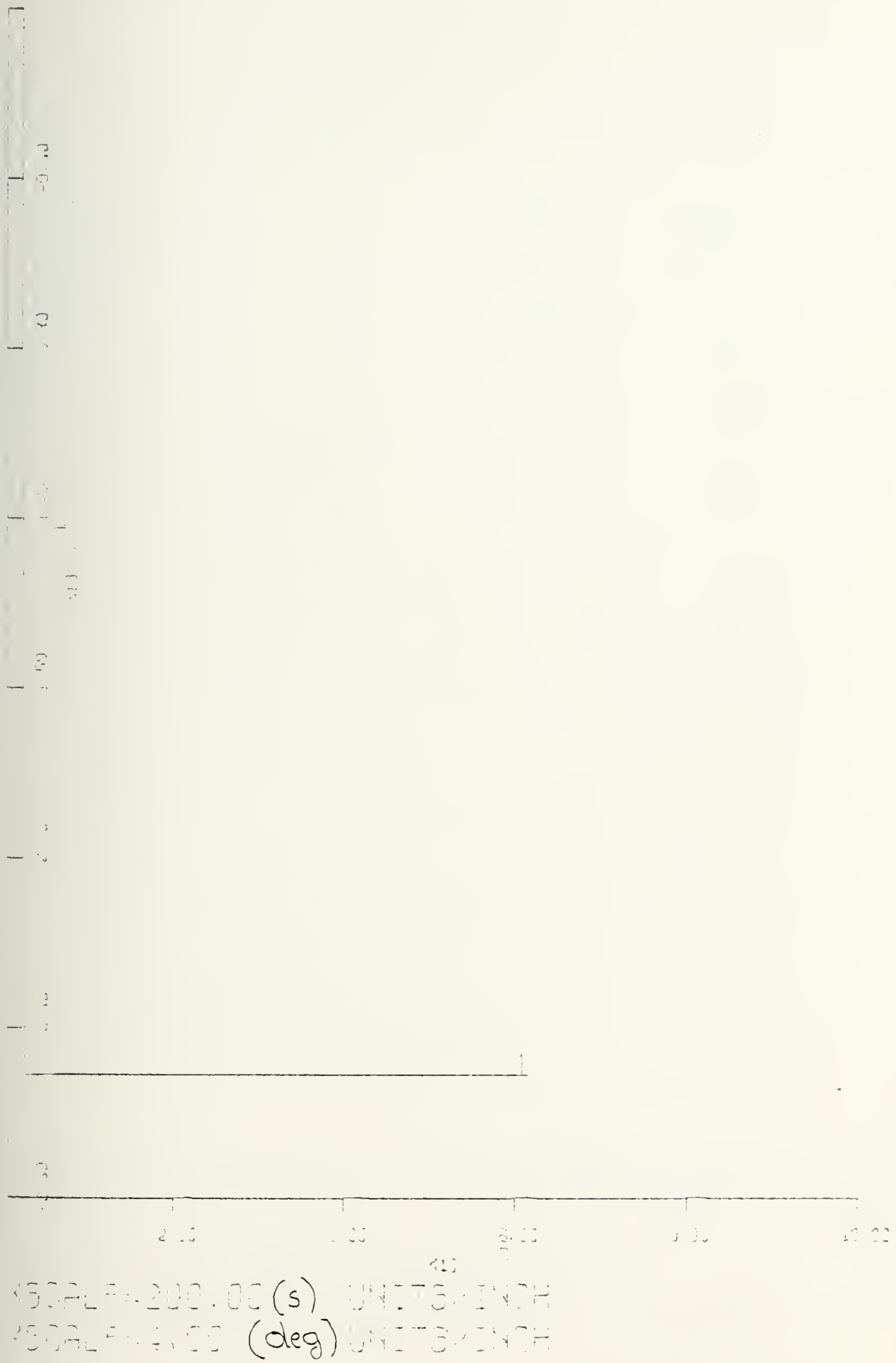


Fig. IV-22d. Fairwater Plane Angle vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.7

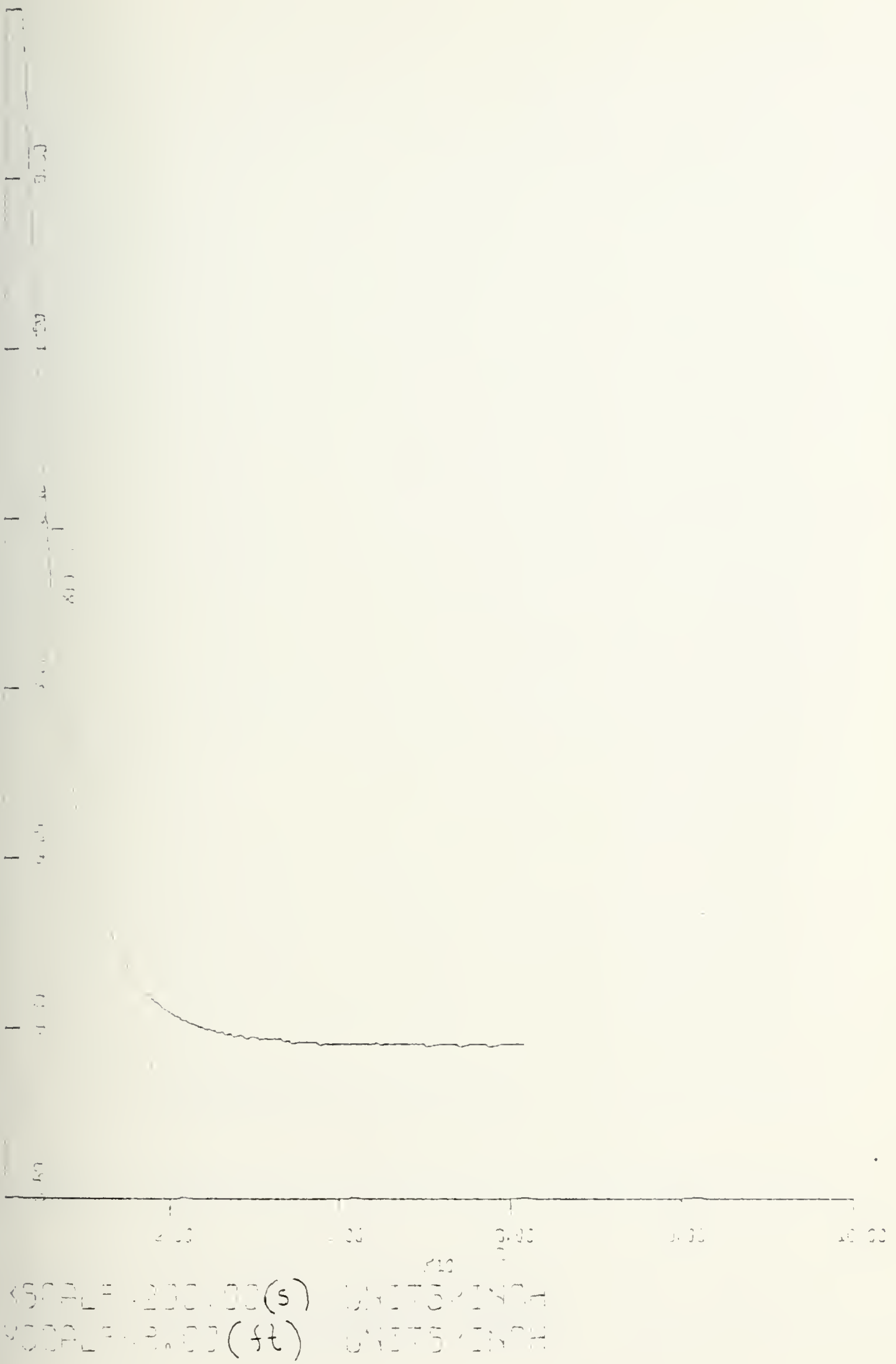


Fig. IV-23a. Depth vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.05

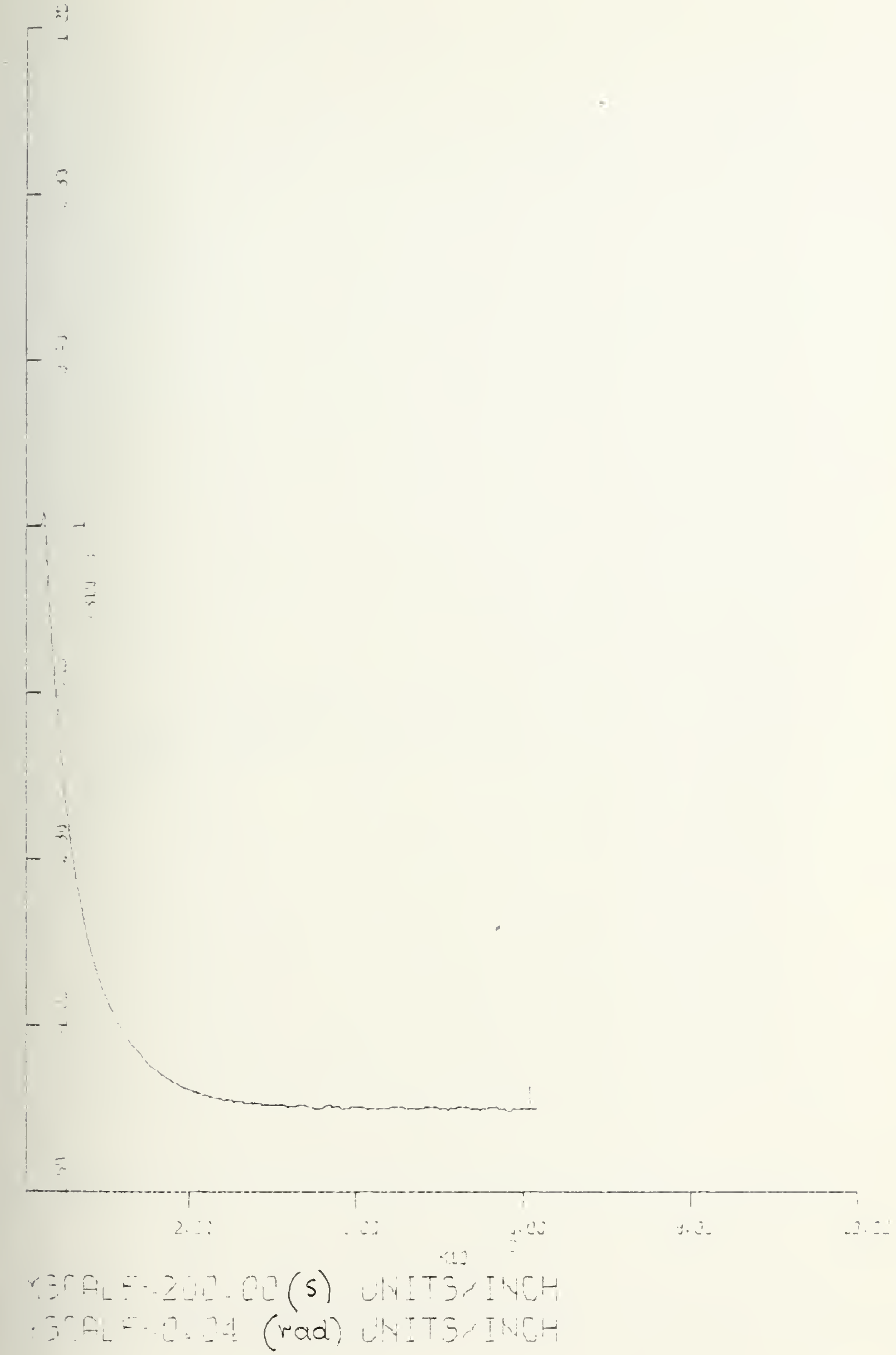


Fig. IV-23b. Pitch vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.05

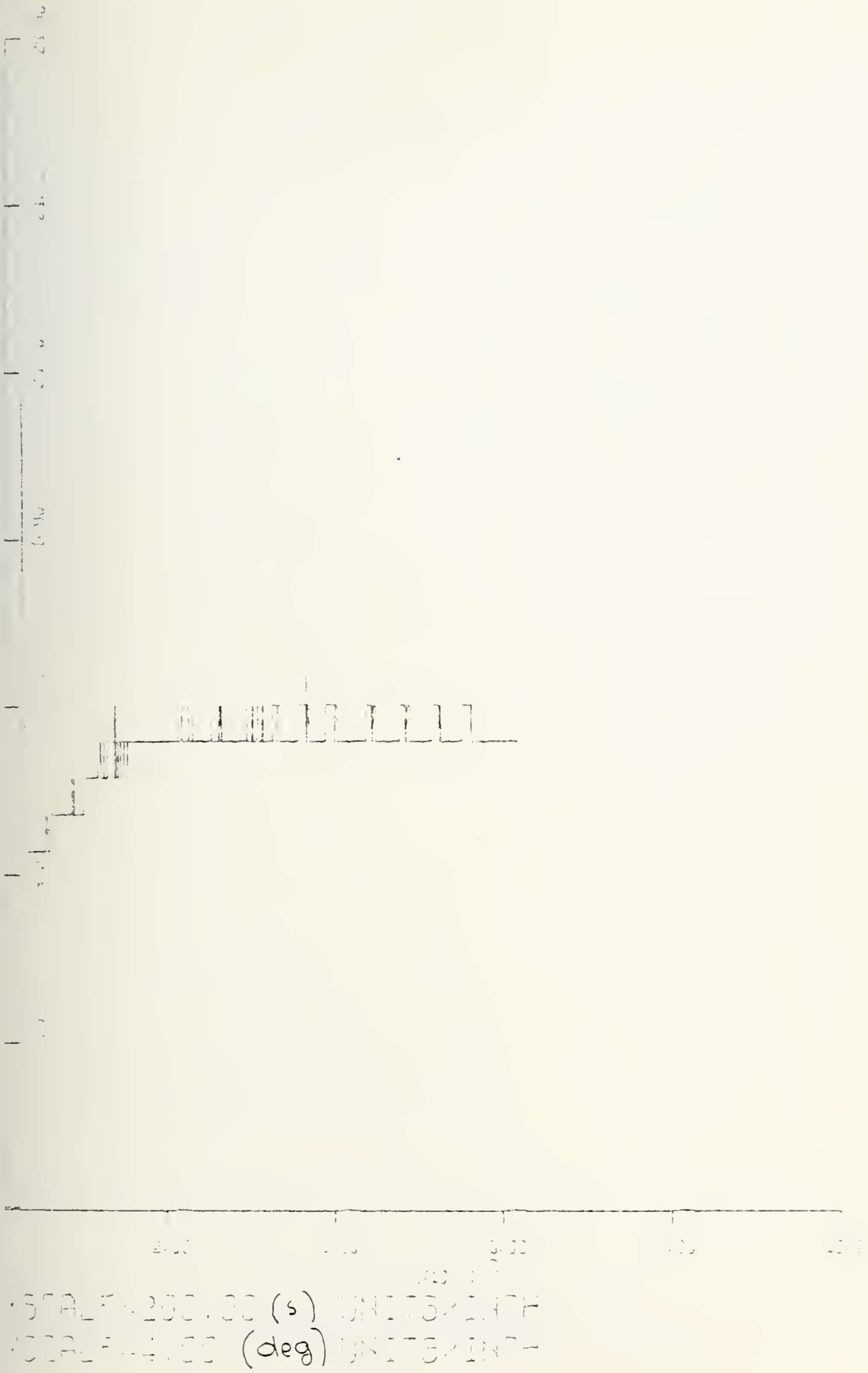


Fig. IV-23c. Stern Plane Angle vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.05

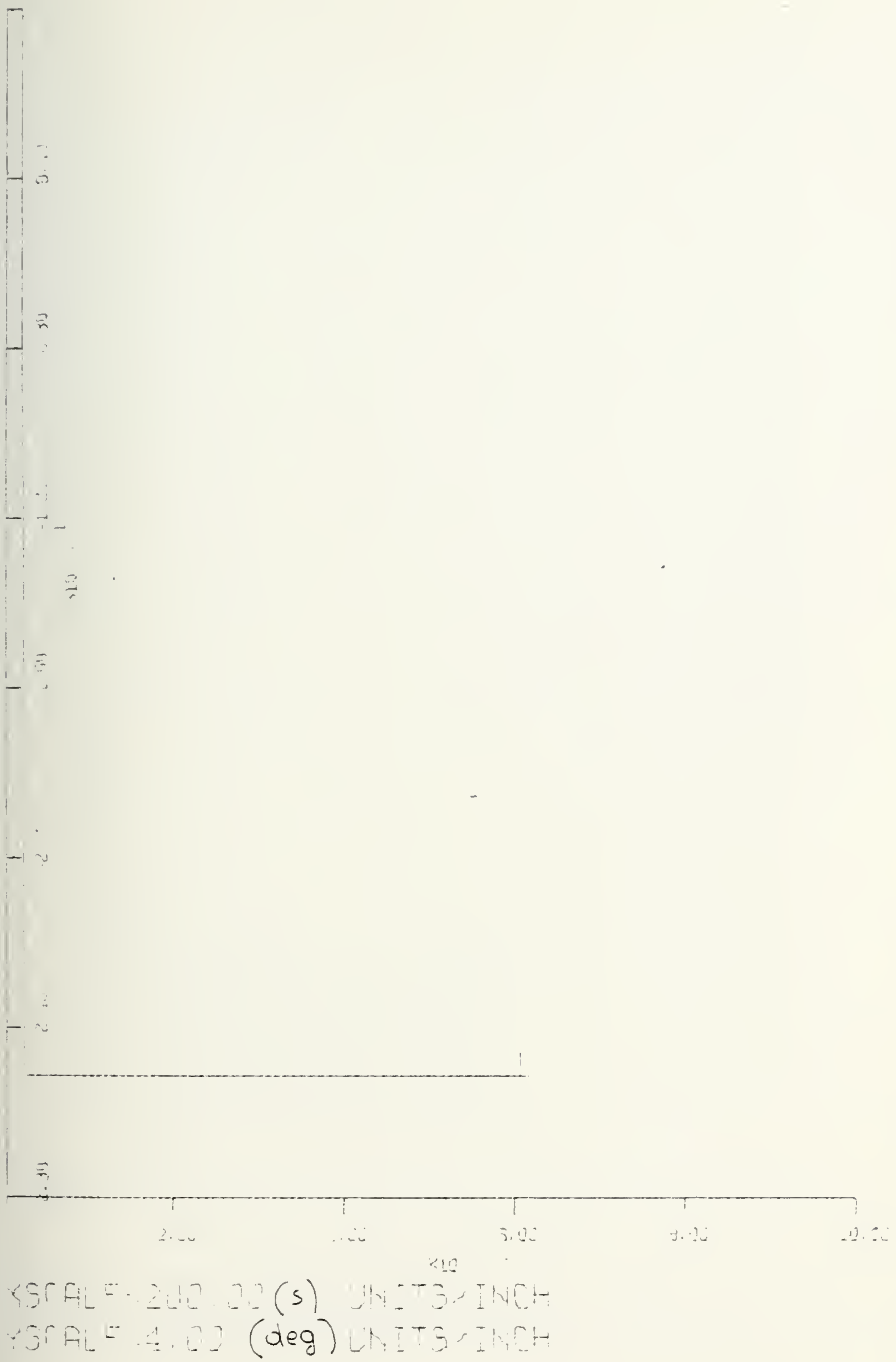


Fig. IV-23d. Fairwater Plane Angle vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.05

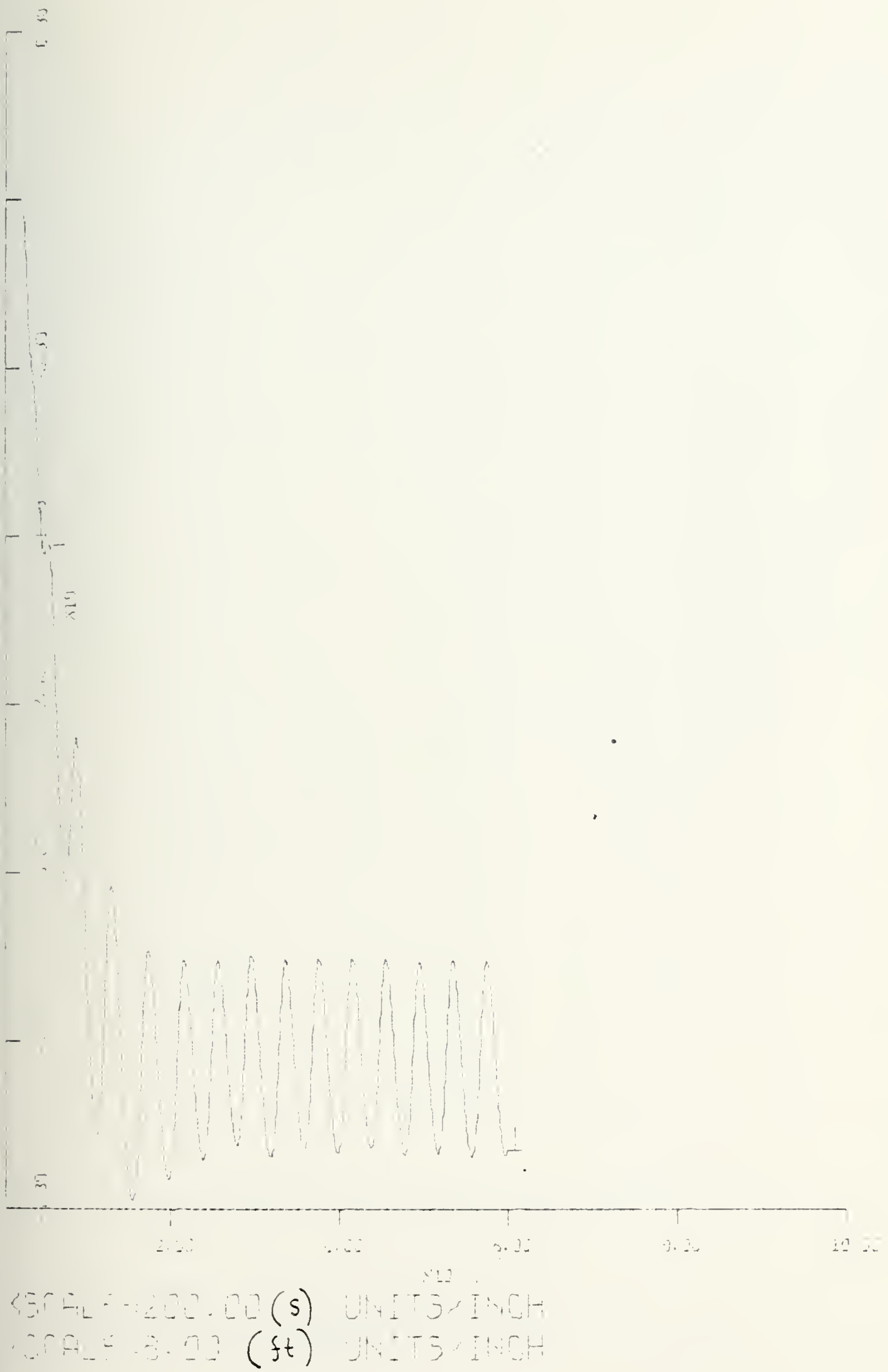


Fig. IV-24a. Depth vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=164, C=0.001, E=0.001. Parameter X=0.9

XSCALE=200.00 (s) UNITS/INCH
YSCALE=0.04 (rad) UNITS/INCH

Fig. IV-24b. Pitch vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=164, C=0.001, E=0.001. Parameter X=0.9

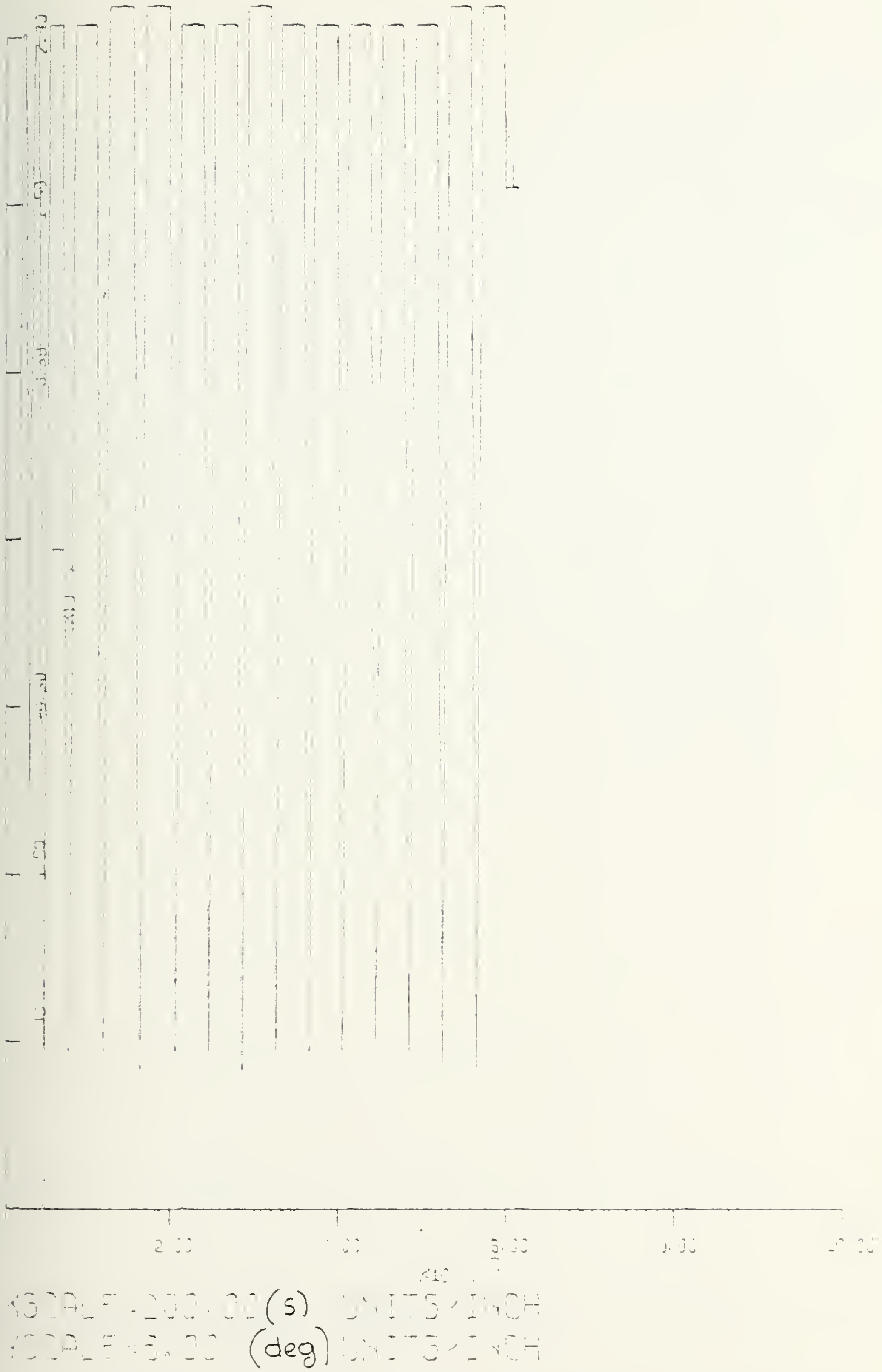


Fig. IV-24c. Stern Plane Angle vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=164, C=0.001, E=0.001. Parameter X=0.9

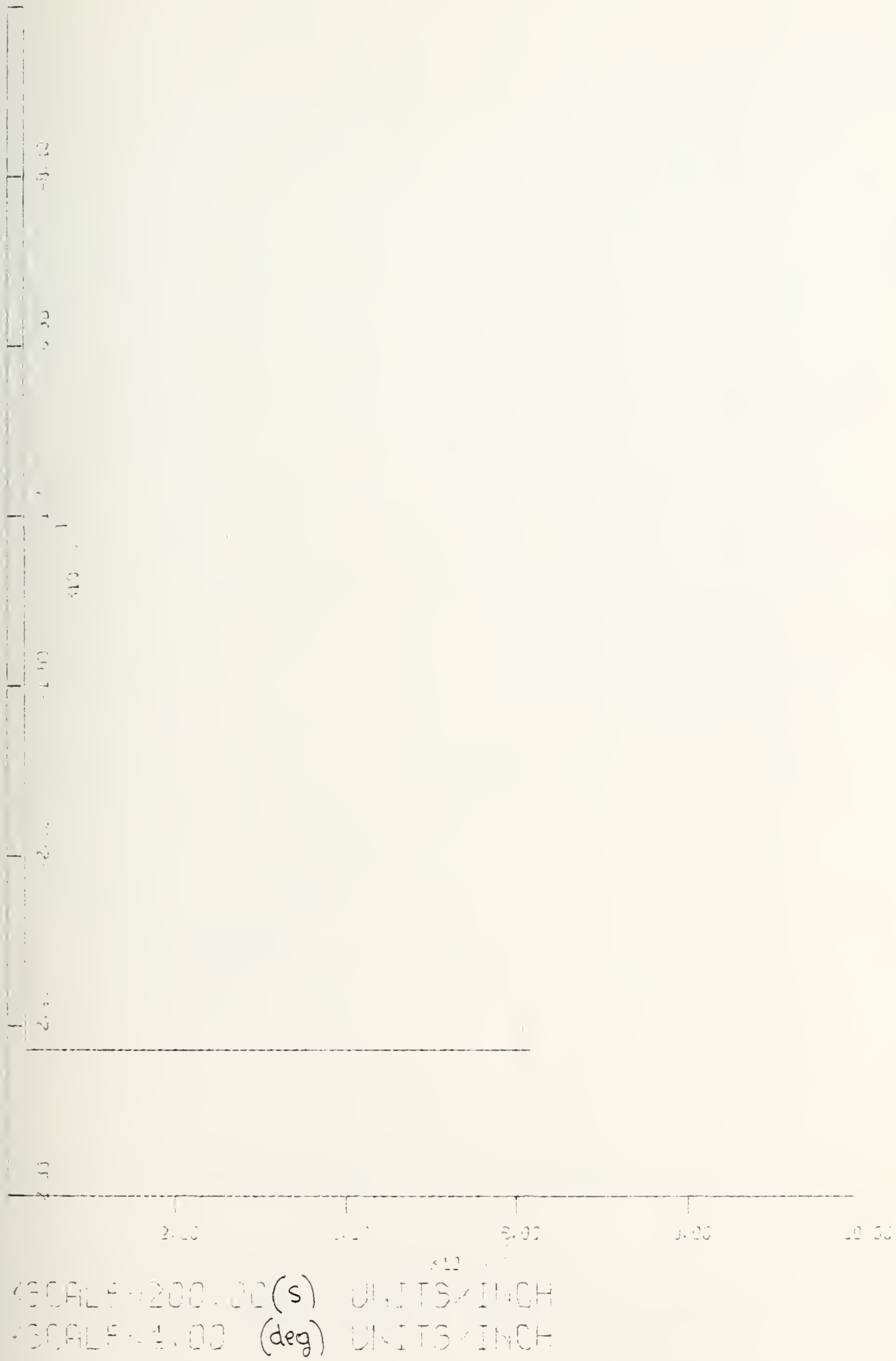


Fig. IV-24d. Fairwater Plane Angle vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=164, C=0.001, E=0.001. Parameter X=0.9

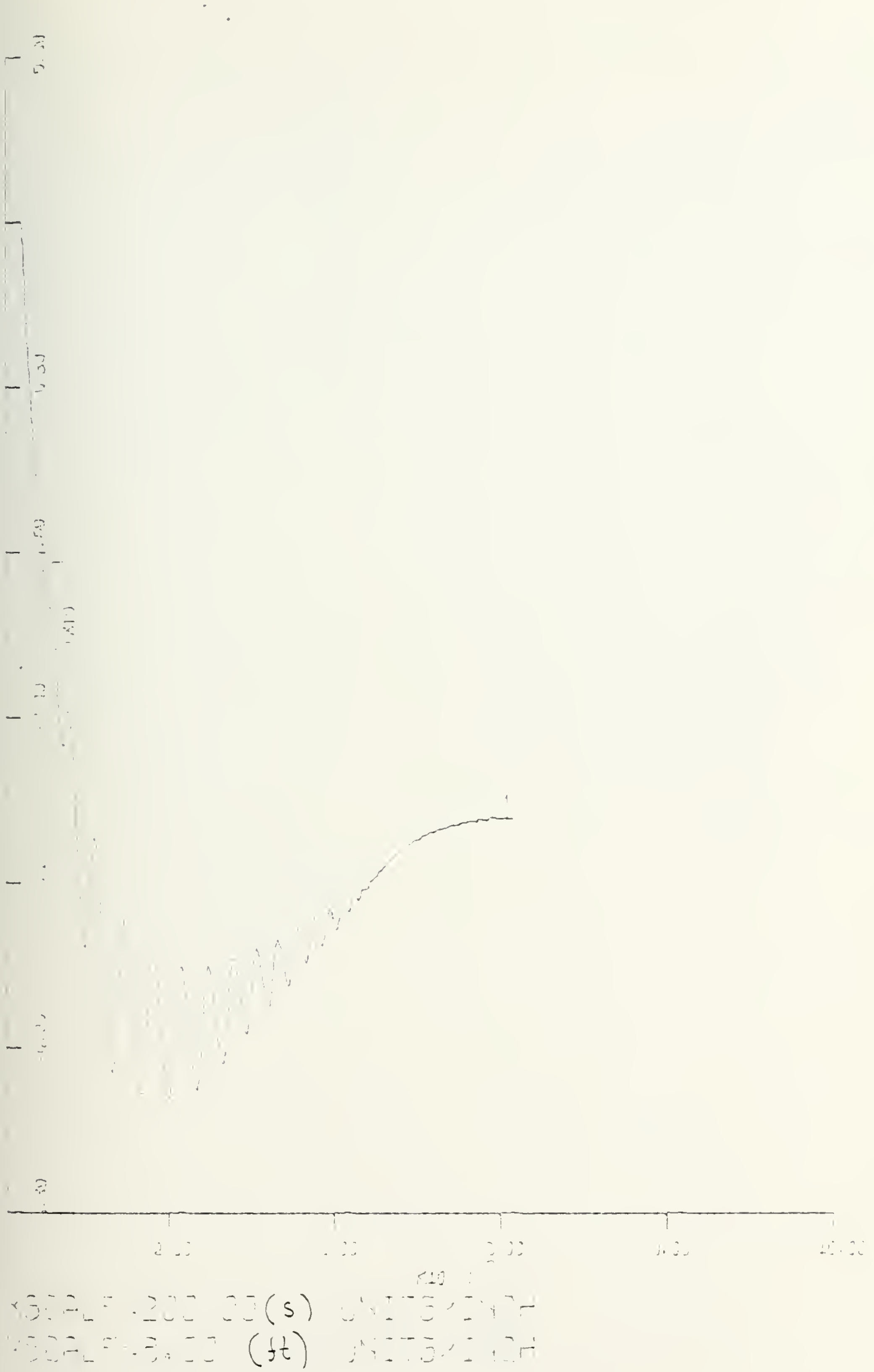


Fig. IV-25a. Depth vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=164, C=0.001, C=0.001. Parameter X=0.5

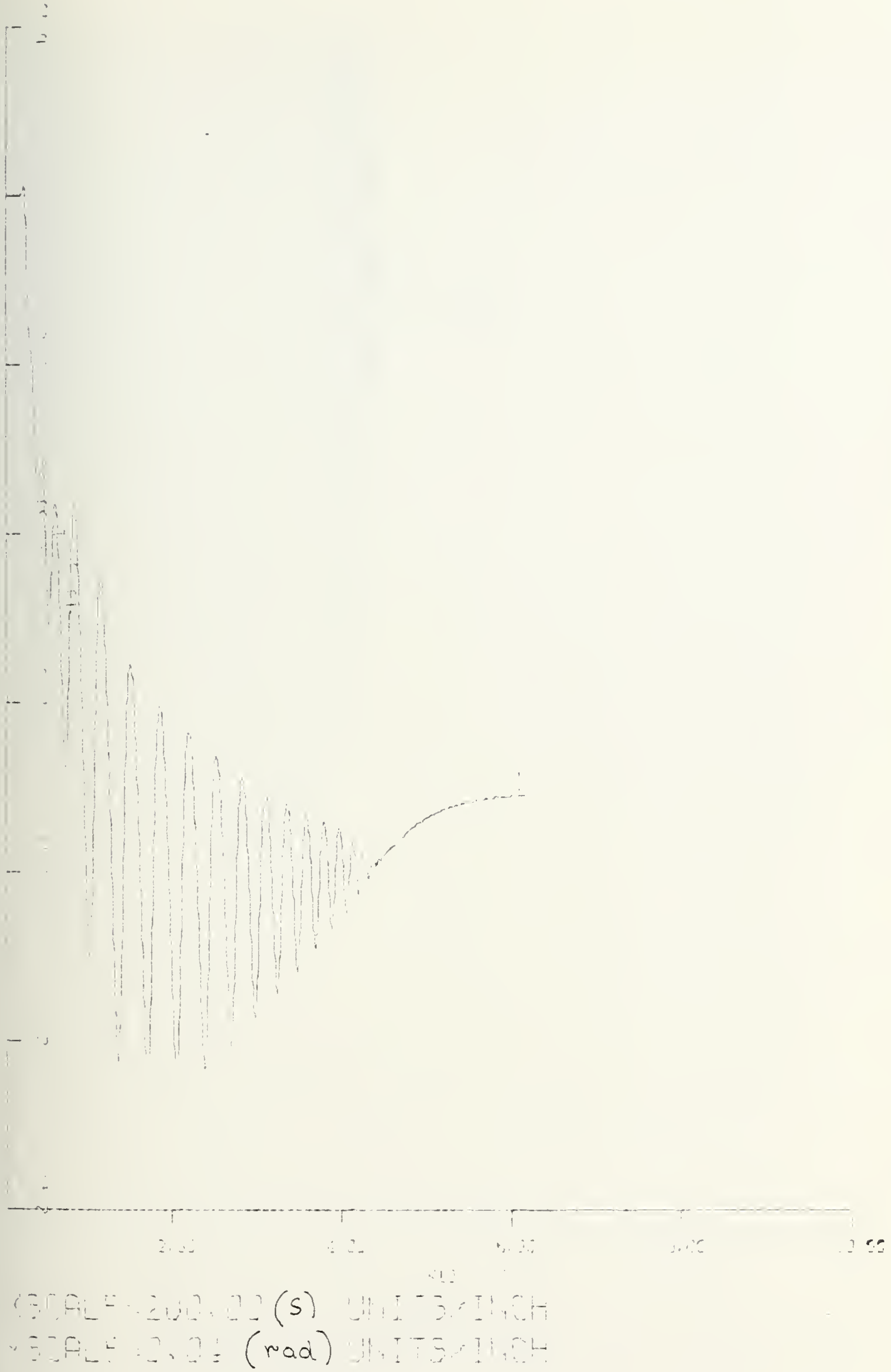


Fig. IV-25b. Pitch vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=164, C=0.001, E=0.001. Parameter X=0.5

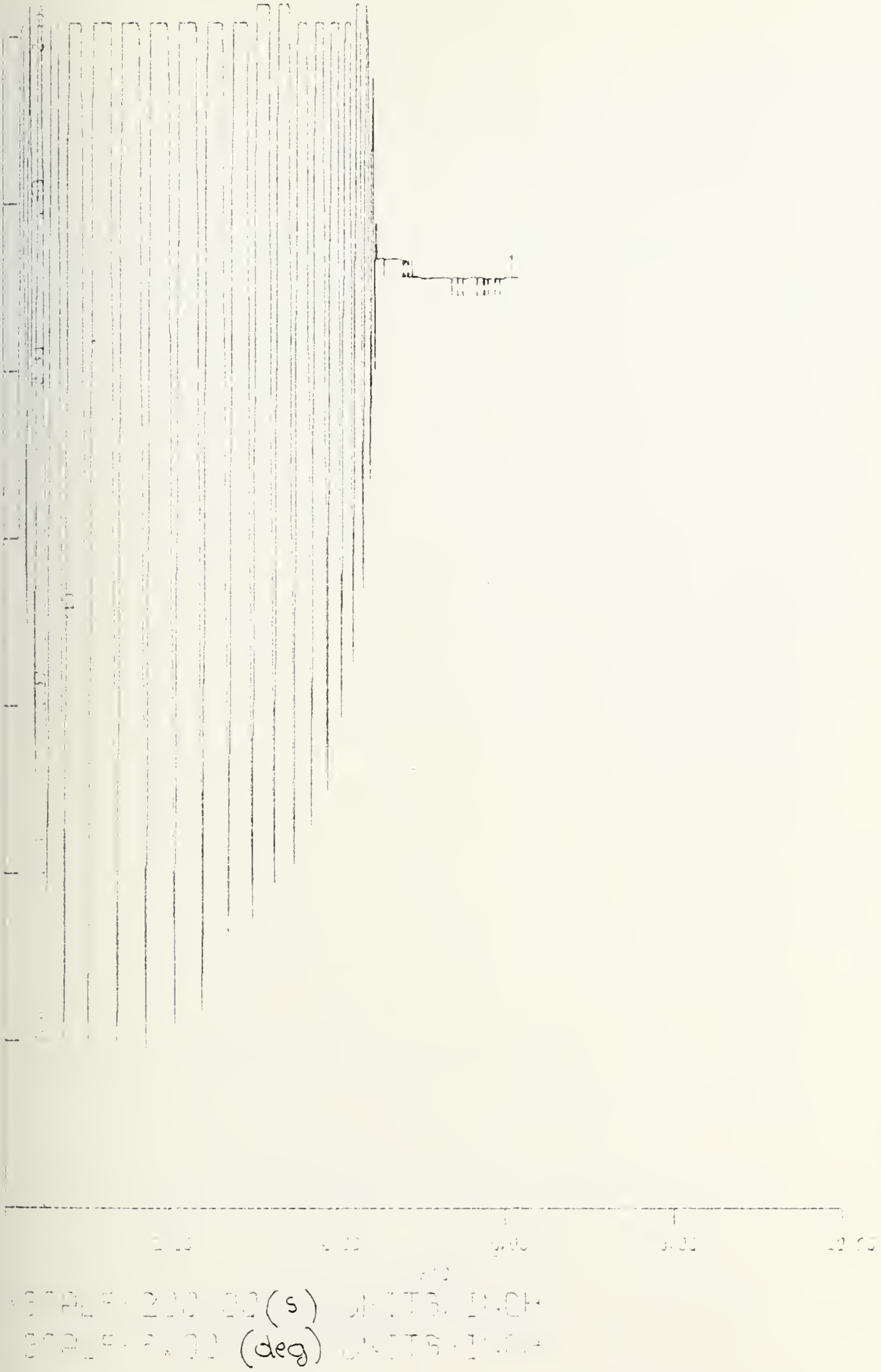


Fig. IV-25c. Stern Plane Angle vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=164, C=0.001, E=0.001. Parameter X=0.5

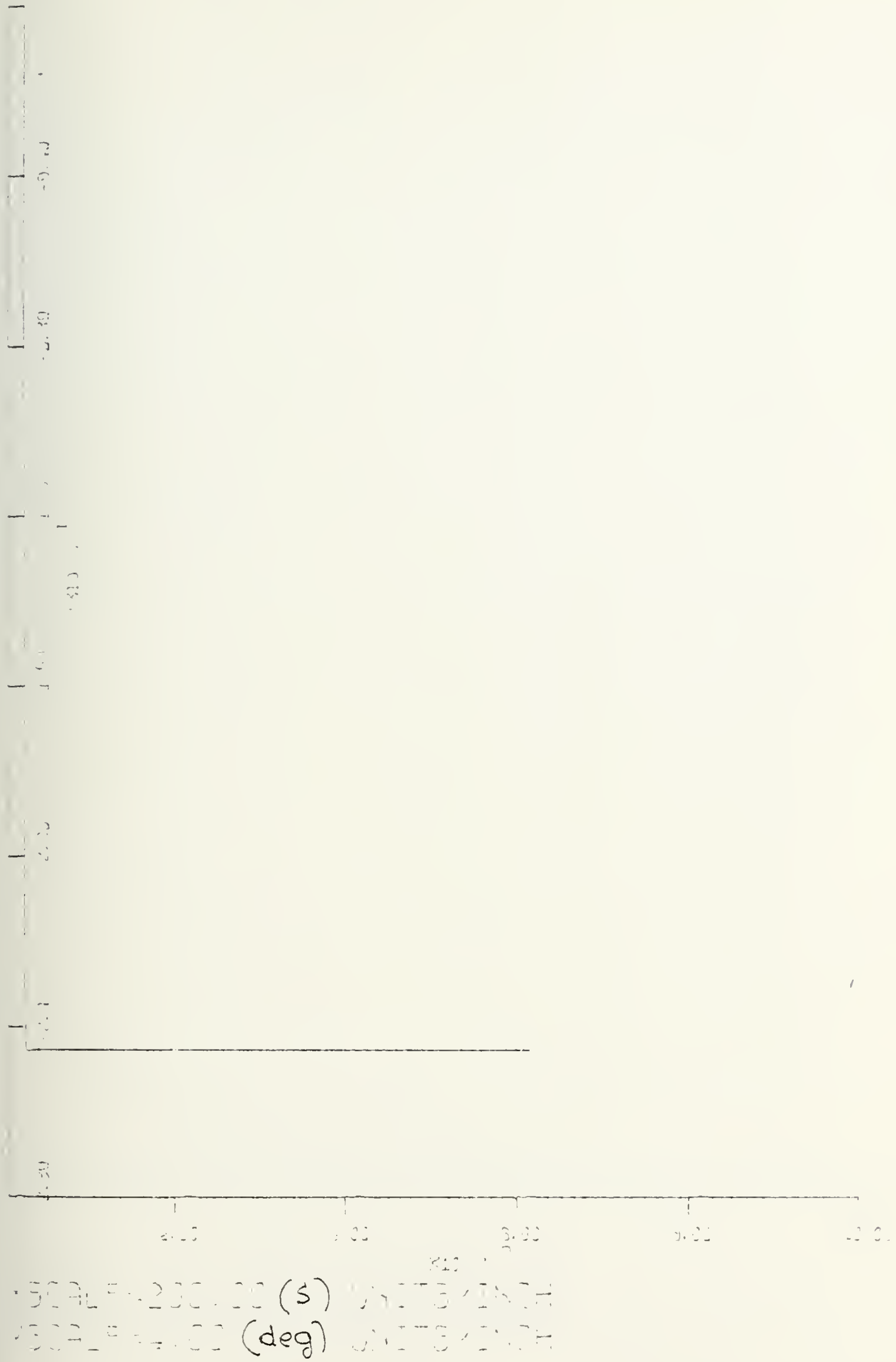


Fig. IV-25d. Fairwater Plane Angle vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers with B=164, C=0.001, E=0.001. Parameter X=0.5

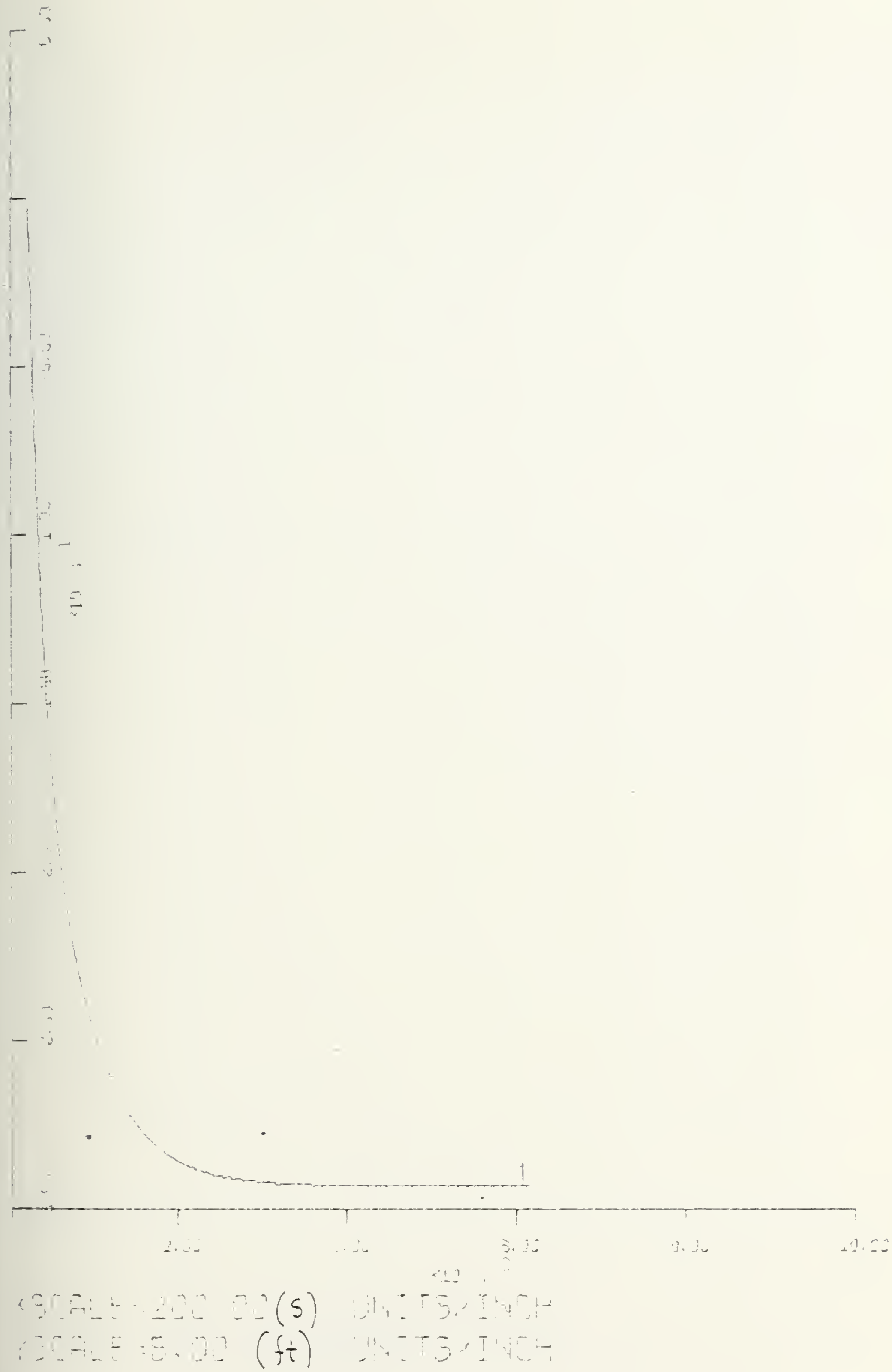


Fig. IV-26a. Depth vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.4

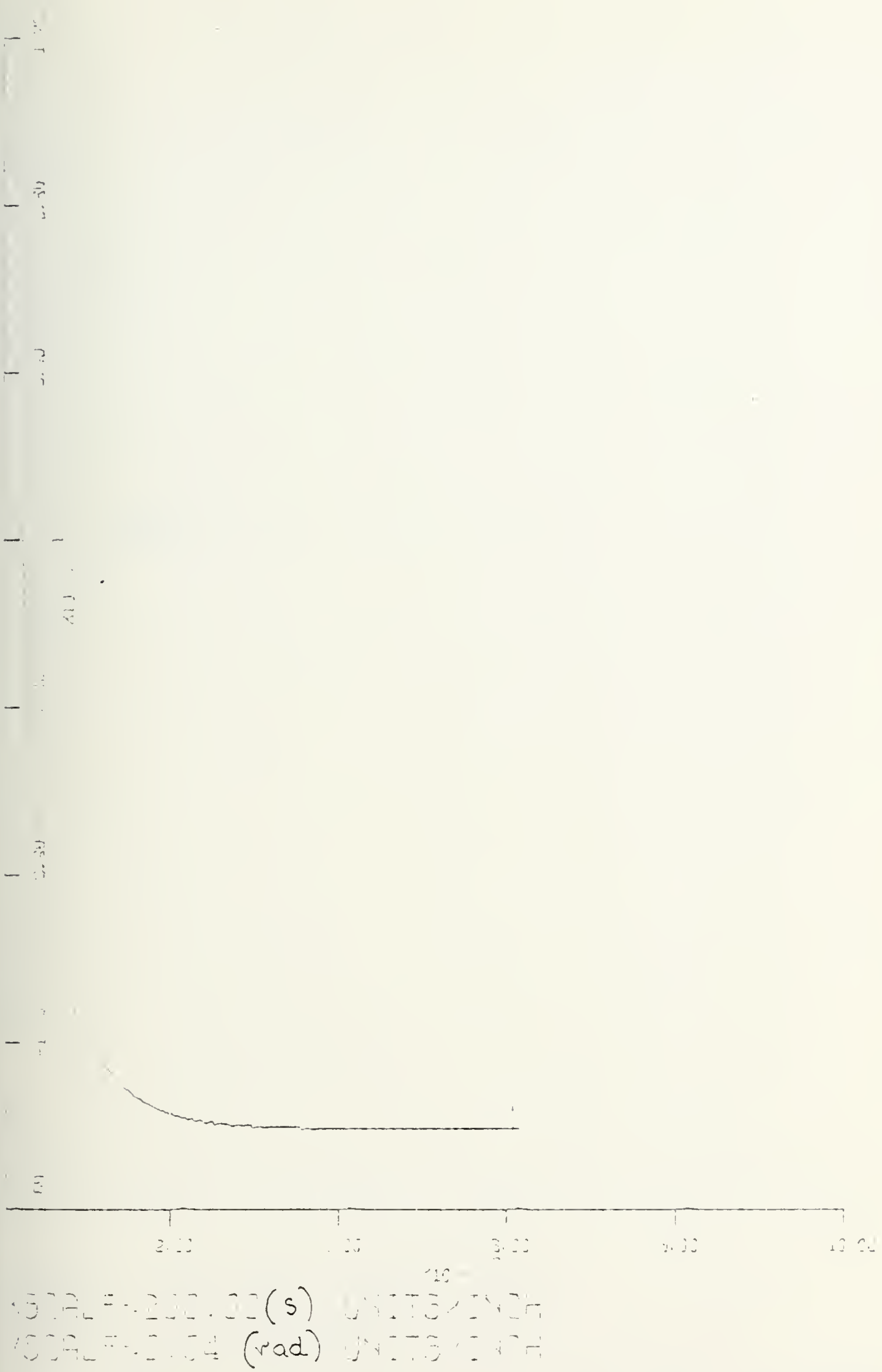


Fig. IV-26b. Pitch vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.4

Fig. IV-16b. Pitch vs. Time. Response to a step [illegible] at [illegible]. [illegible] case [illegible] design [illegible] combined controller [illegible] with [illegible], c=0.001 [illegible] .001. Parameter [illegible]

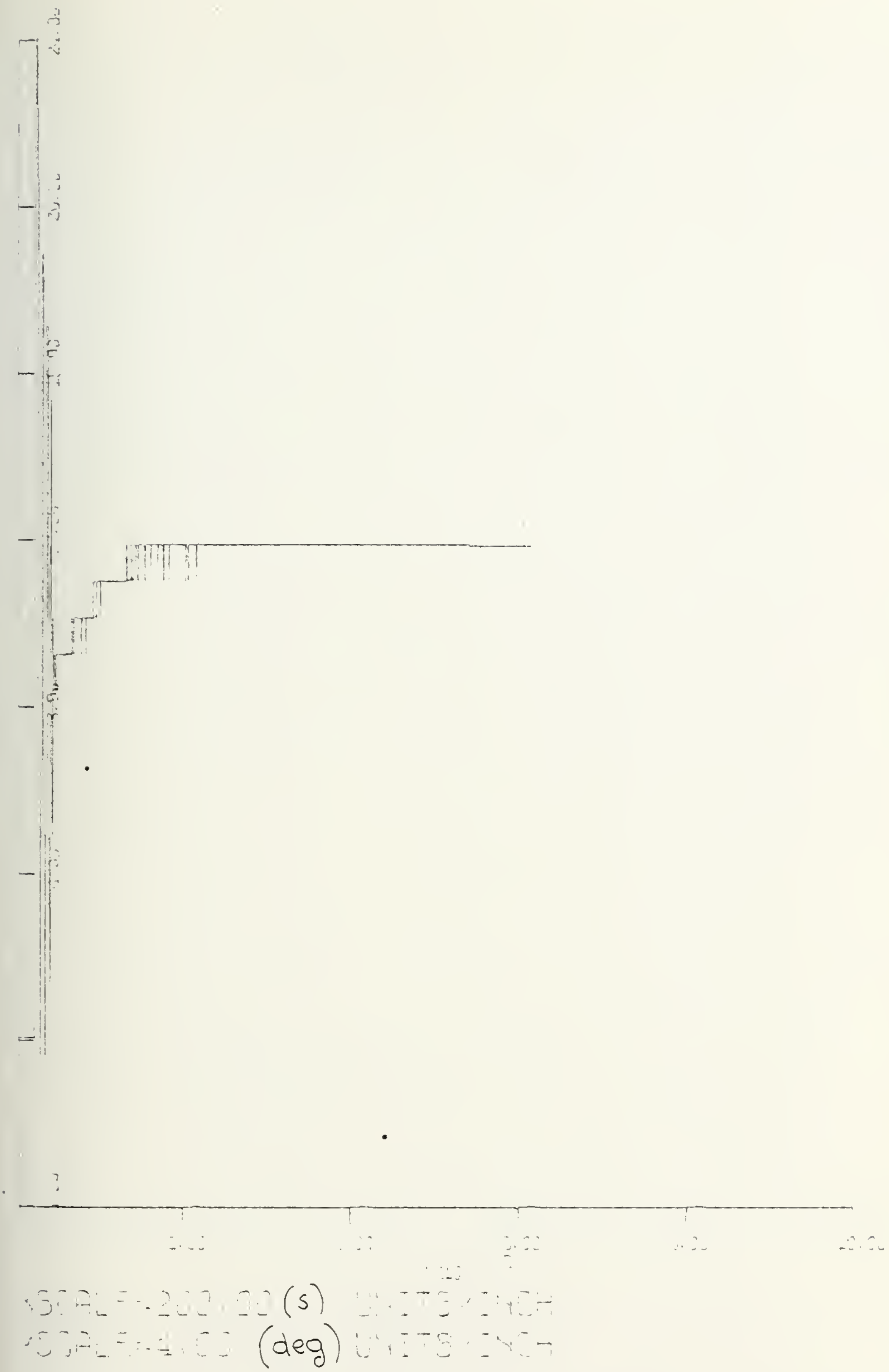


Fig. IV-26c. Stern Plane Angle vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.4

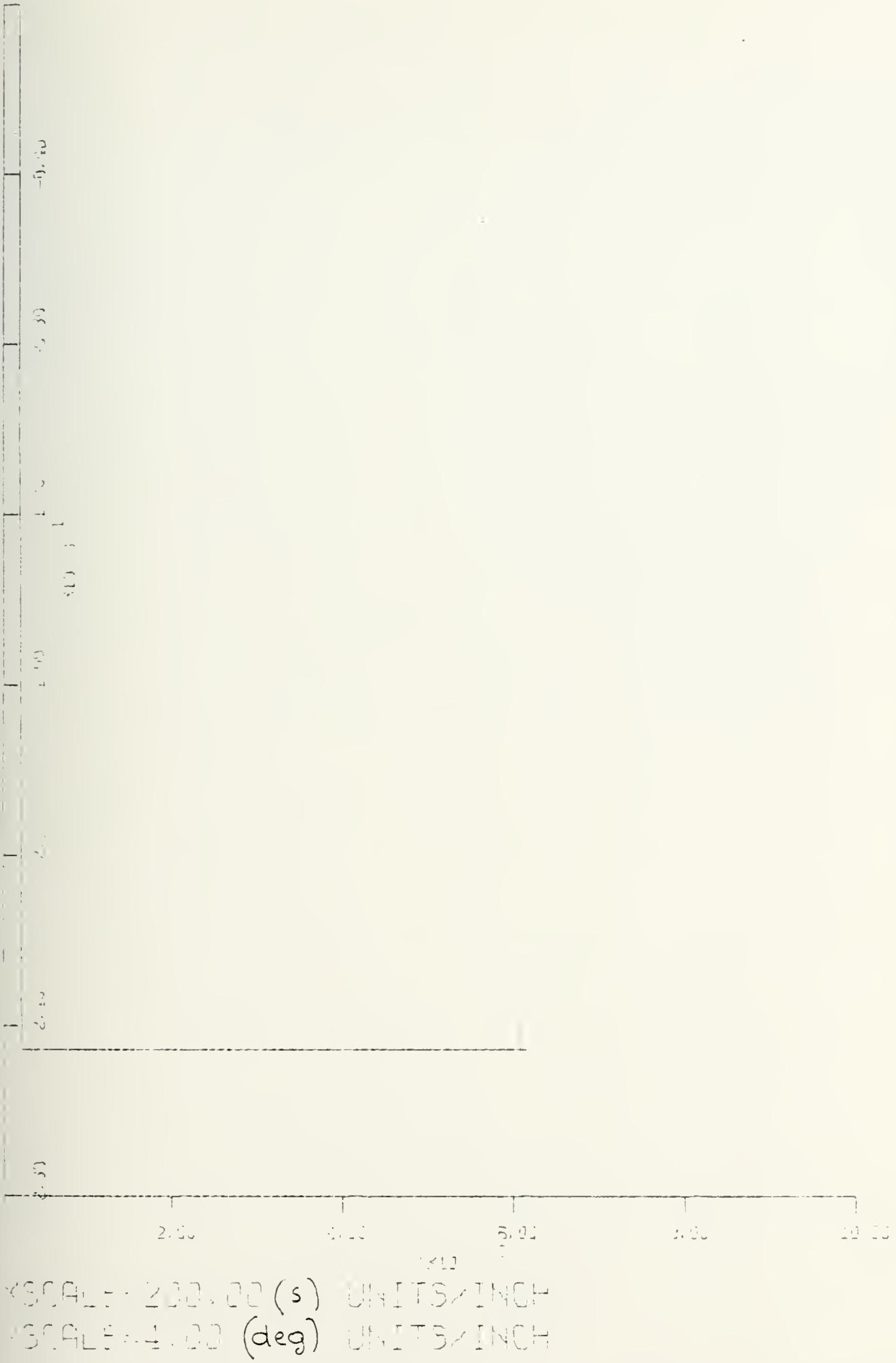


Fig. IV-26d. Fairwater Plane Angle vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.4

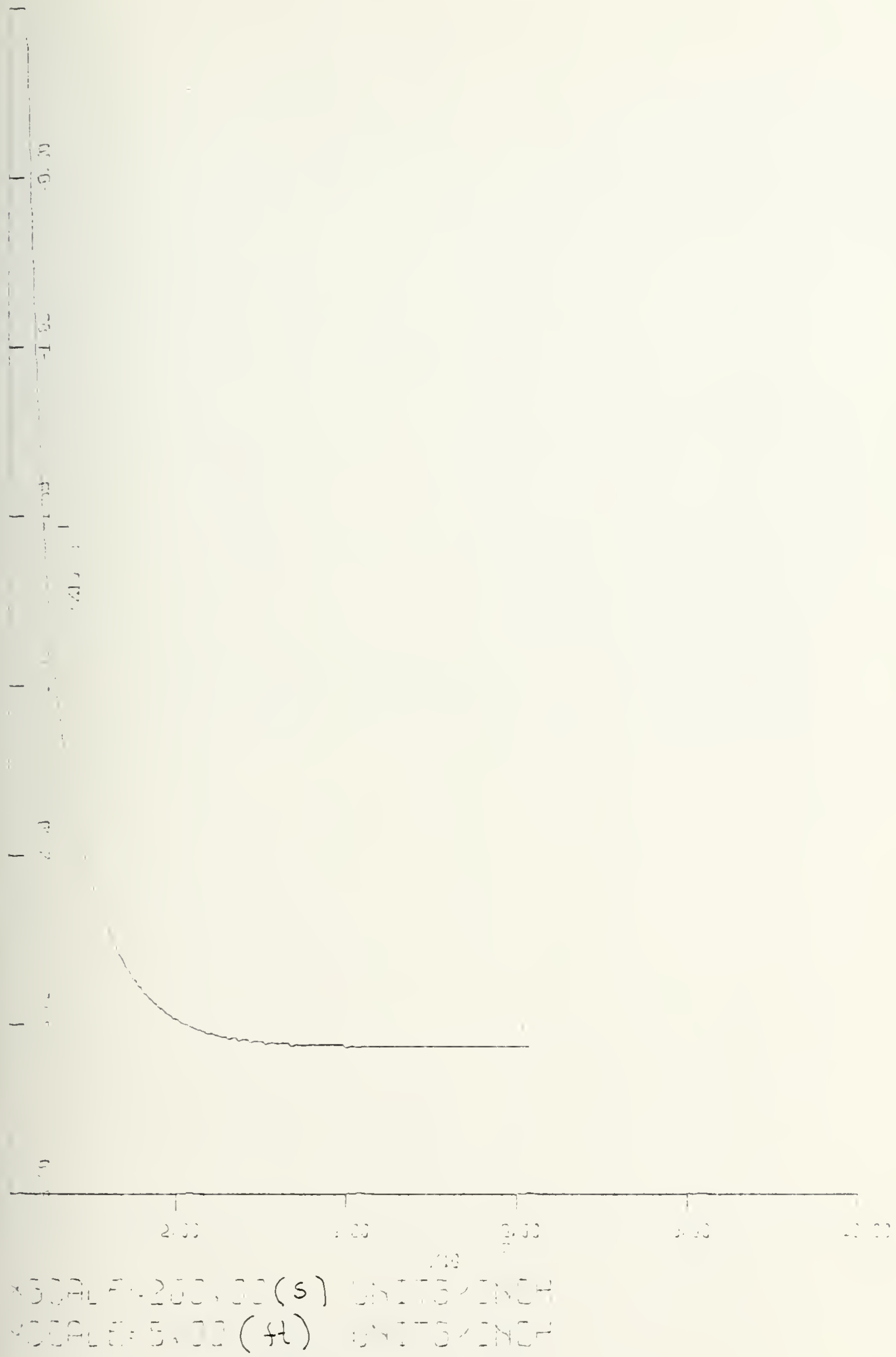


Fig. IV-27a. Depth vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.3

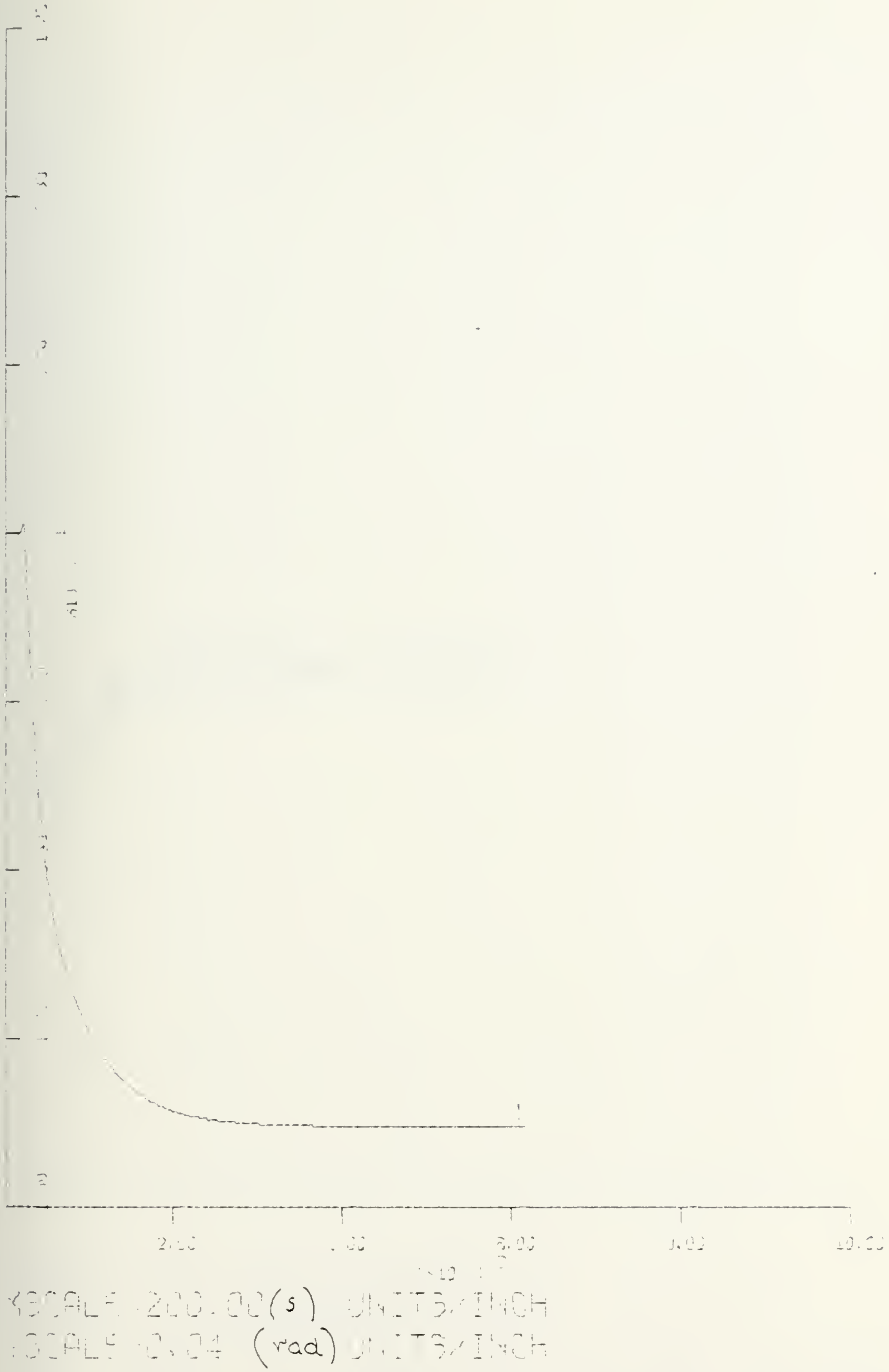


Fig. IV-27b. Pitch vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.3

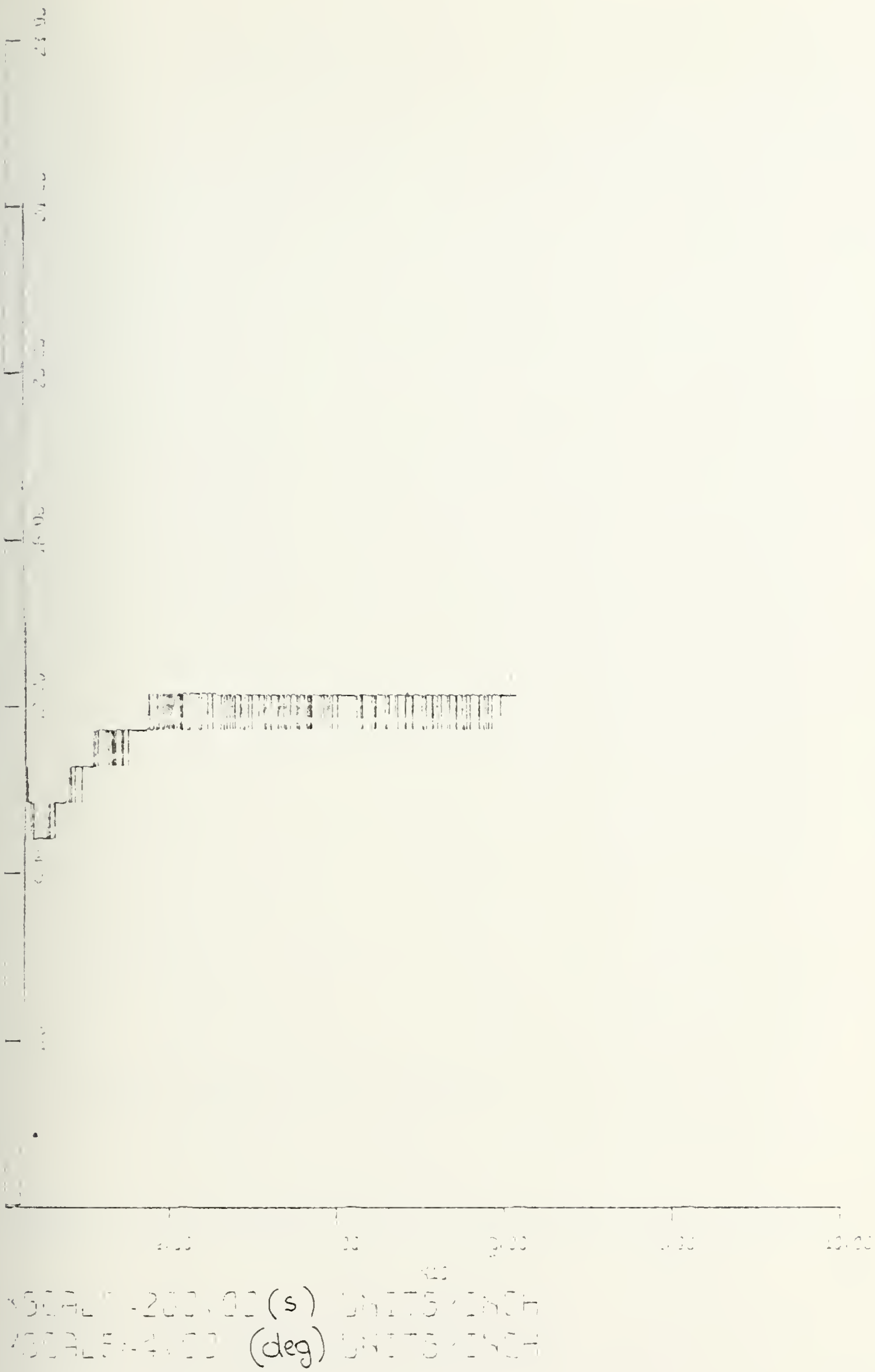


Fig. IV-27c. Stern Plane Angle vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.3

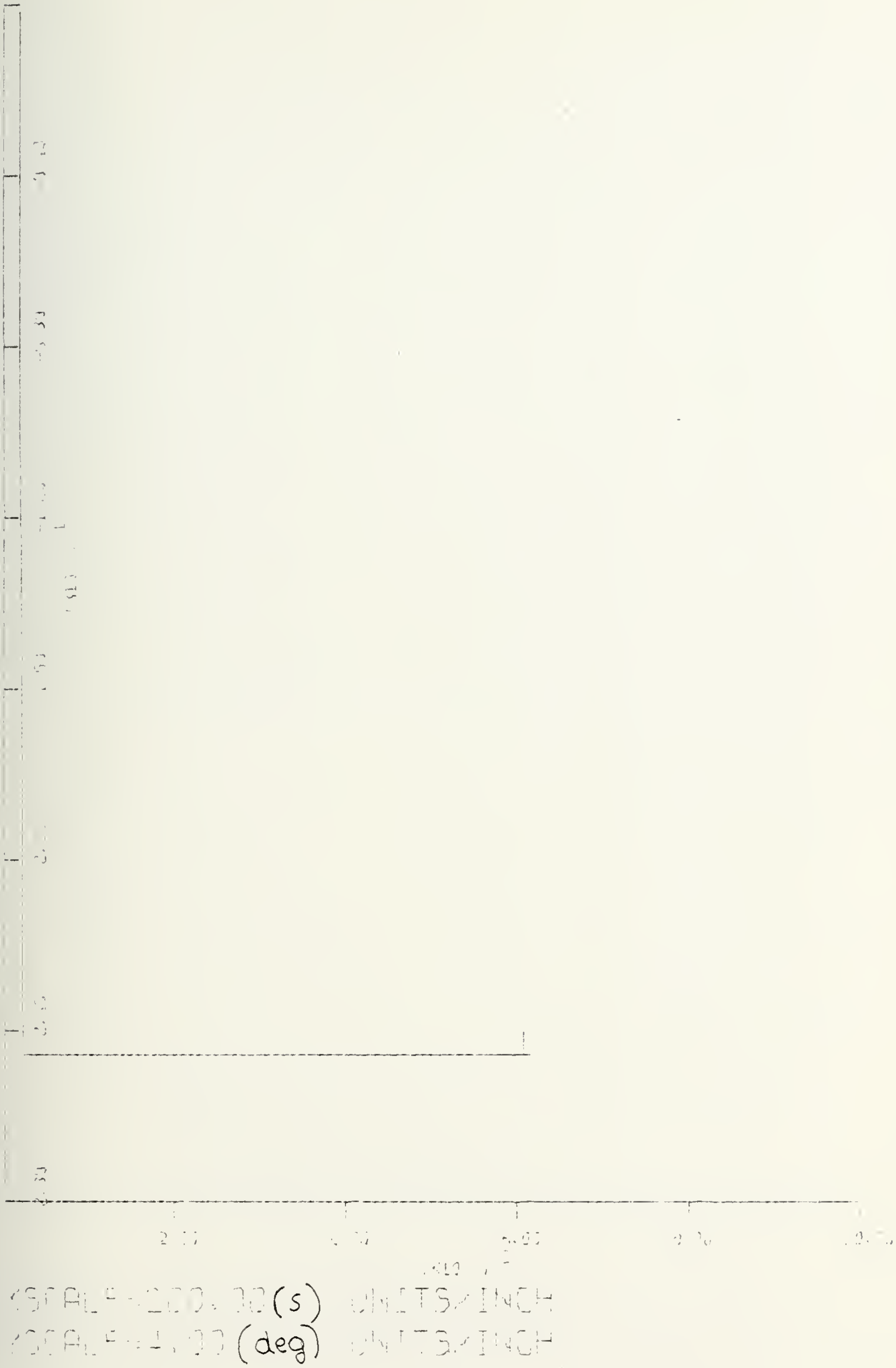


Fig. IV-27d. Fairwater Plane Angle vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.3

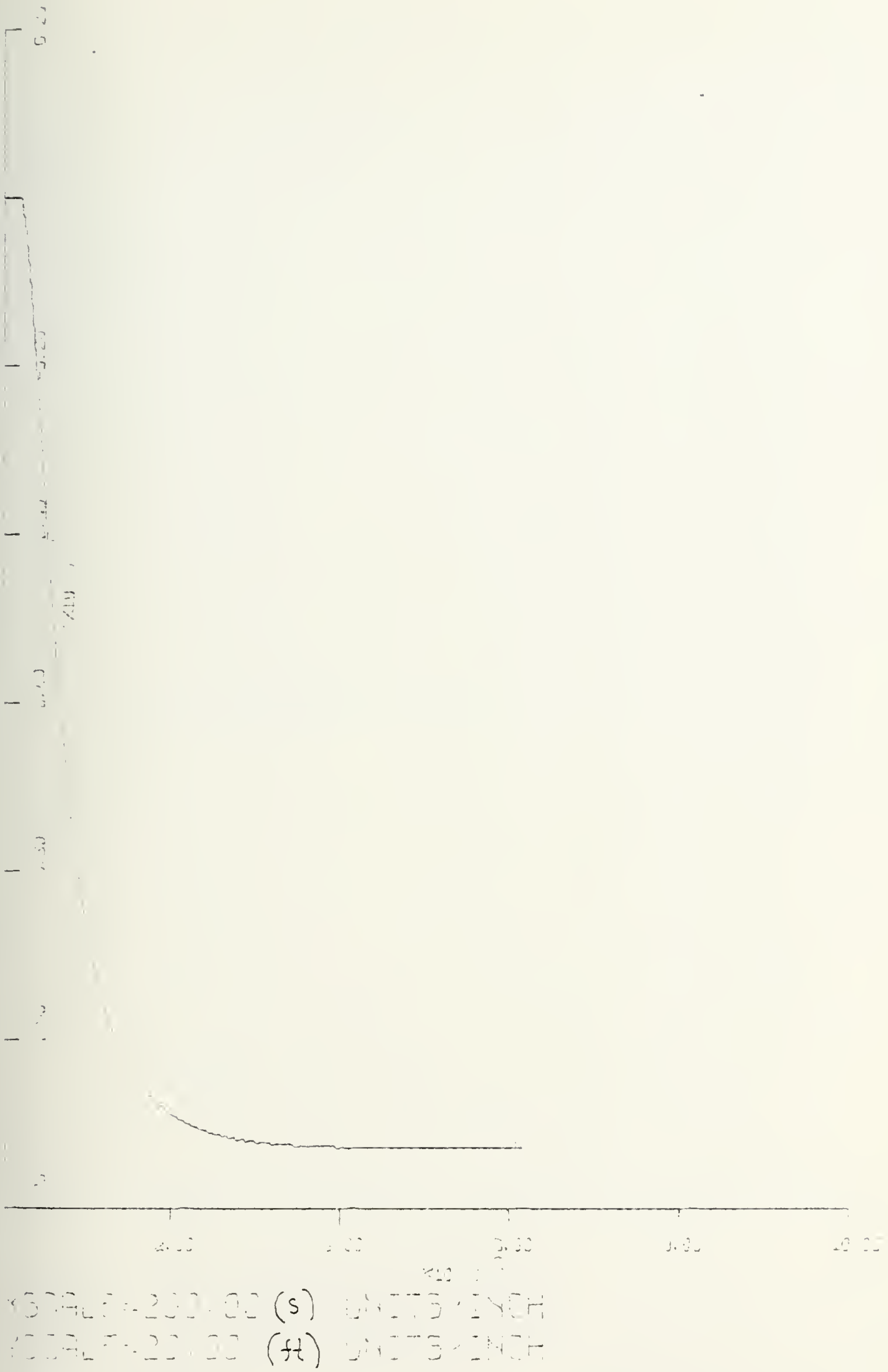


Fig. IV-28a. Depth vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.05

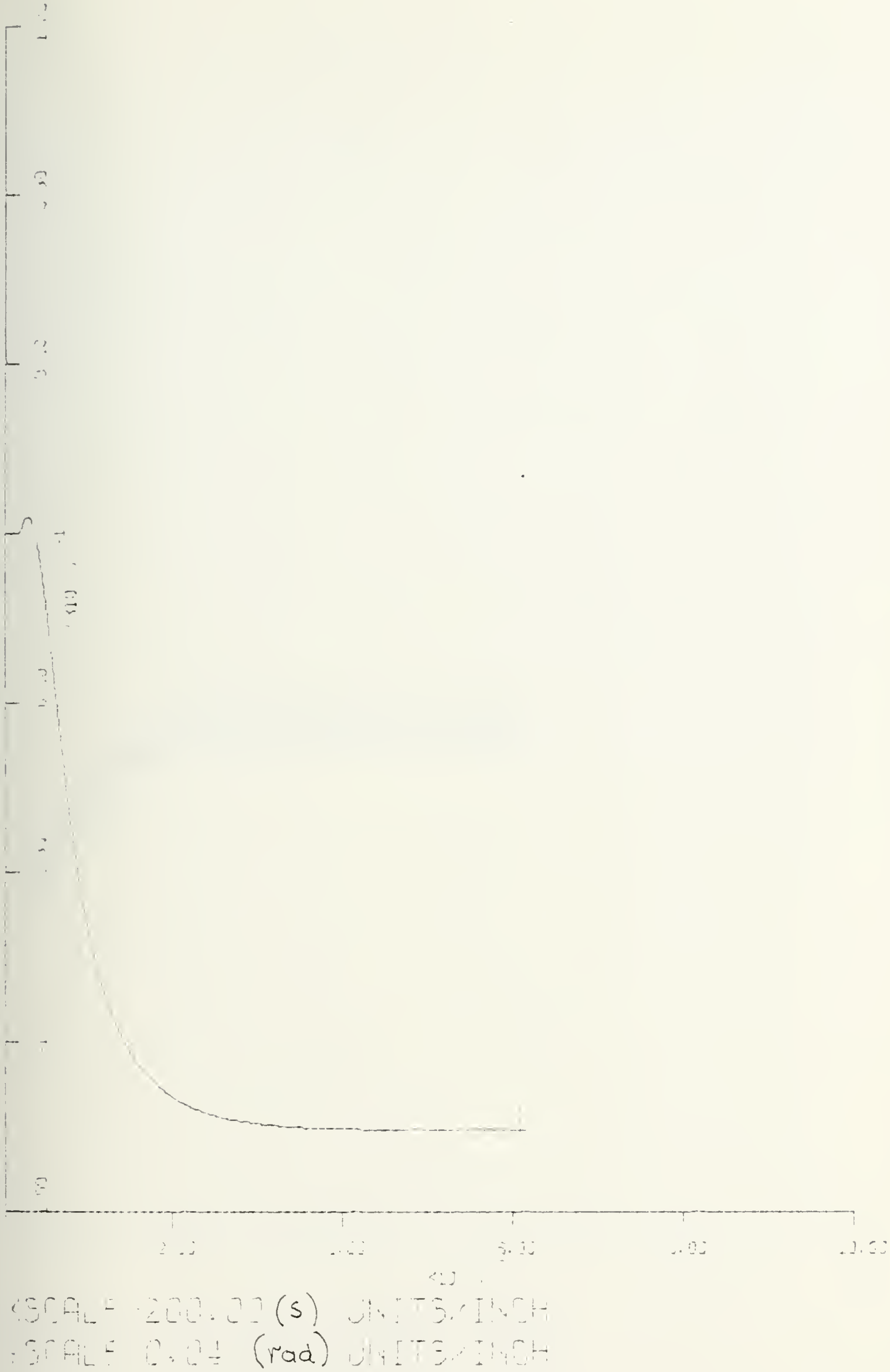


Fig. IV-28b. Pitch vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.05

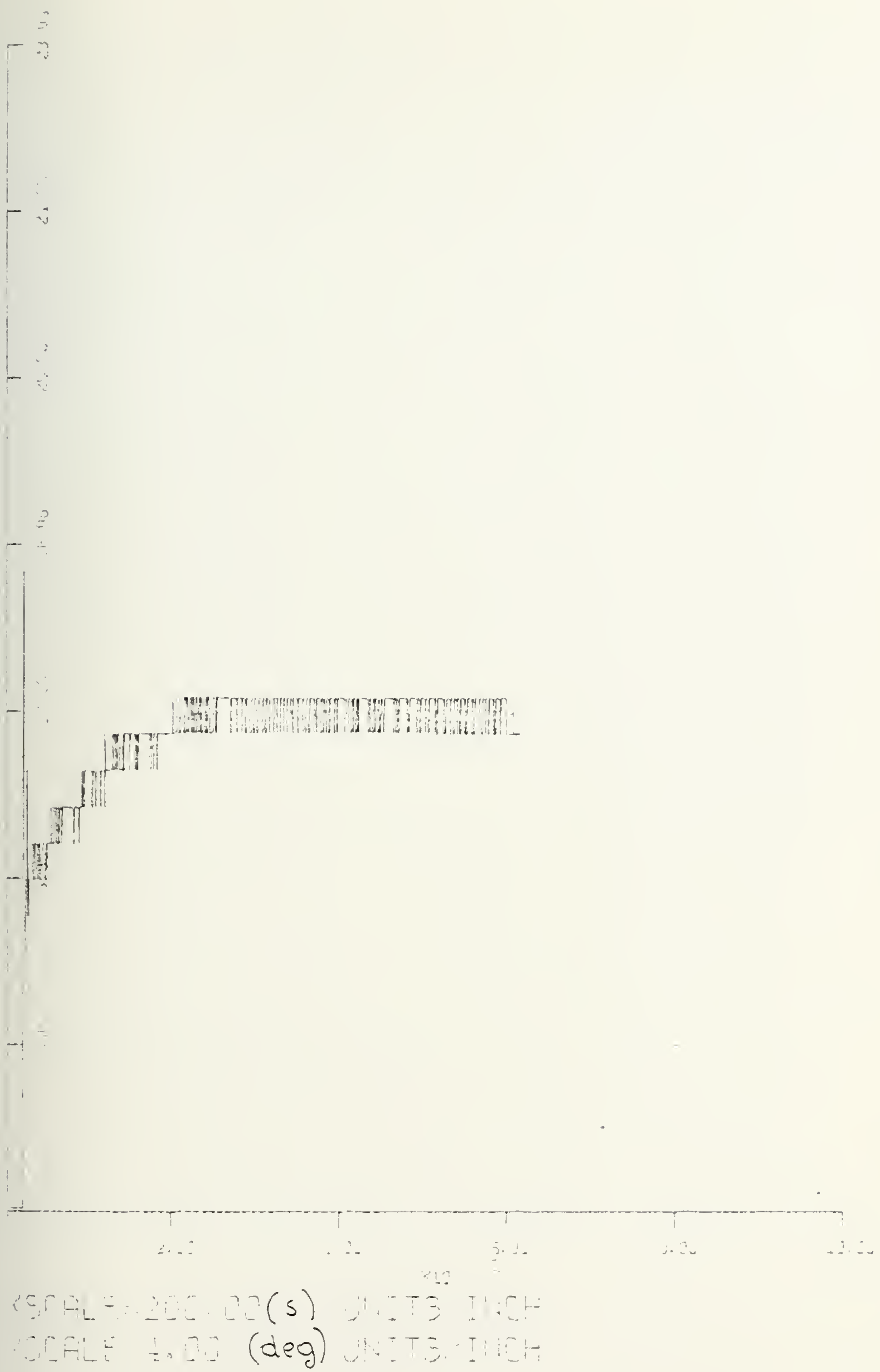


Fig. IV-28c. Stern Plane Angle vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.05

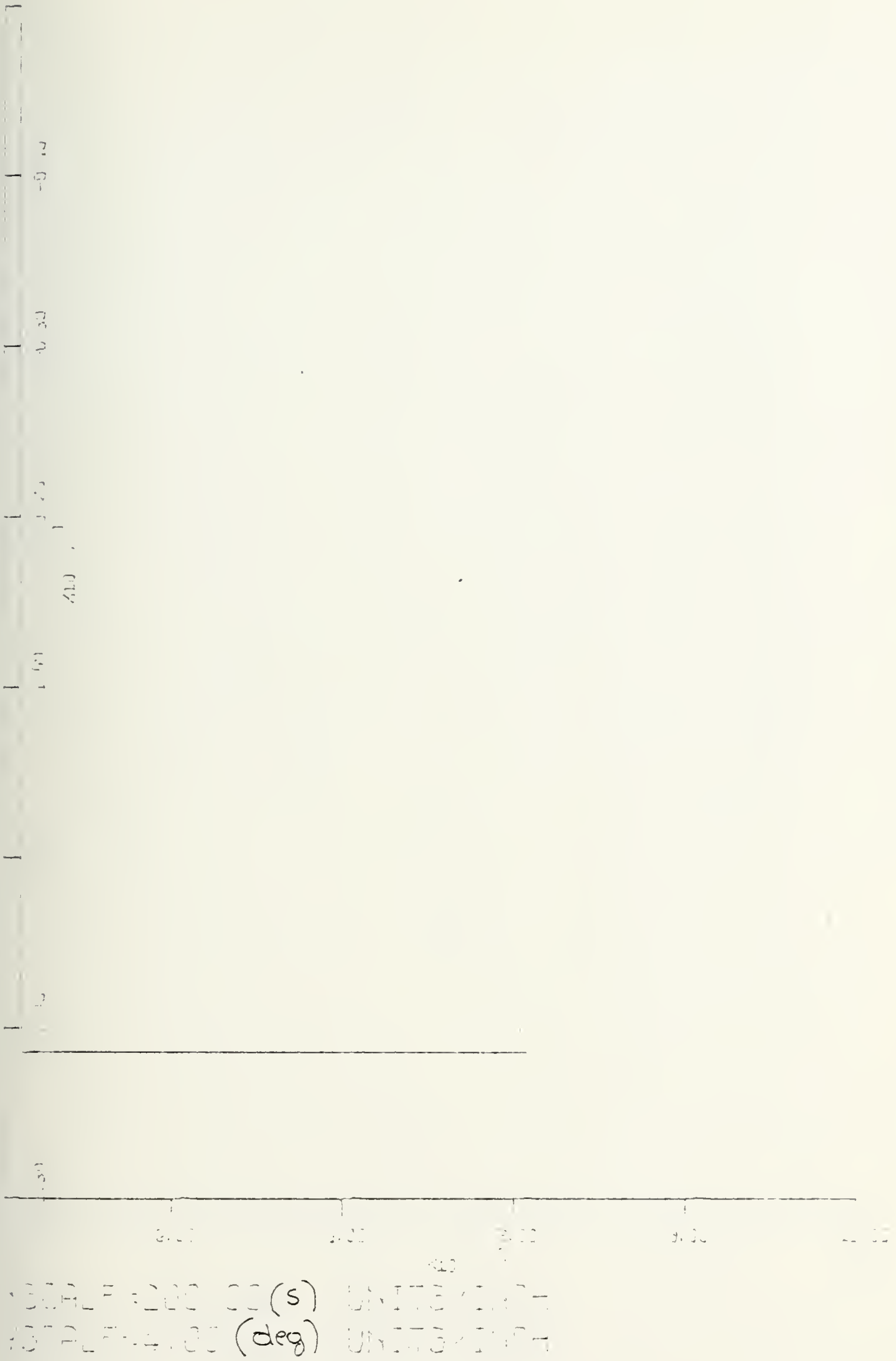


Fig. IV-28d. Fairwater Plane Angle vs. Time. Response to a step force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.05

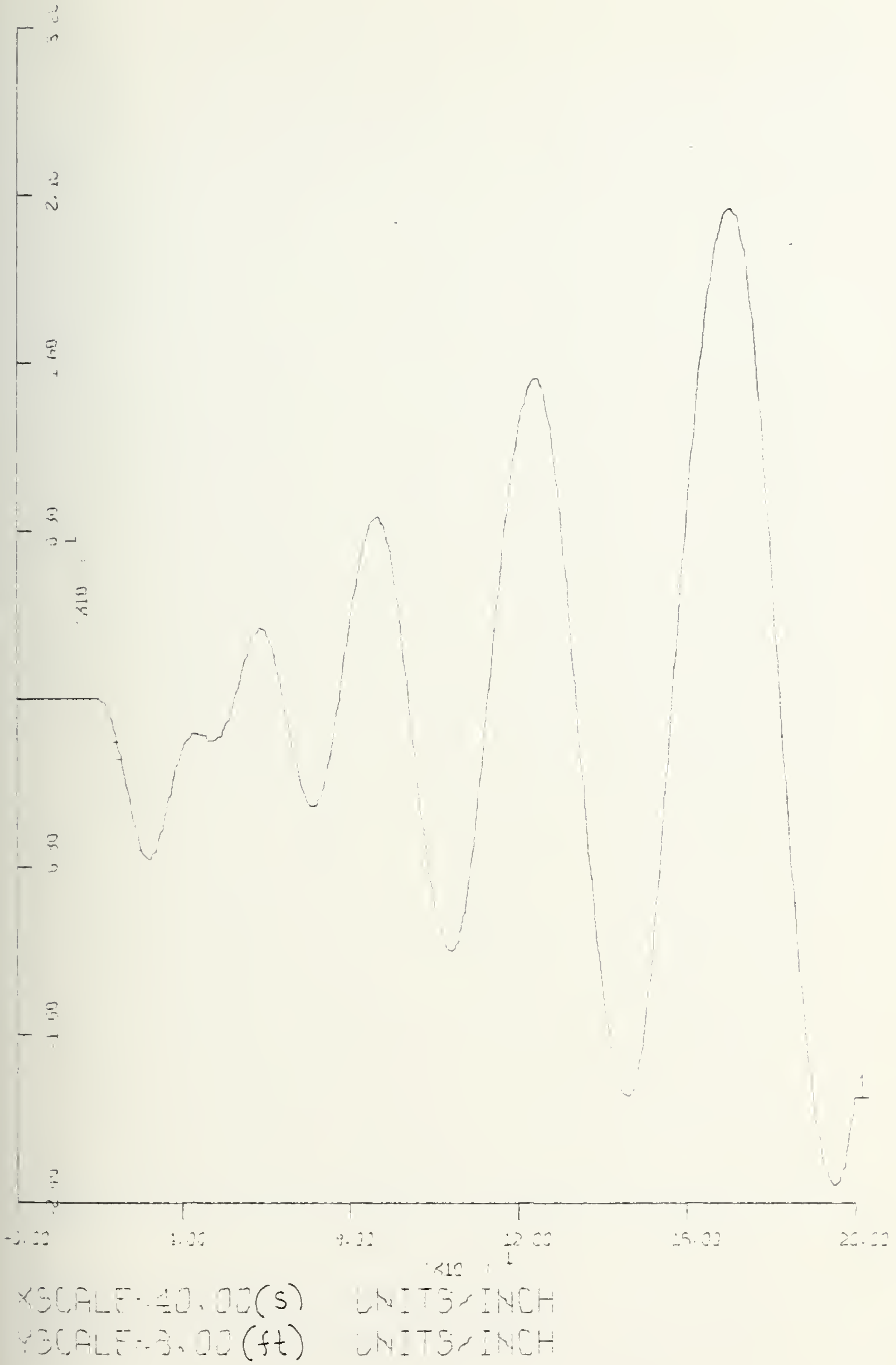


Fig. IV-29a. Depth vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.7

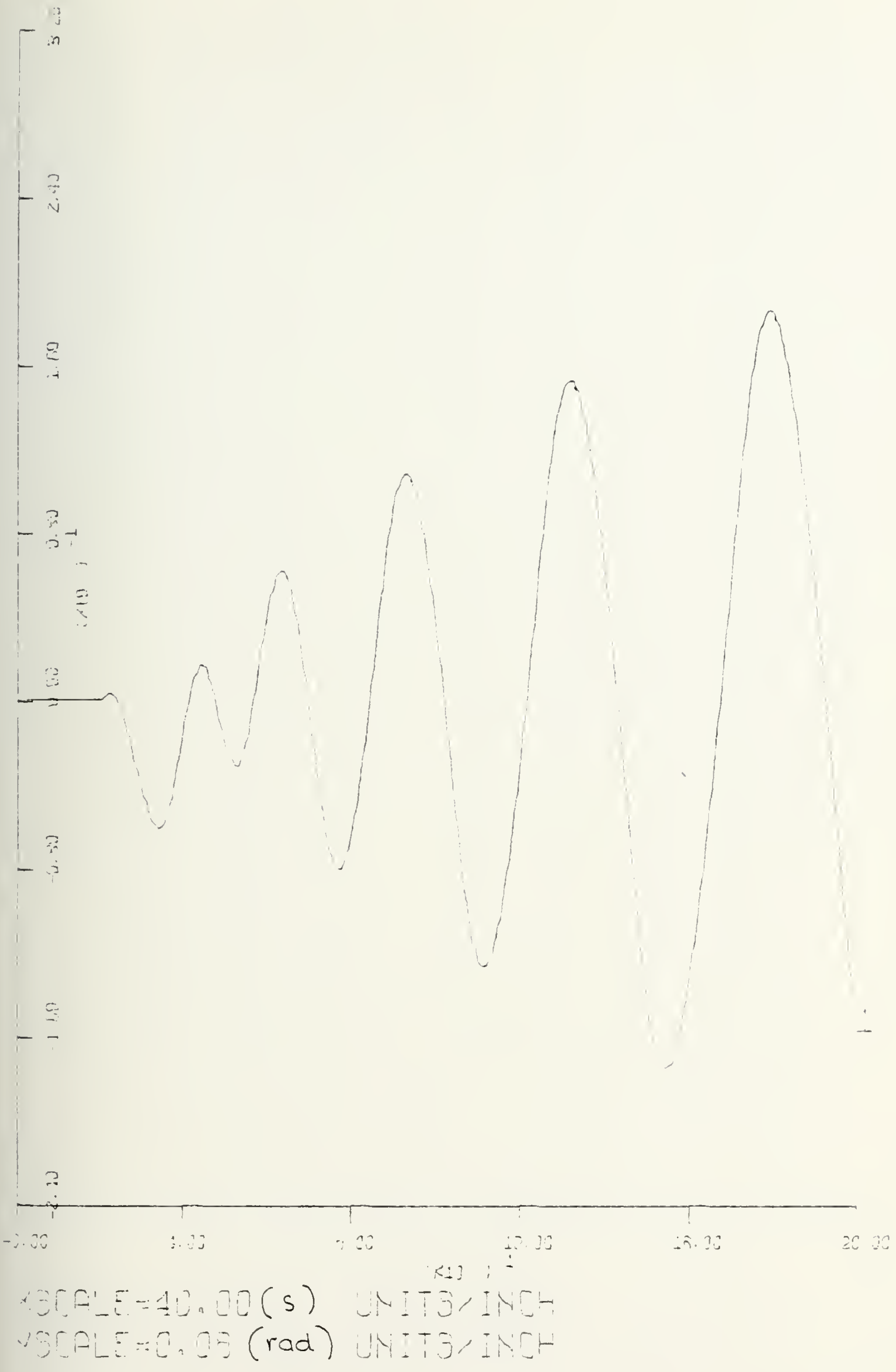


Fig. IV-29b. Pitch vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.7

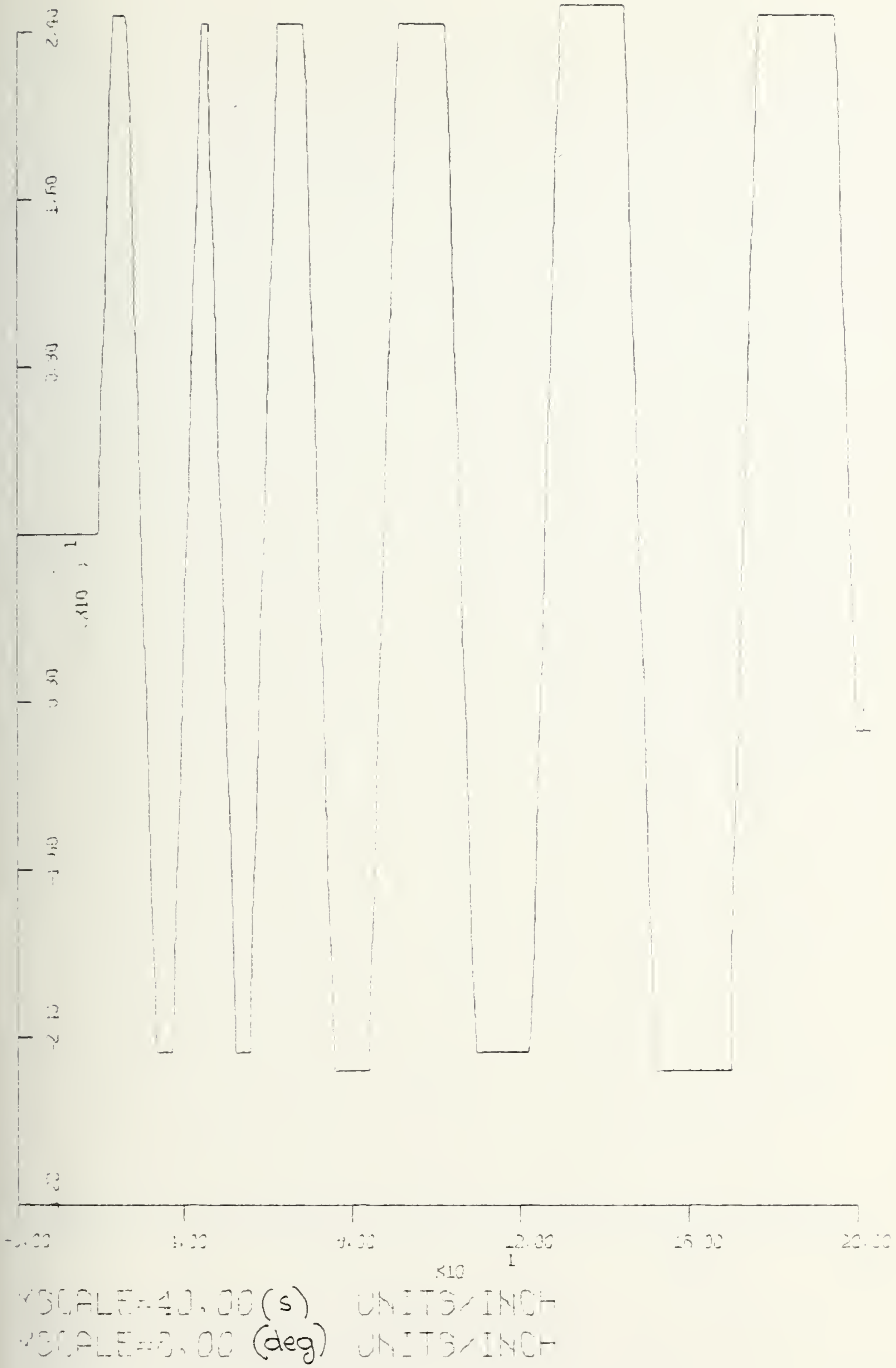


Fig. IV-29c. Stern Plane Angle vs. Time. Response to a pulse fotce at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.7

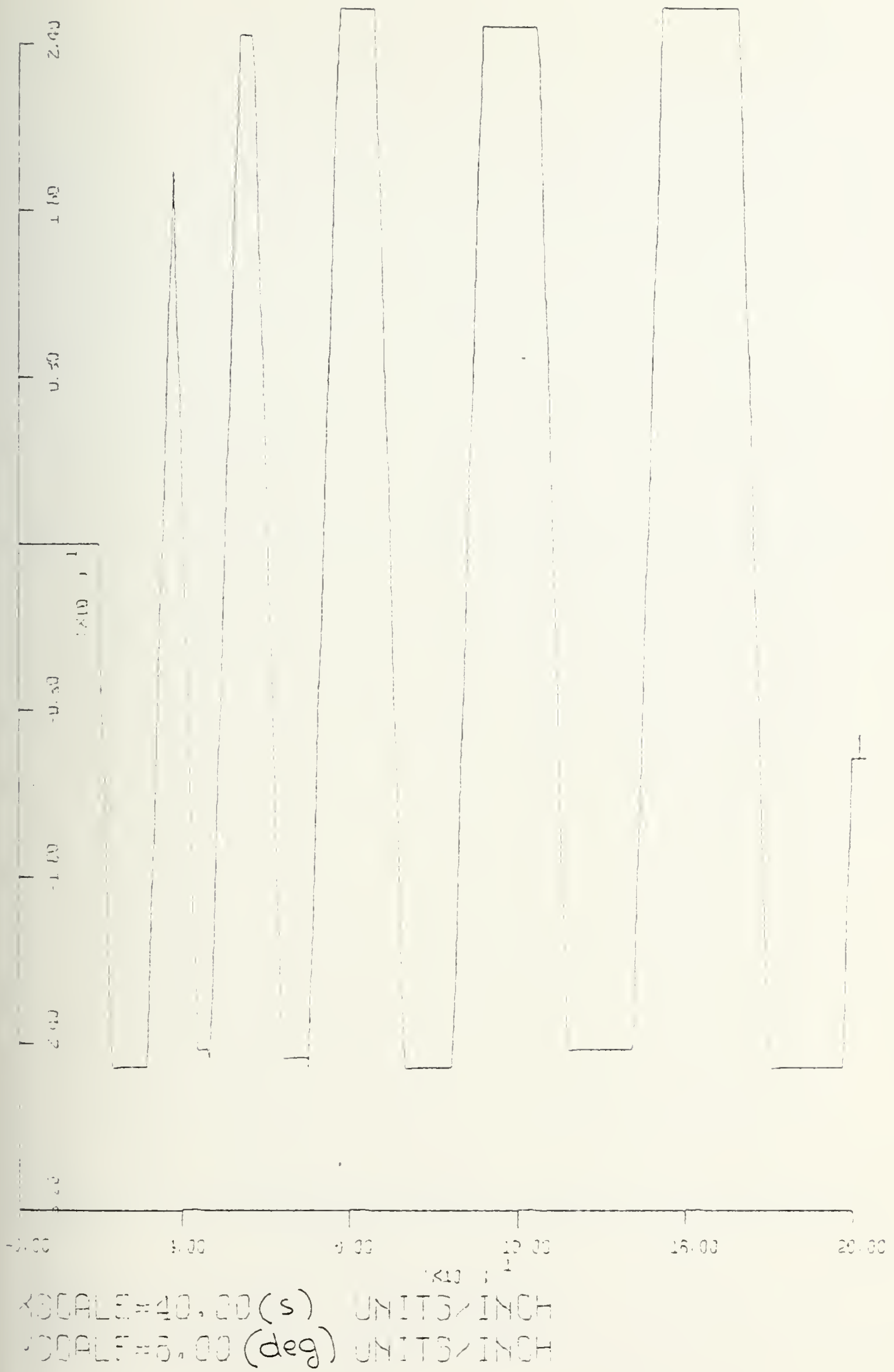


Fig. IV-29d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.7

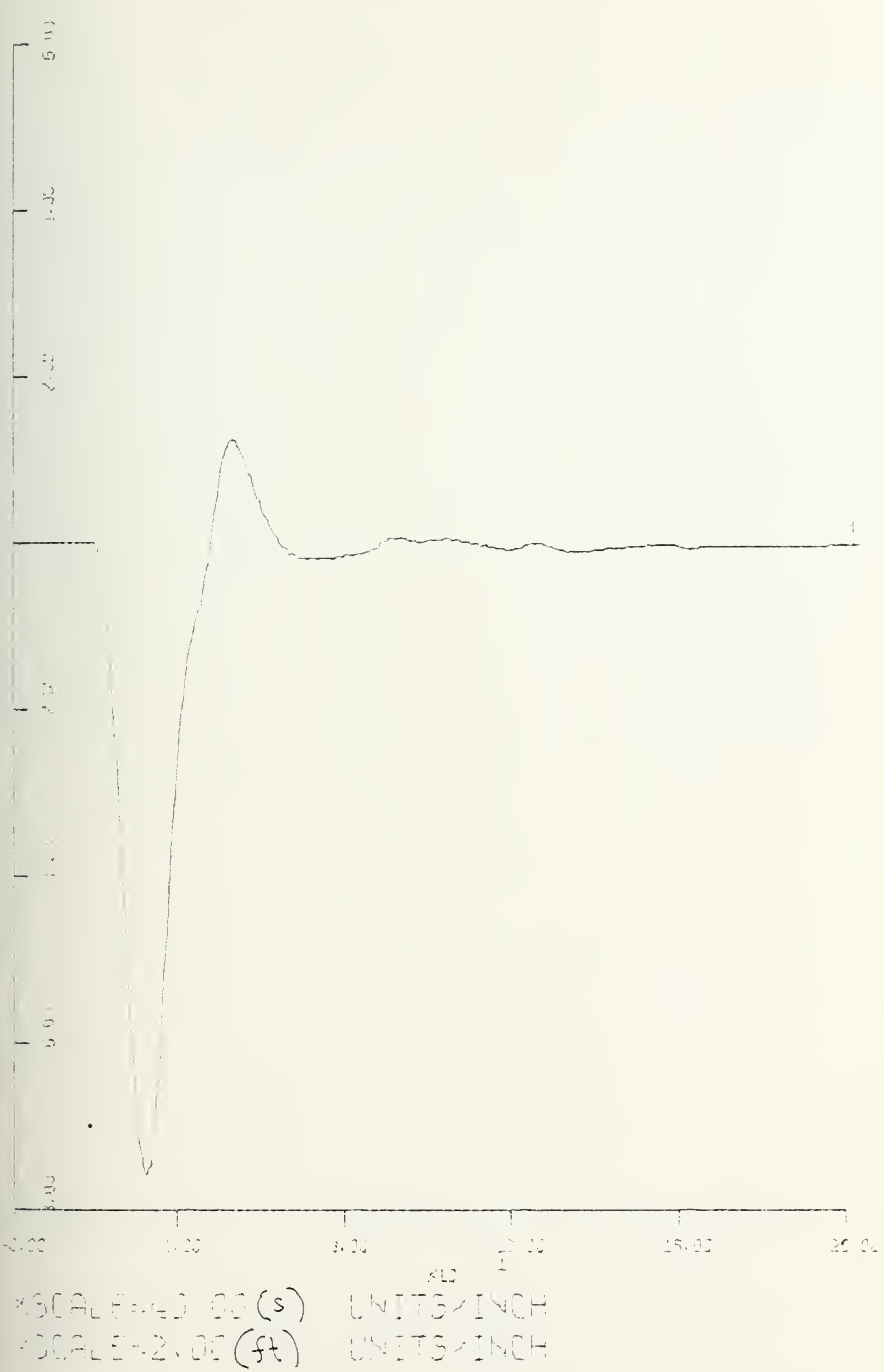


Fig. IV-30a. Depth vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.6

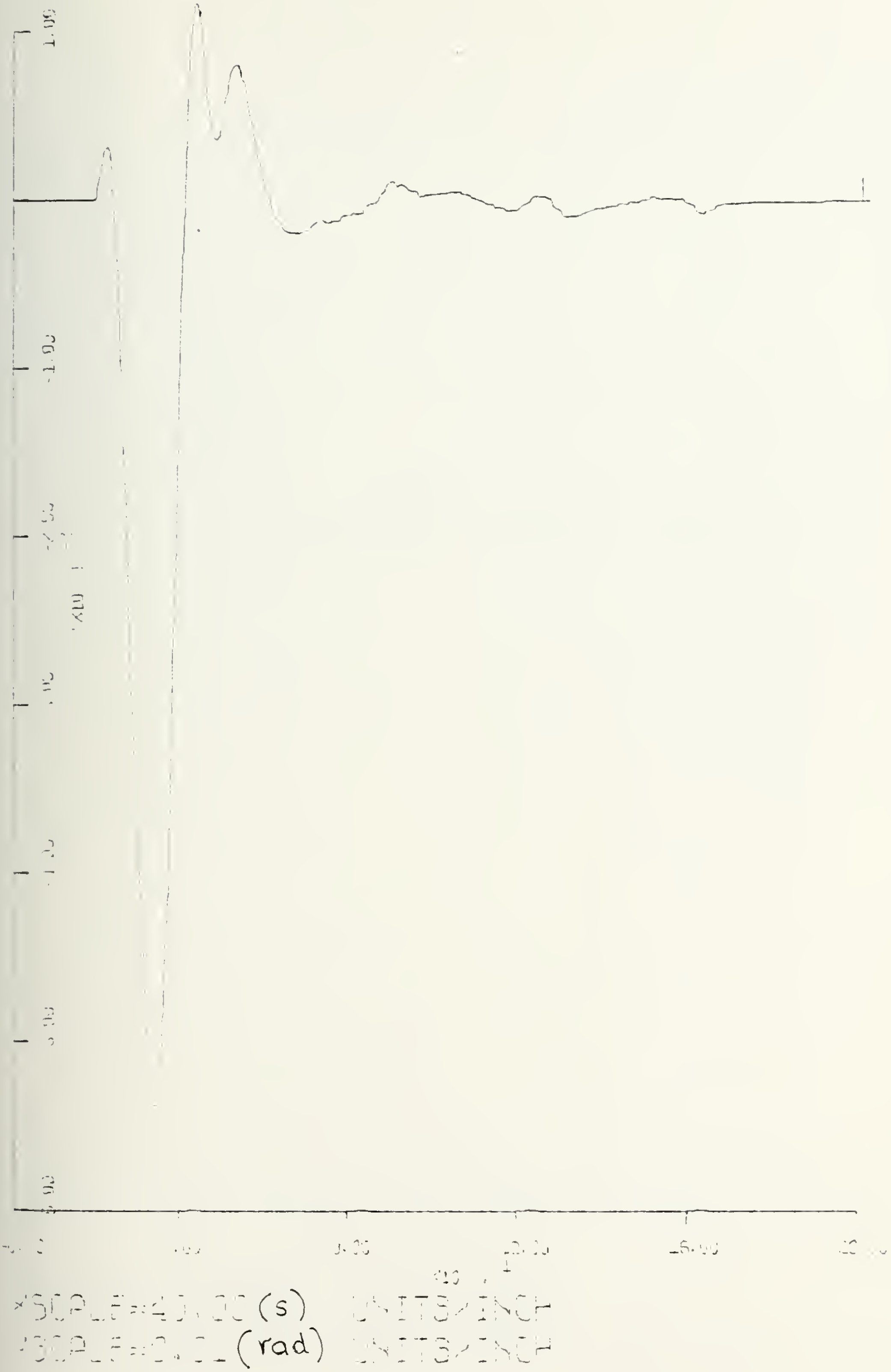


Fig. IV-30b. Pitch vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.6

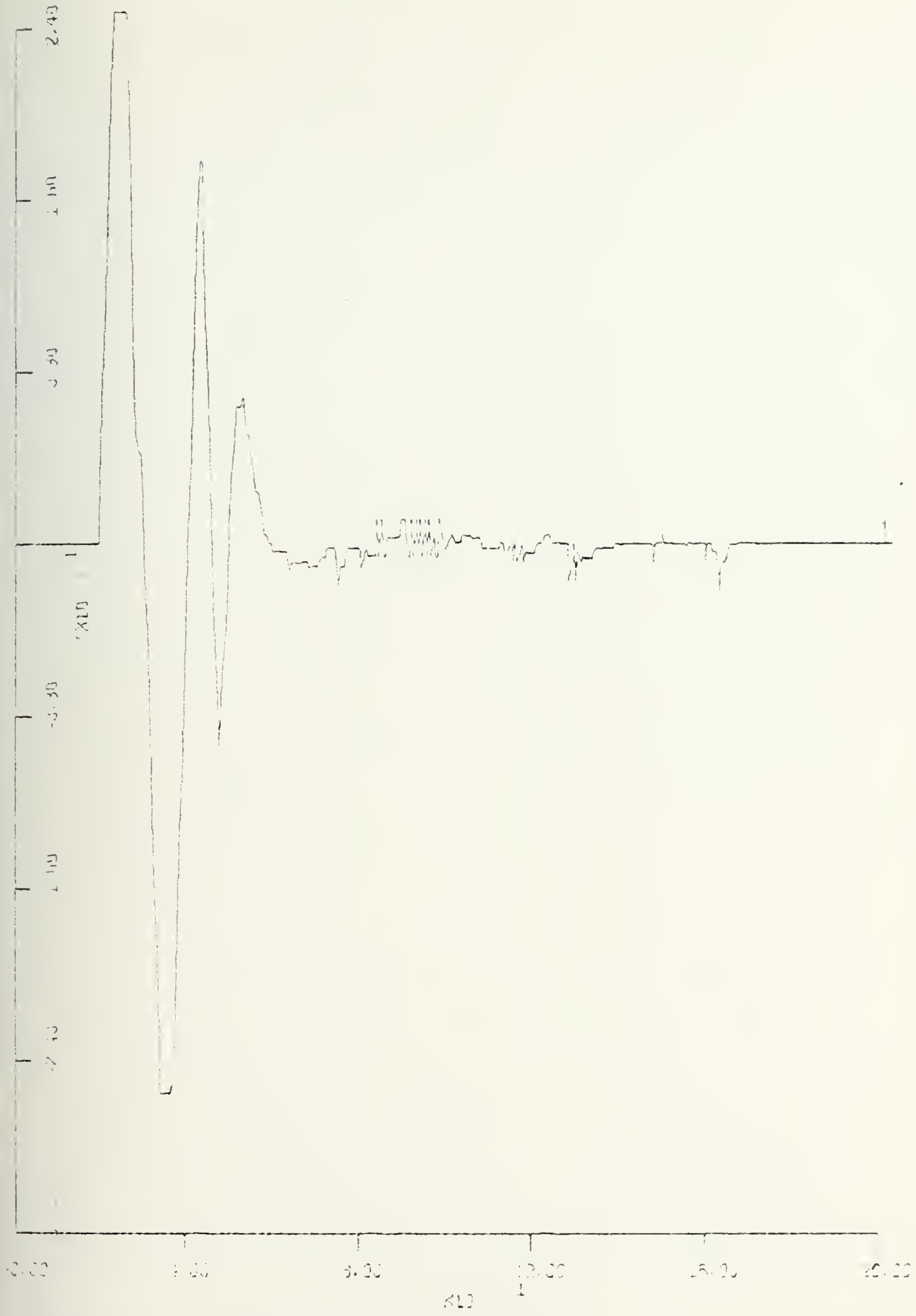


Fig. IV-30c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.6

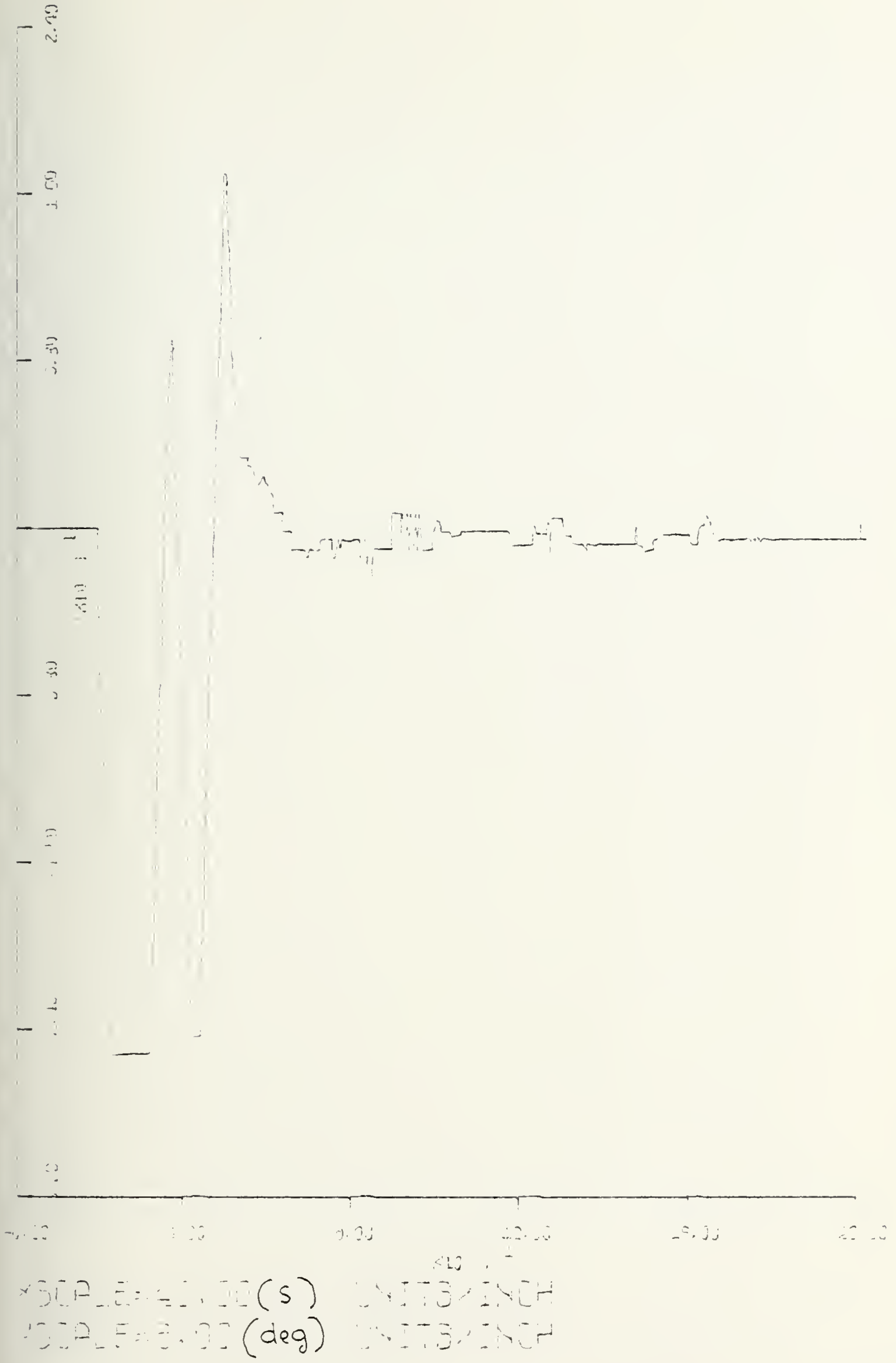


Fig. IV-30d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.6

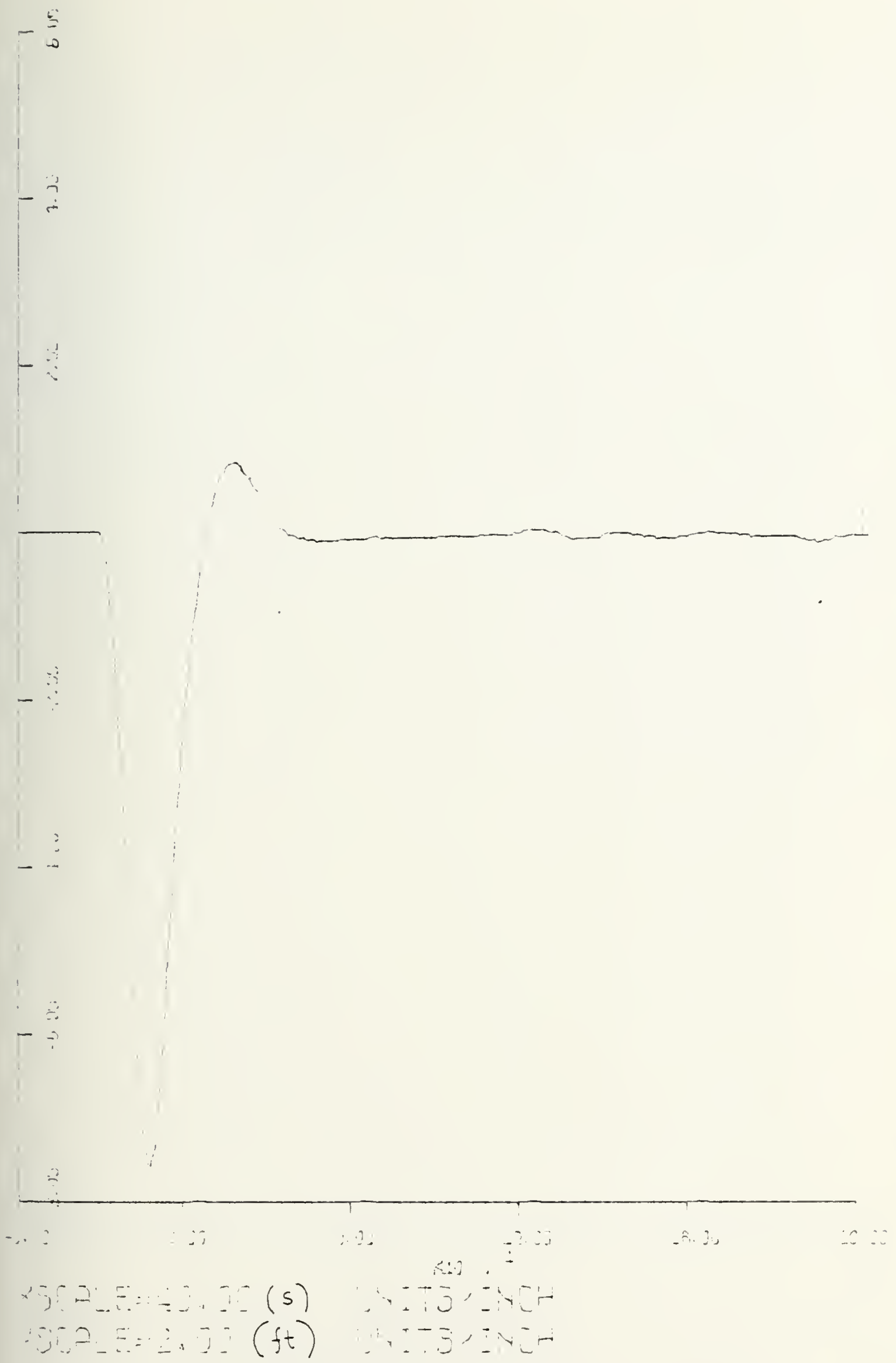


Fig. IV-31a. Depth vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.55

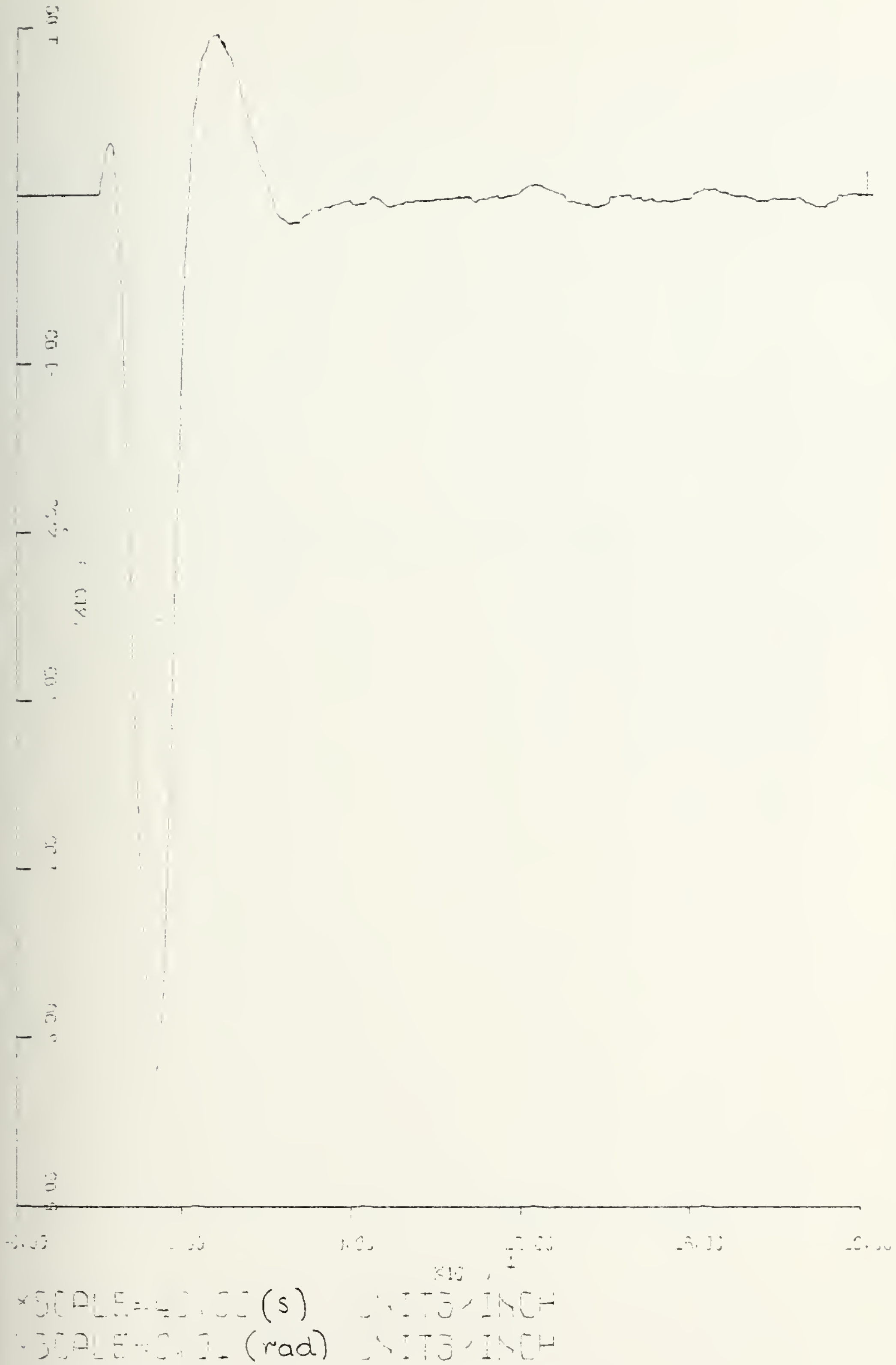


Fig. IV-31b. Pitch vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.55

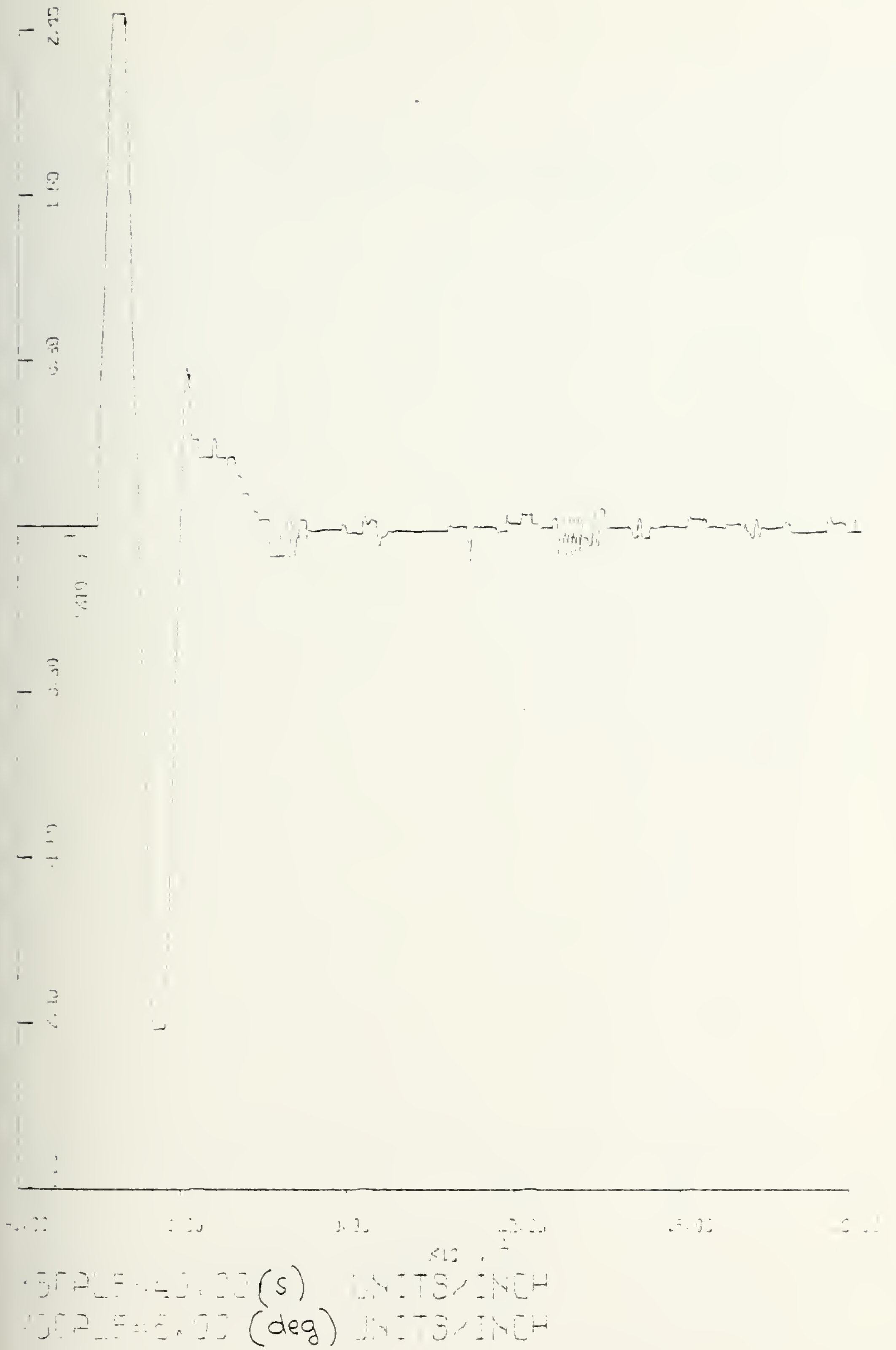


Fig. IV-31c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.55

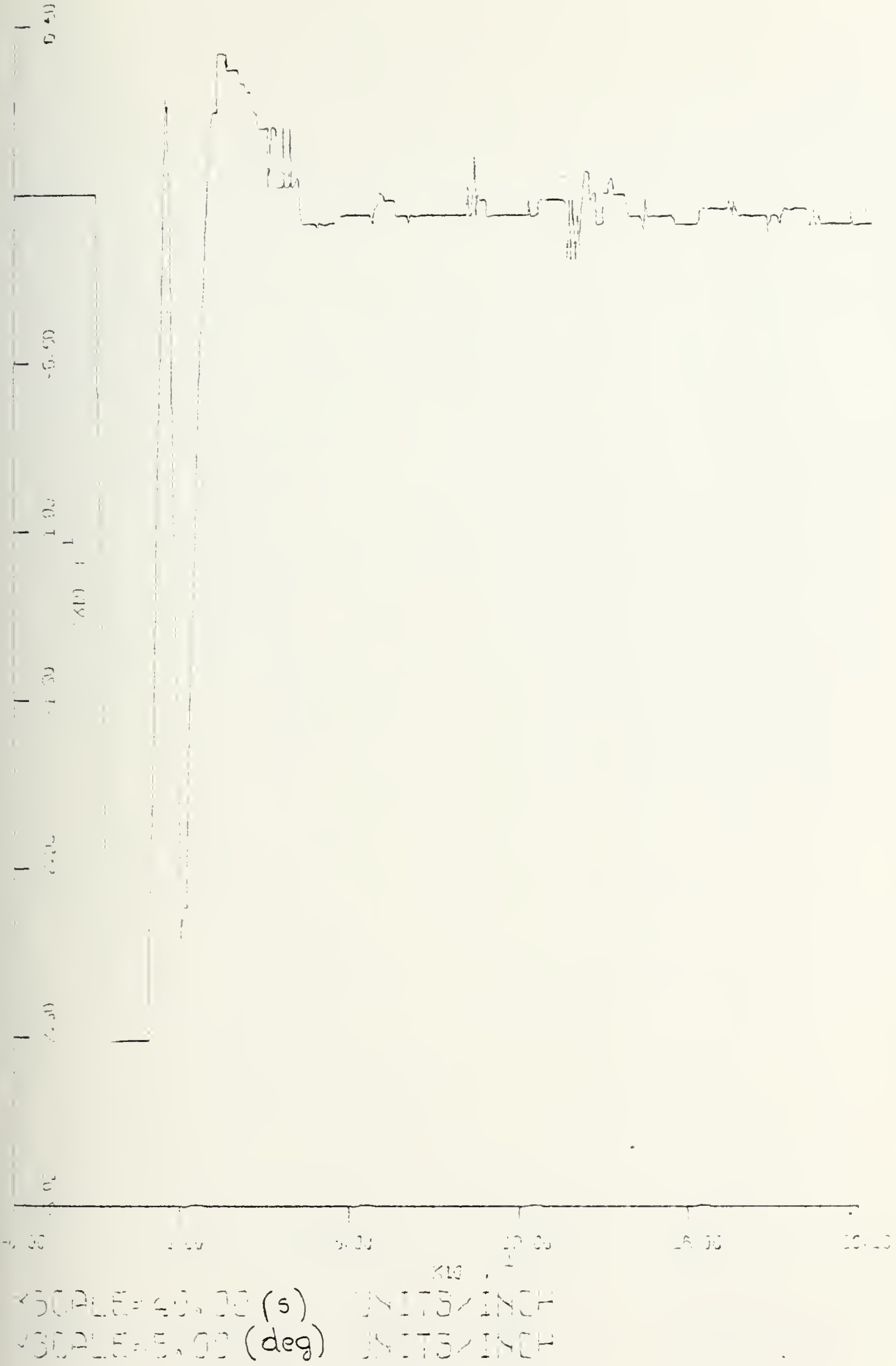


Fig. IV-31d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.55

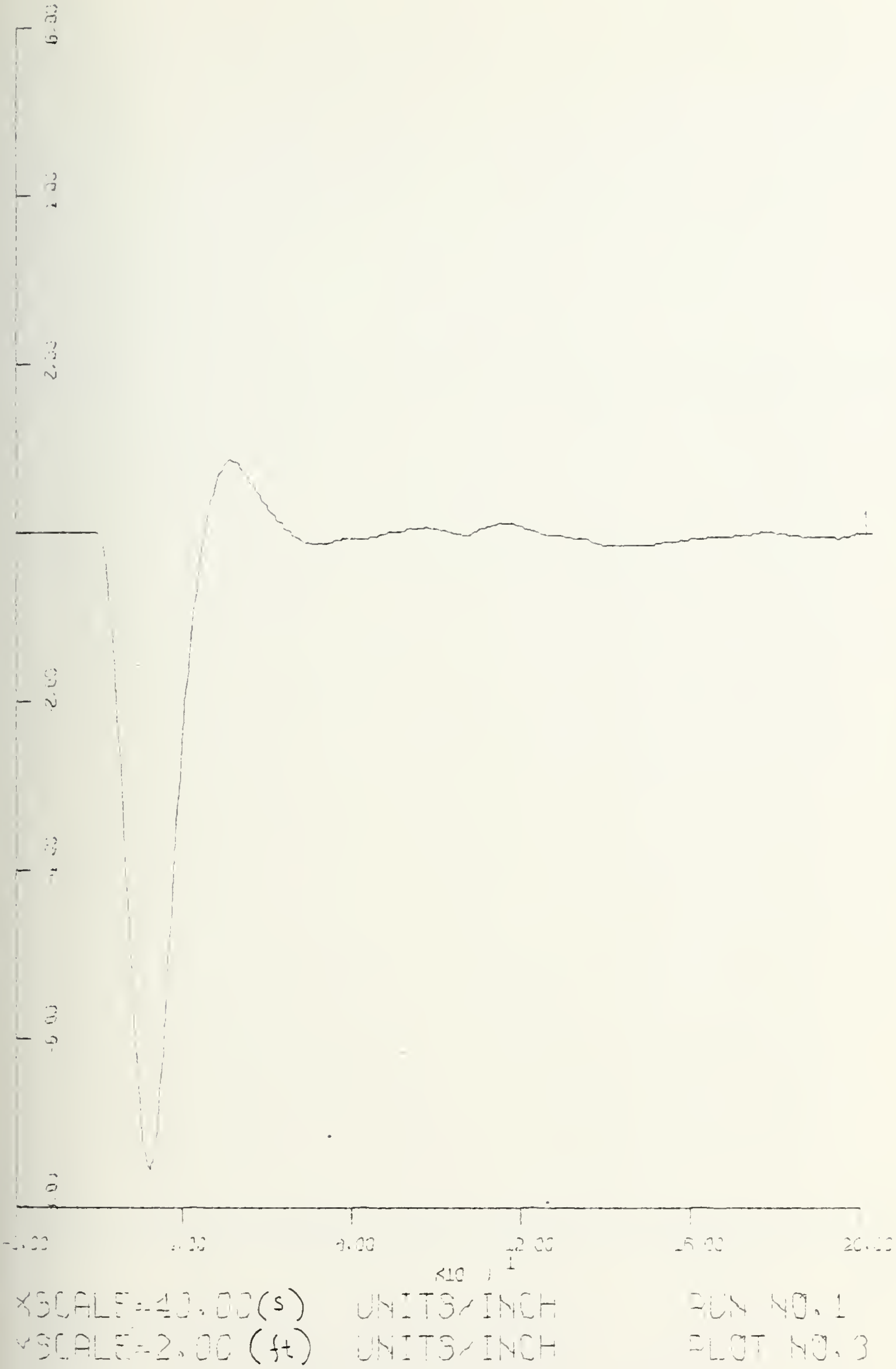
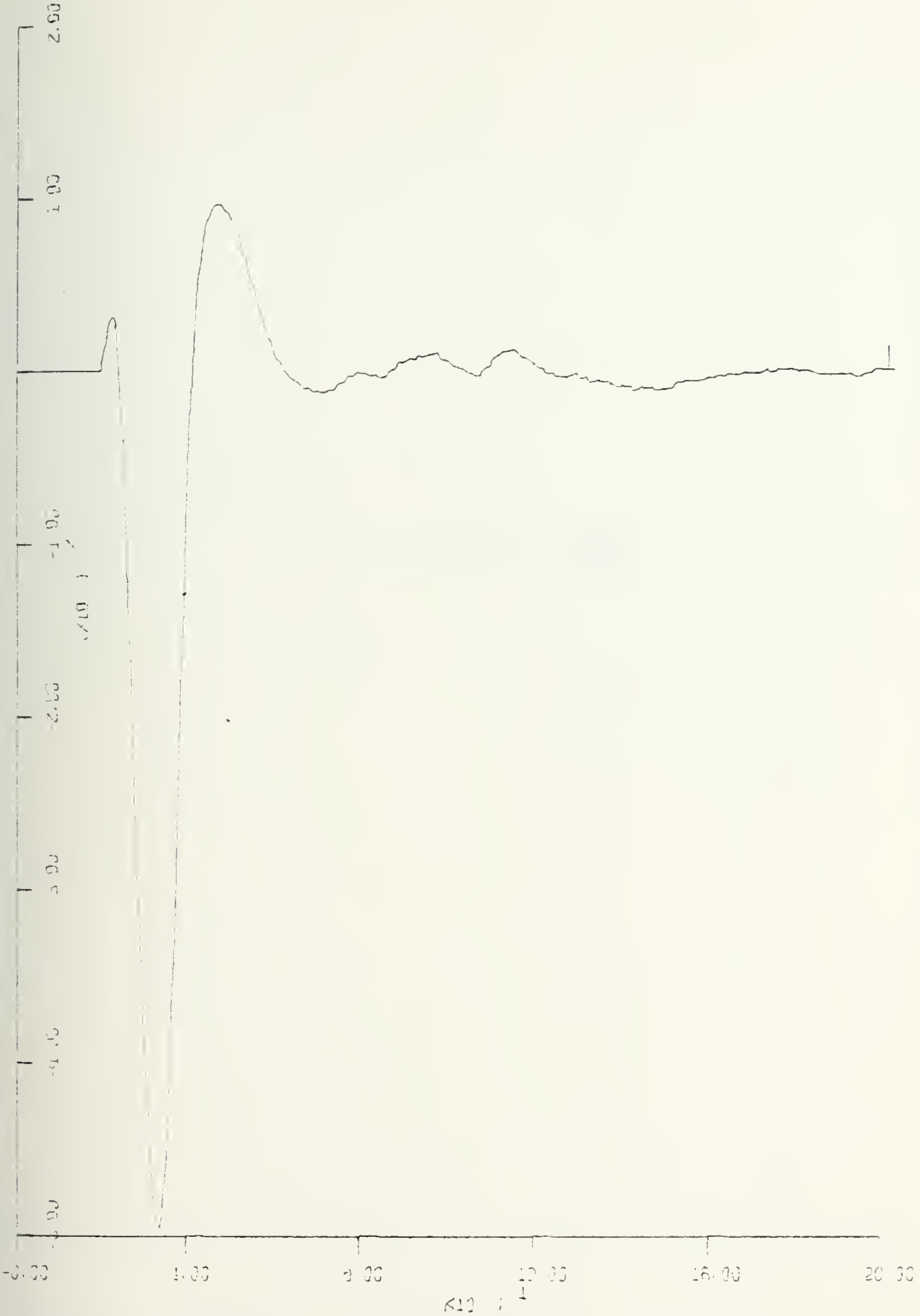


Fig. IV-32a. Depth vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers with B=800, C=10, E=1. Parameter X=0.5



XSCALE=40.00(s) UNITS/INCH
YSCALE=0.01 (rad) UNITS/INCH

Fig. IV-32b. Pitch vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.5

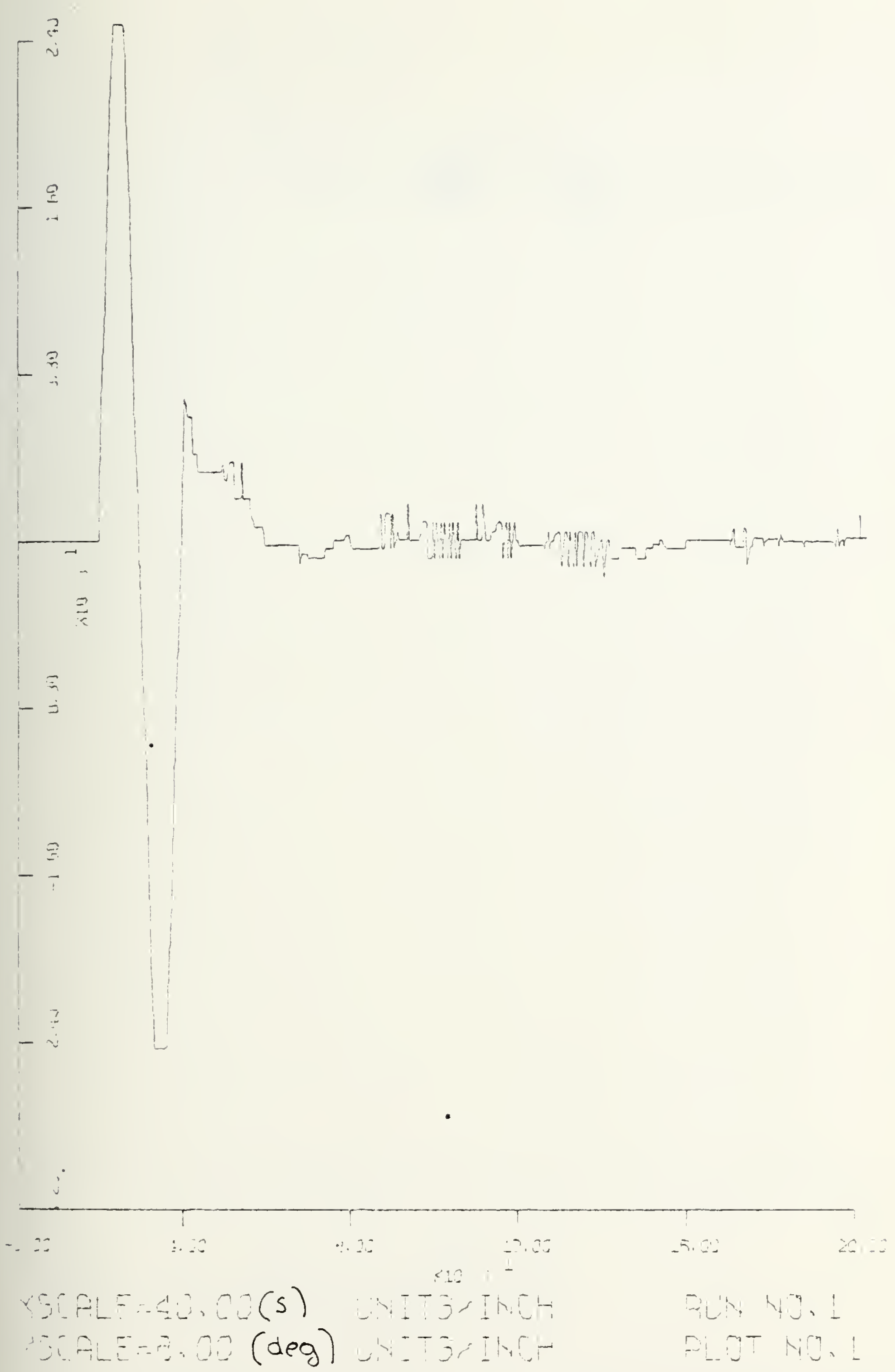


Fig. IV-32c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.5

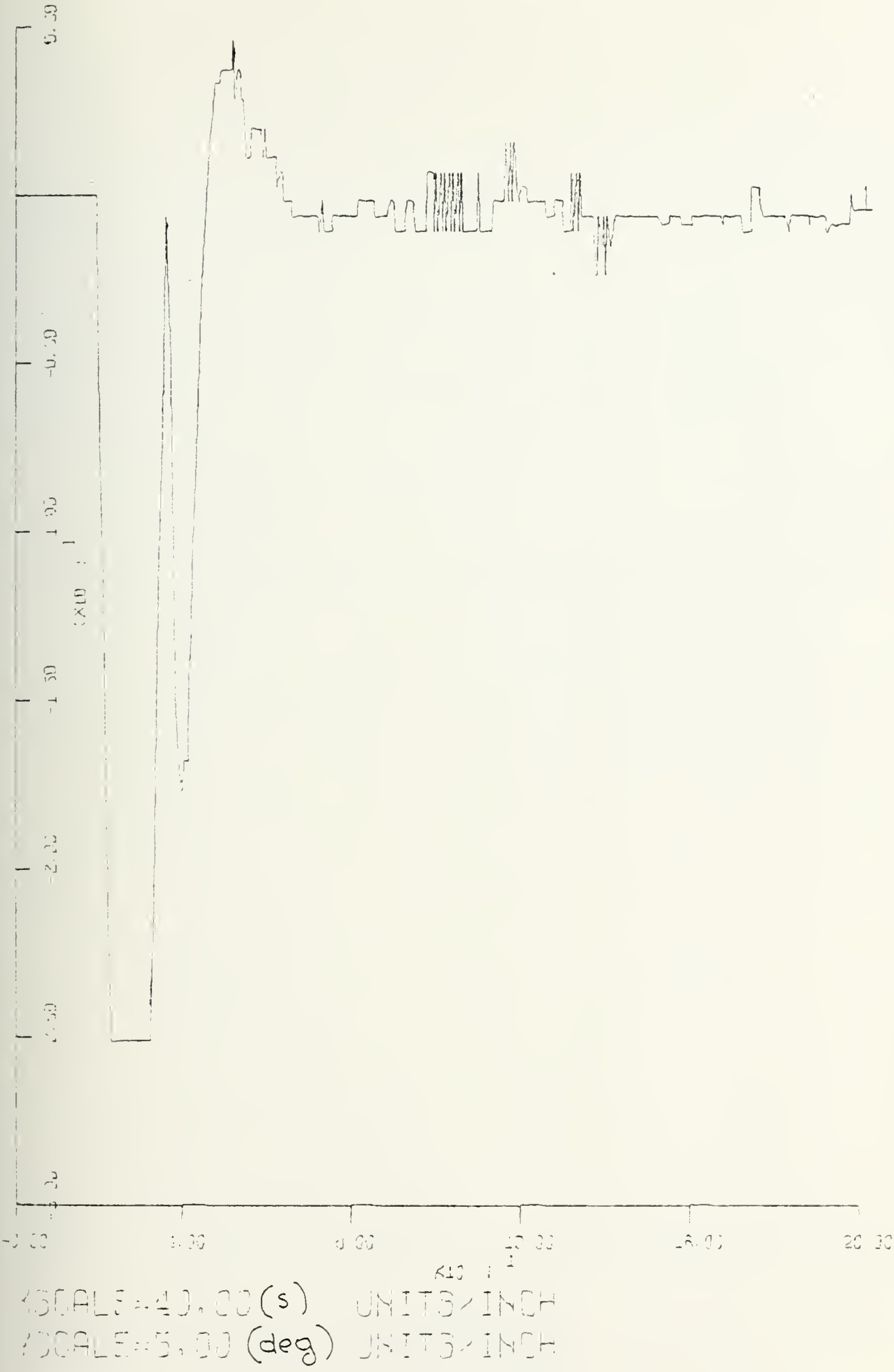


Fig. IV-32d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.5

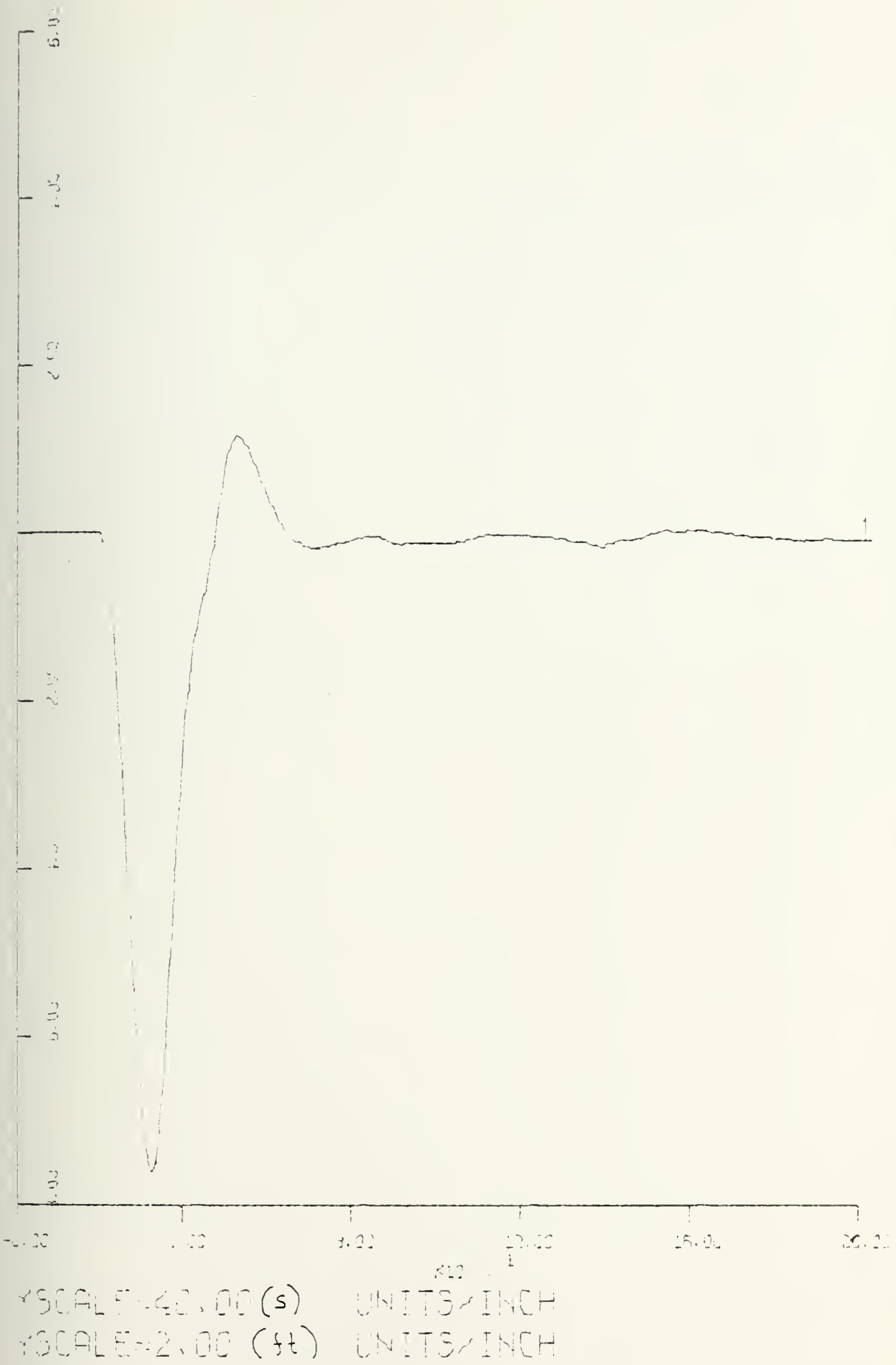


Fig. IV-33a. Depth vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.45

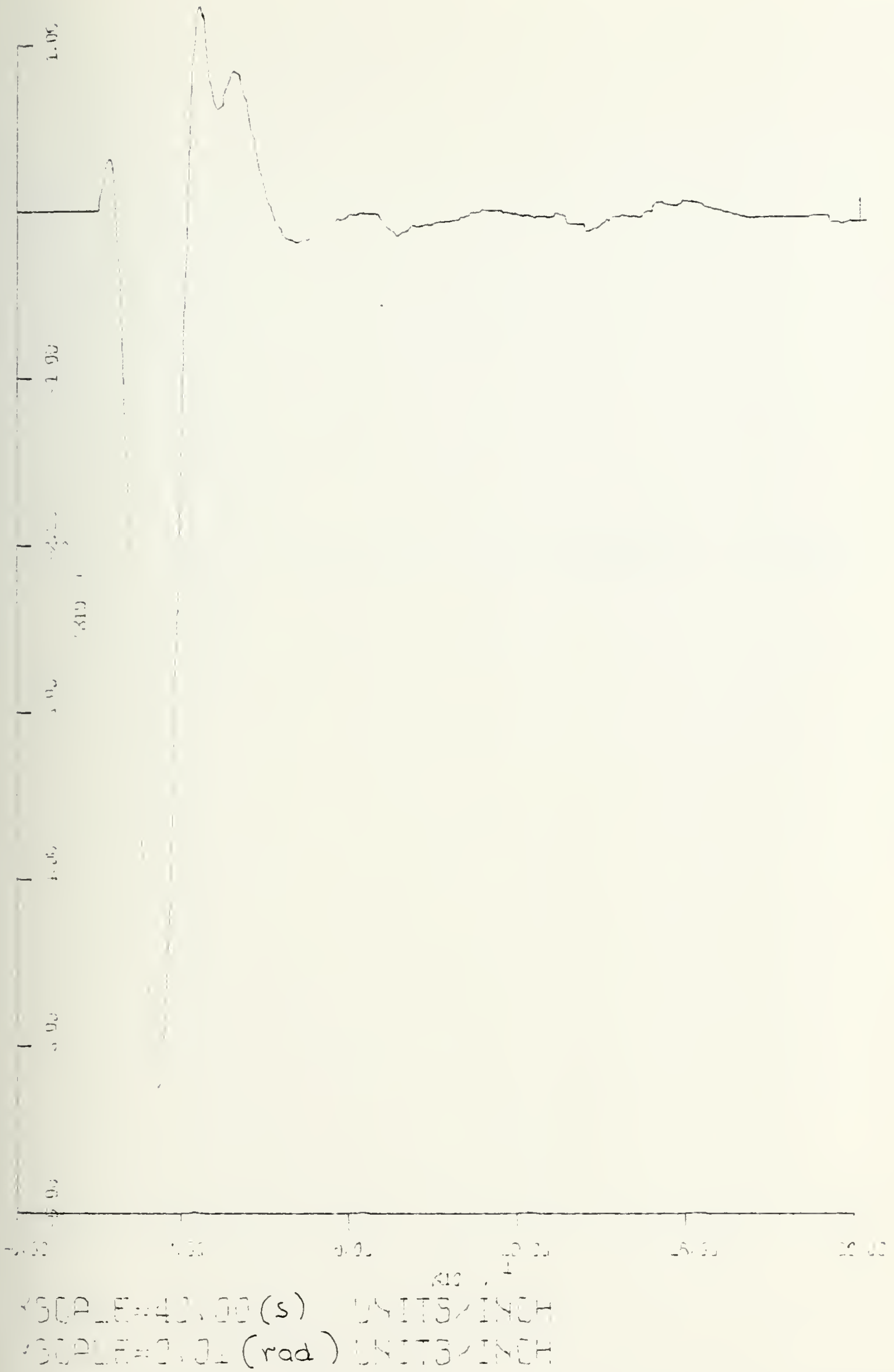


Fig. IV-33b. Pitch vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.45

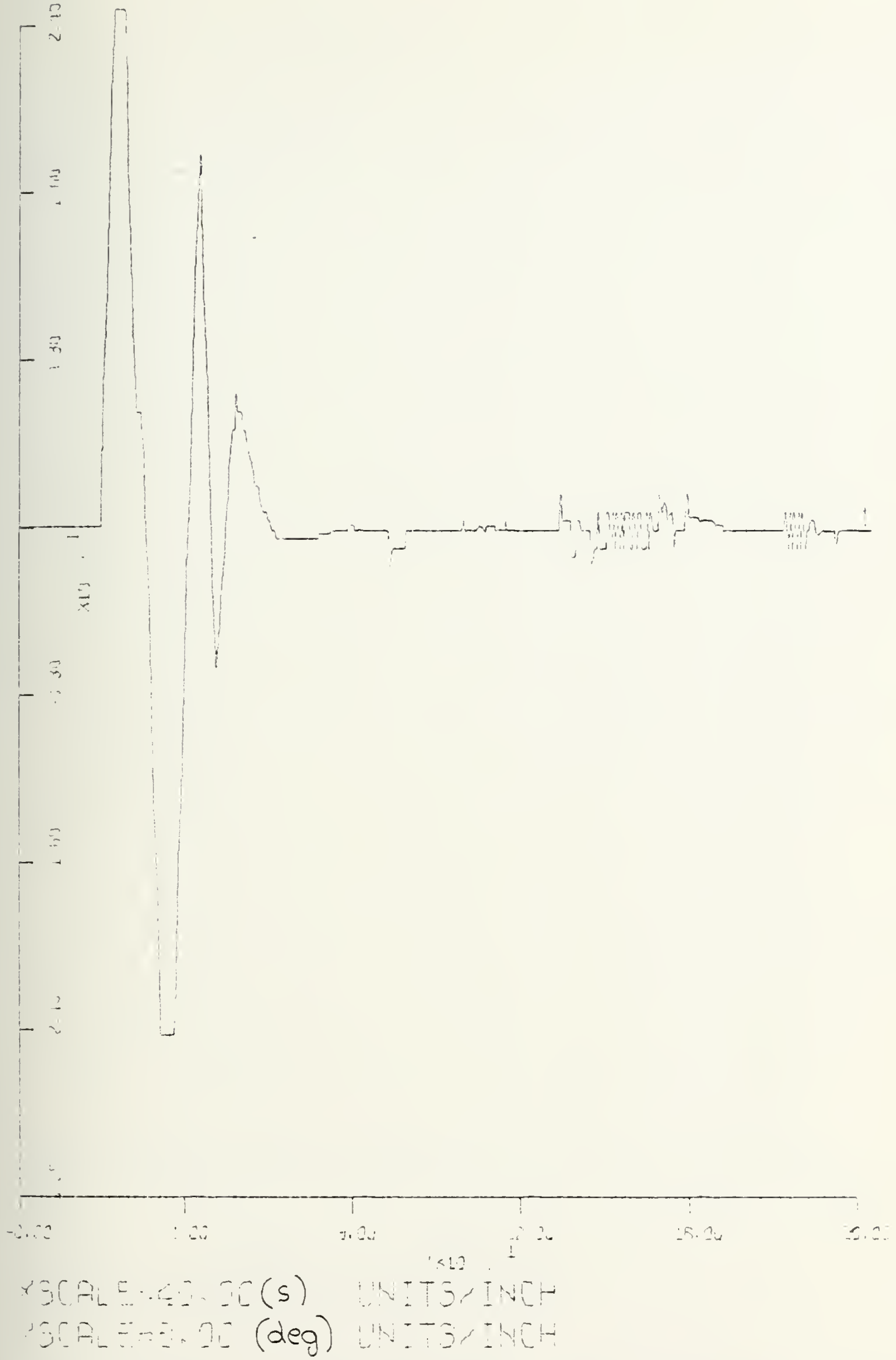


Fig. IV-33c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.45

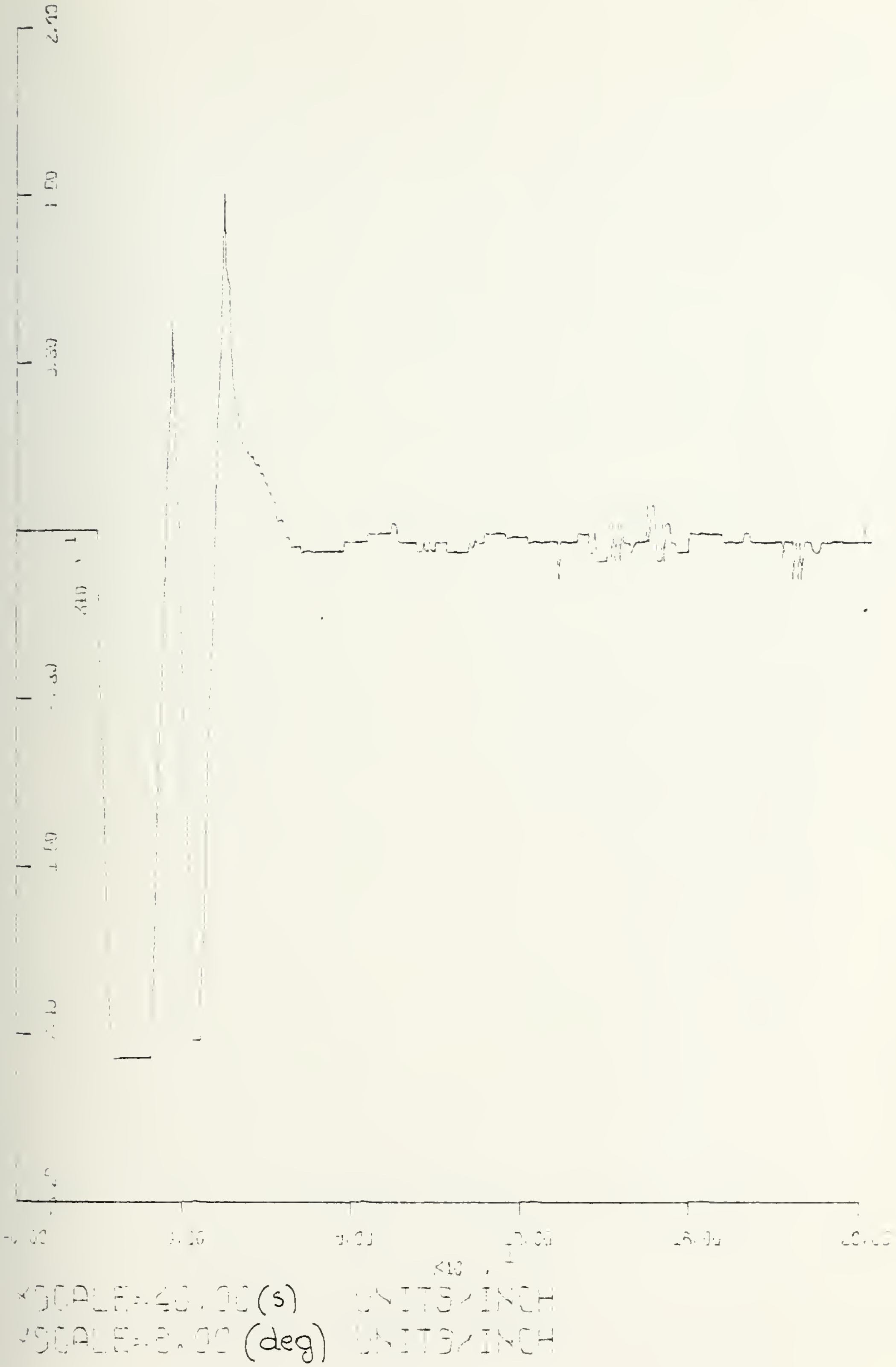


Fig. IV-33d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.45

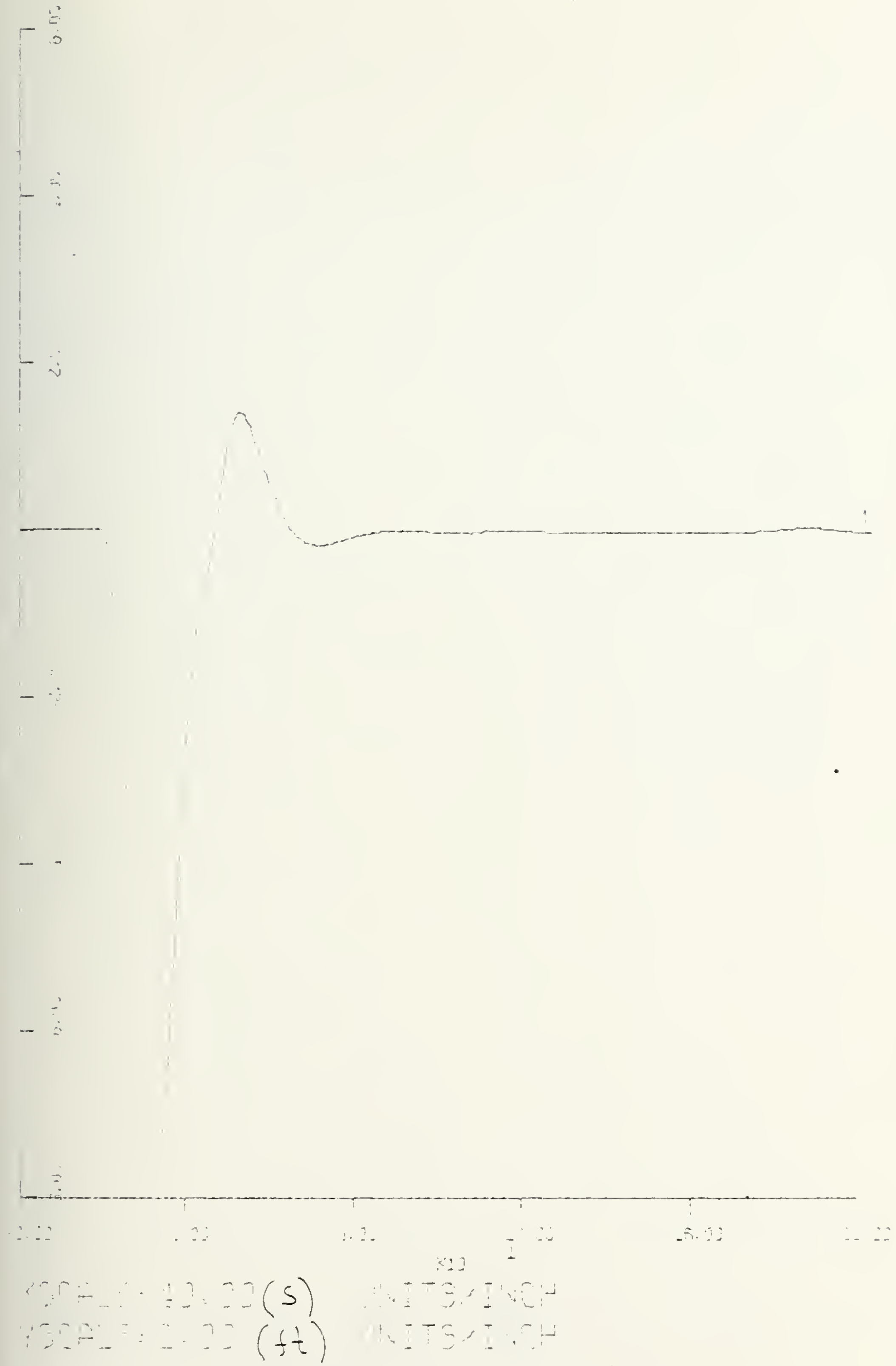


Fig. IV-34a. Depth vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.3

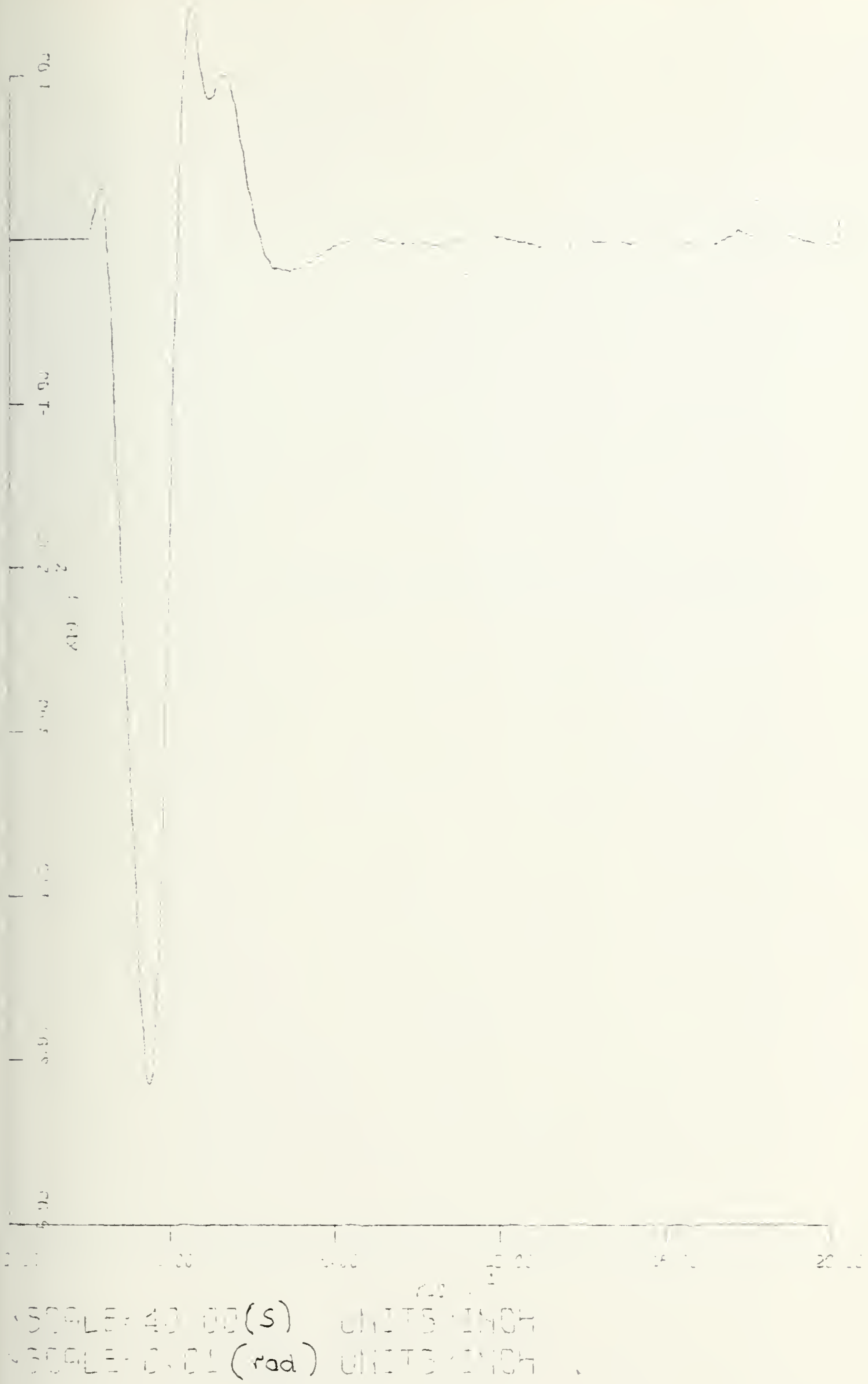


Fig. IV-34b. Pitch vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.3

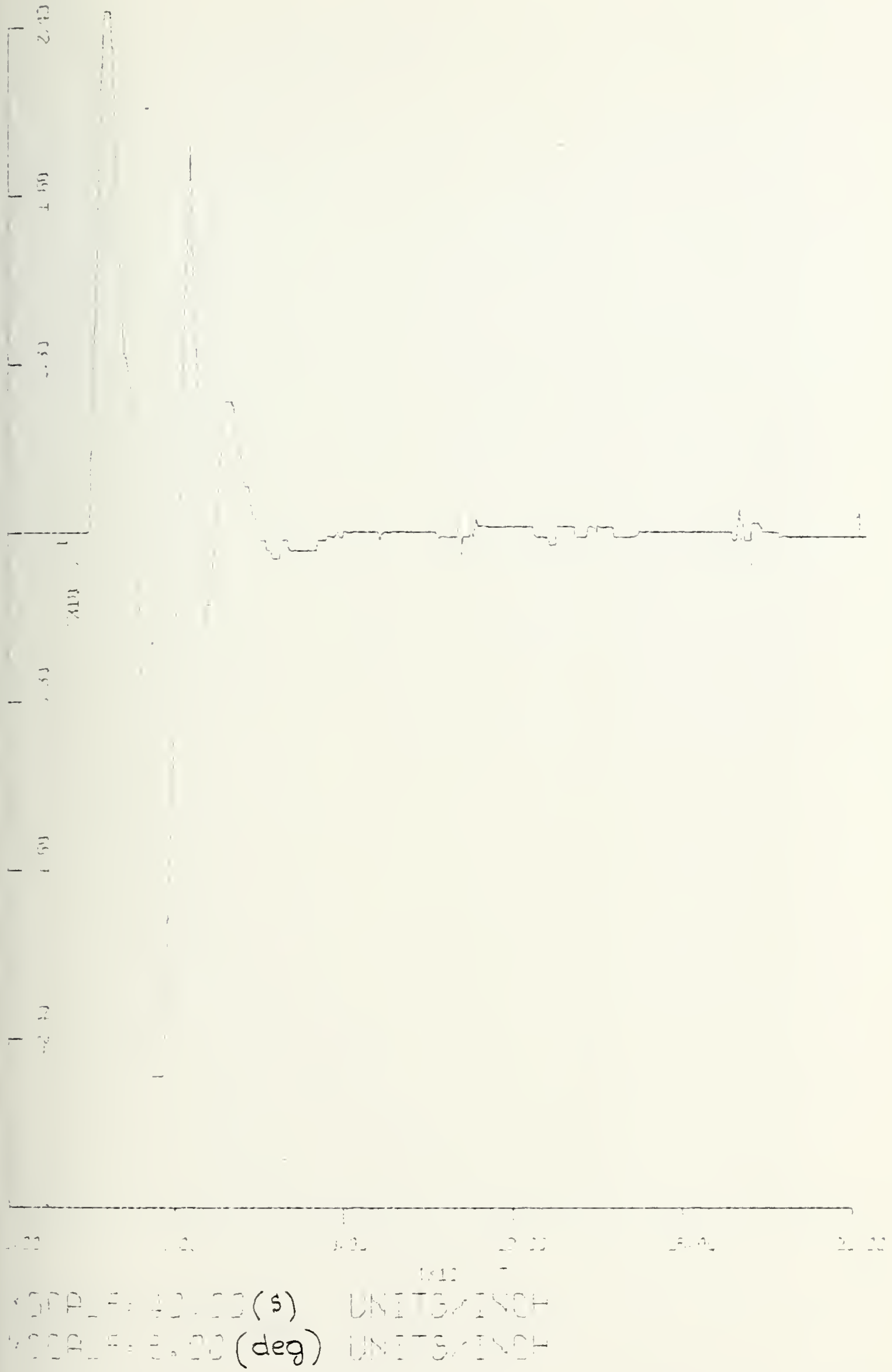


Fig. IV-34c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.3

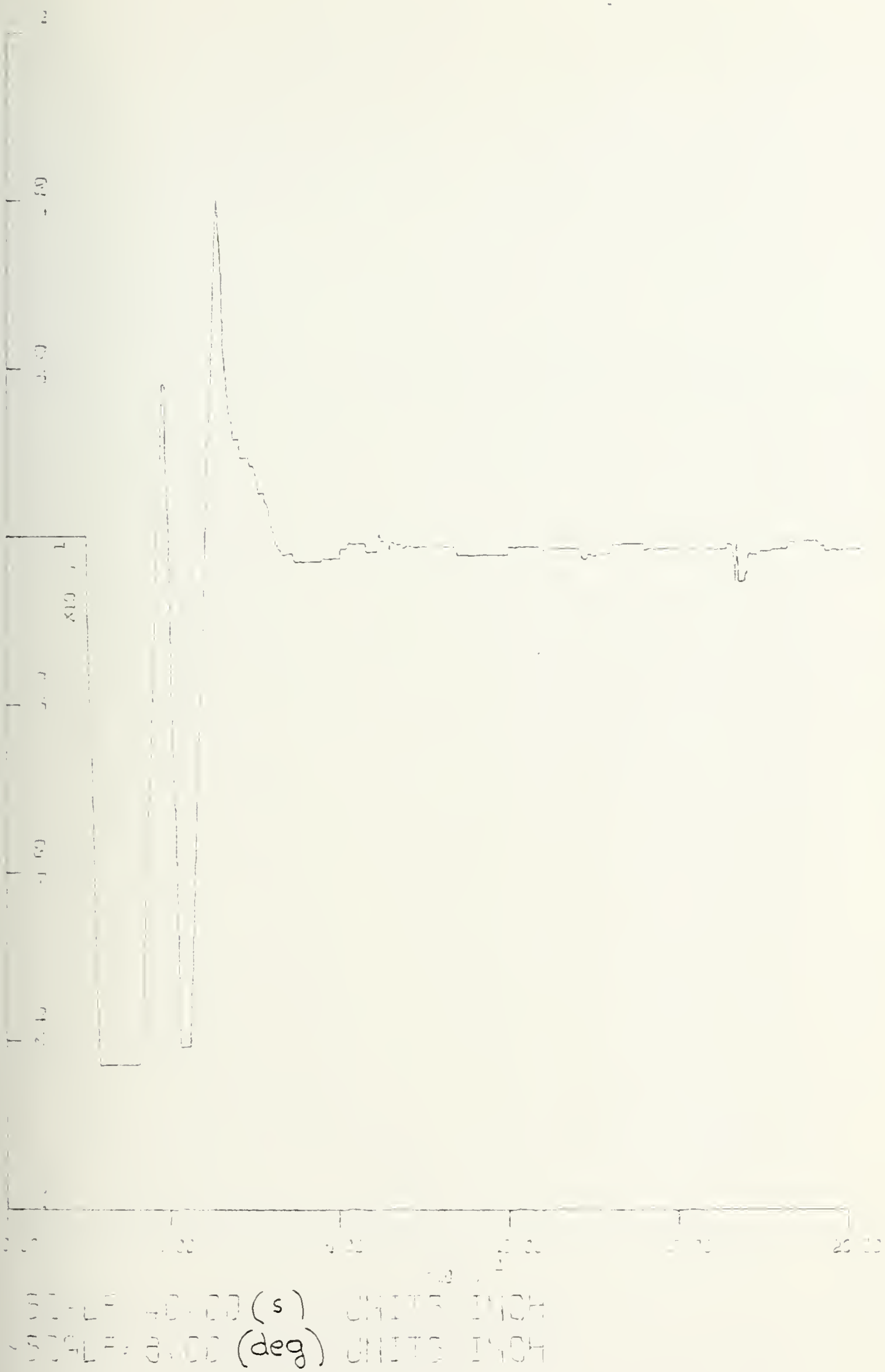


Fig. IV-34d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.3

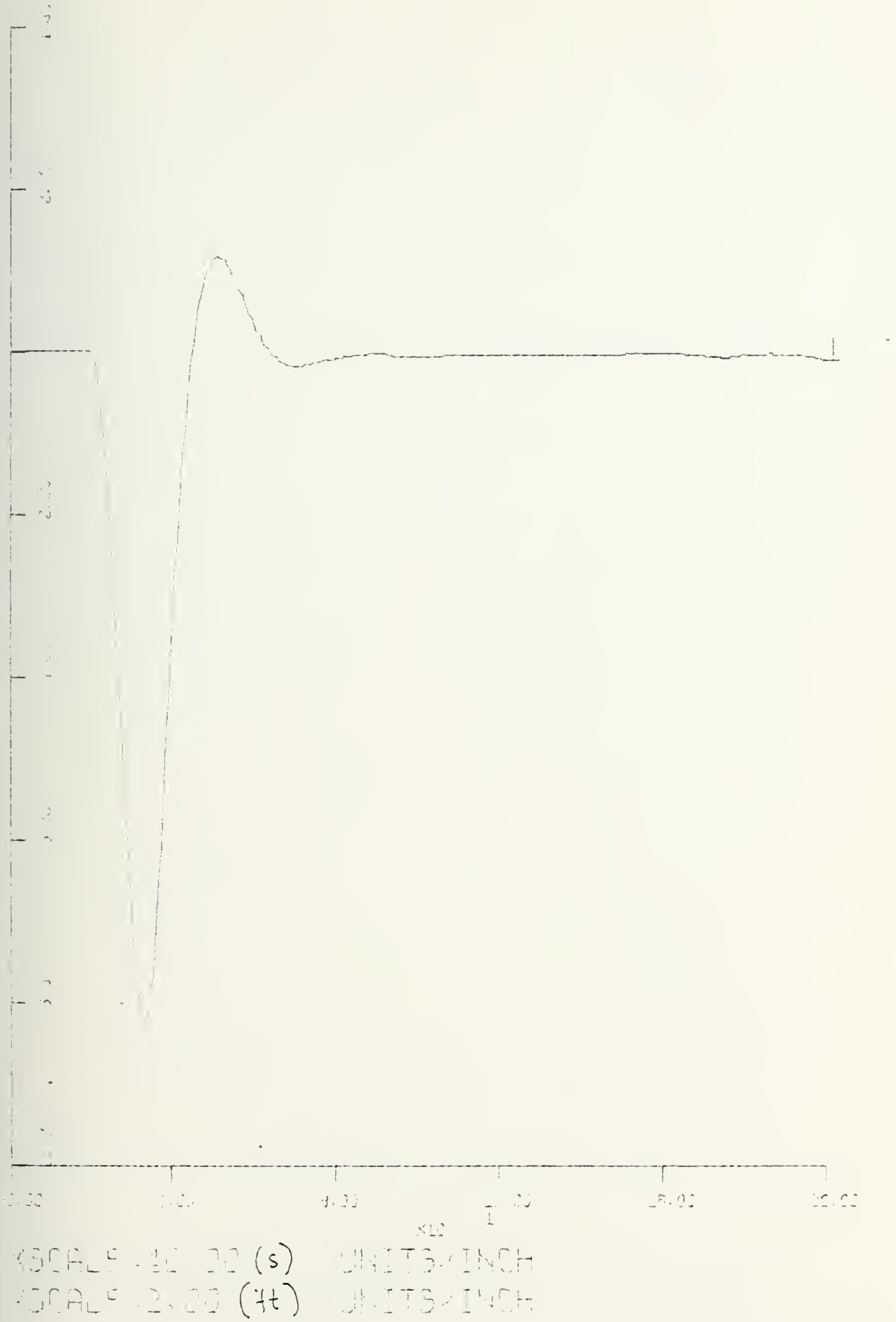


Fig. IV-35a. Depth vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.05

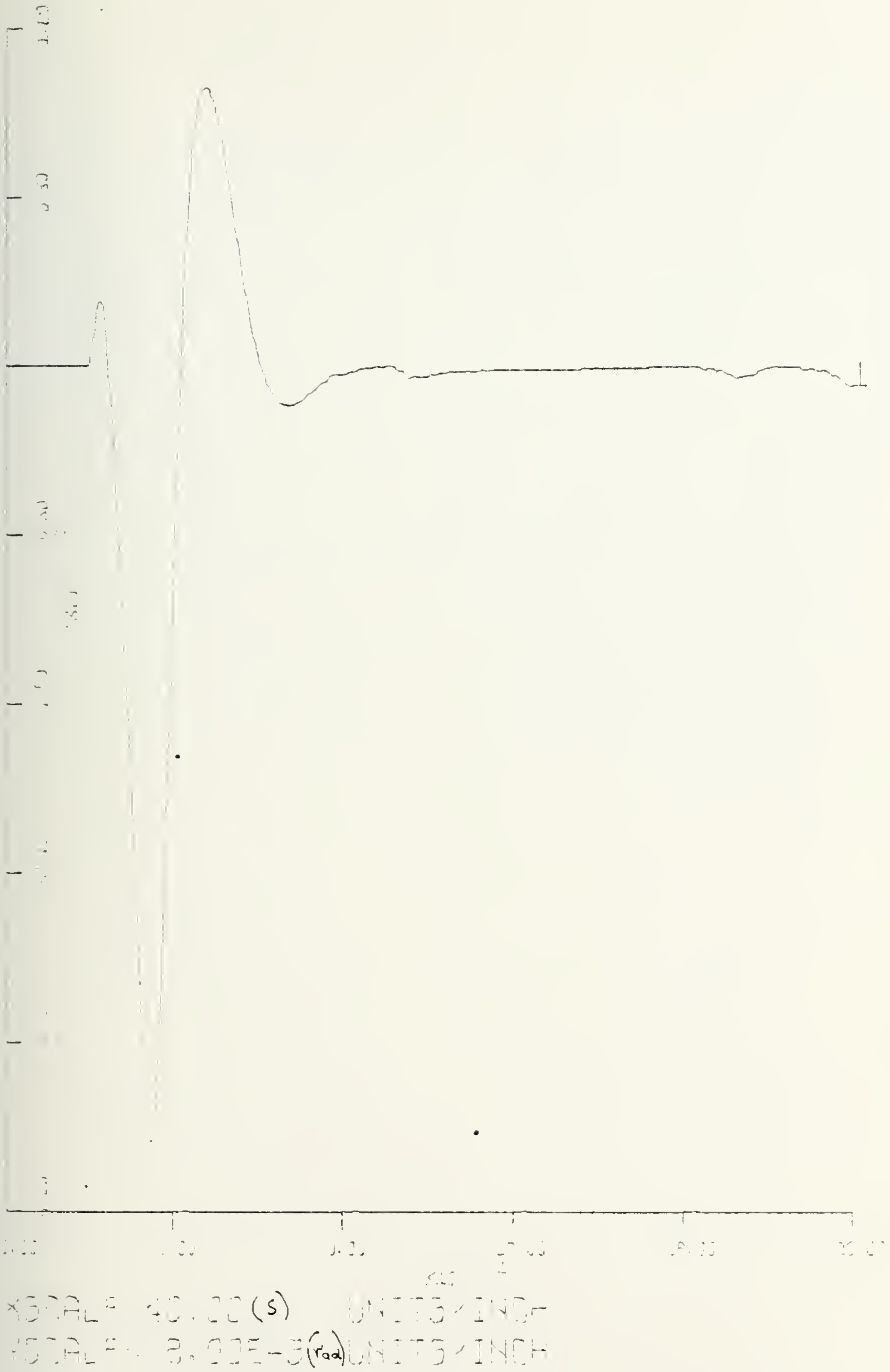


Fig. IV-35b. Pitch vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.05

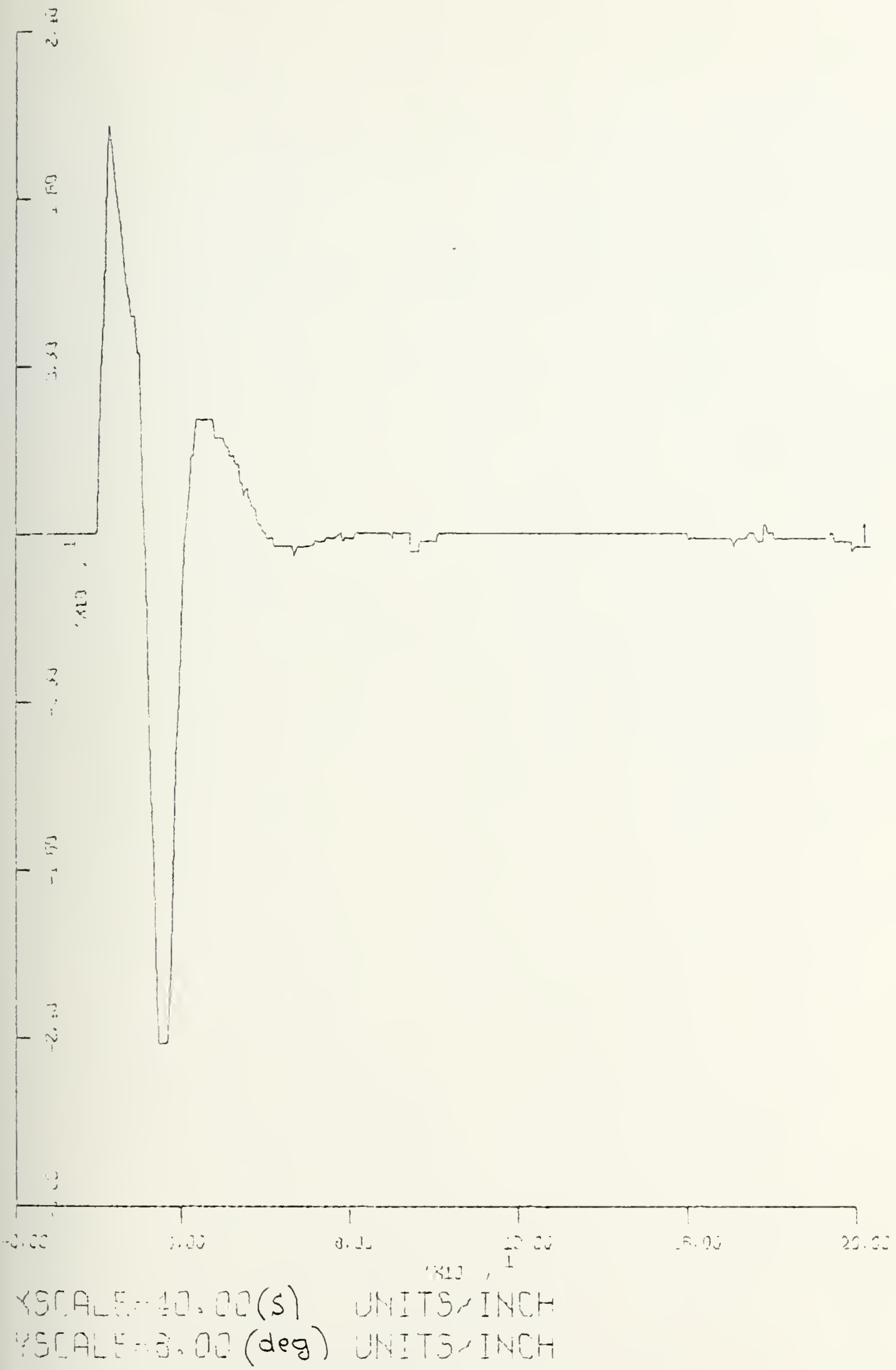


Fig. IV-35c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.05

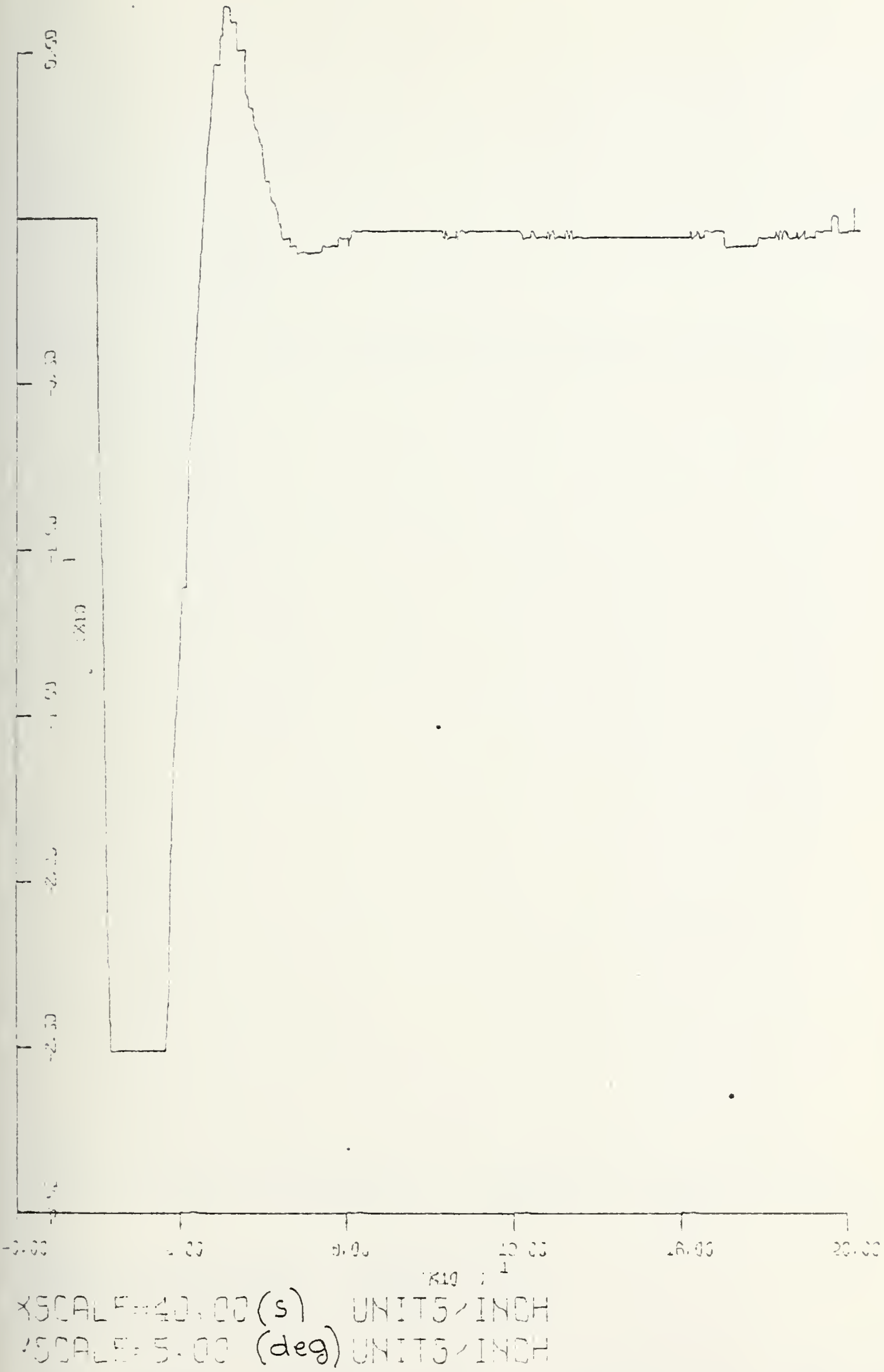


Fig. IV-35d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.05

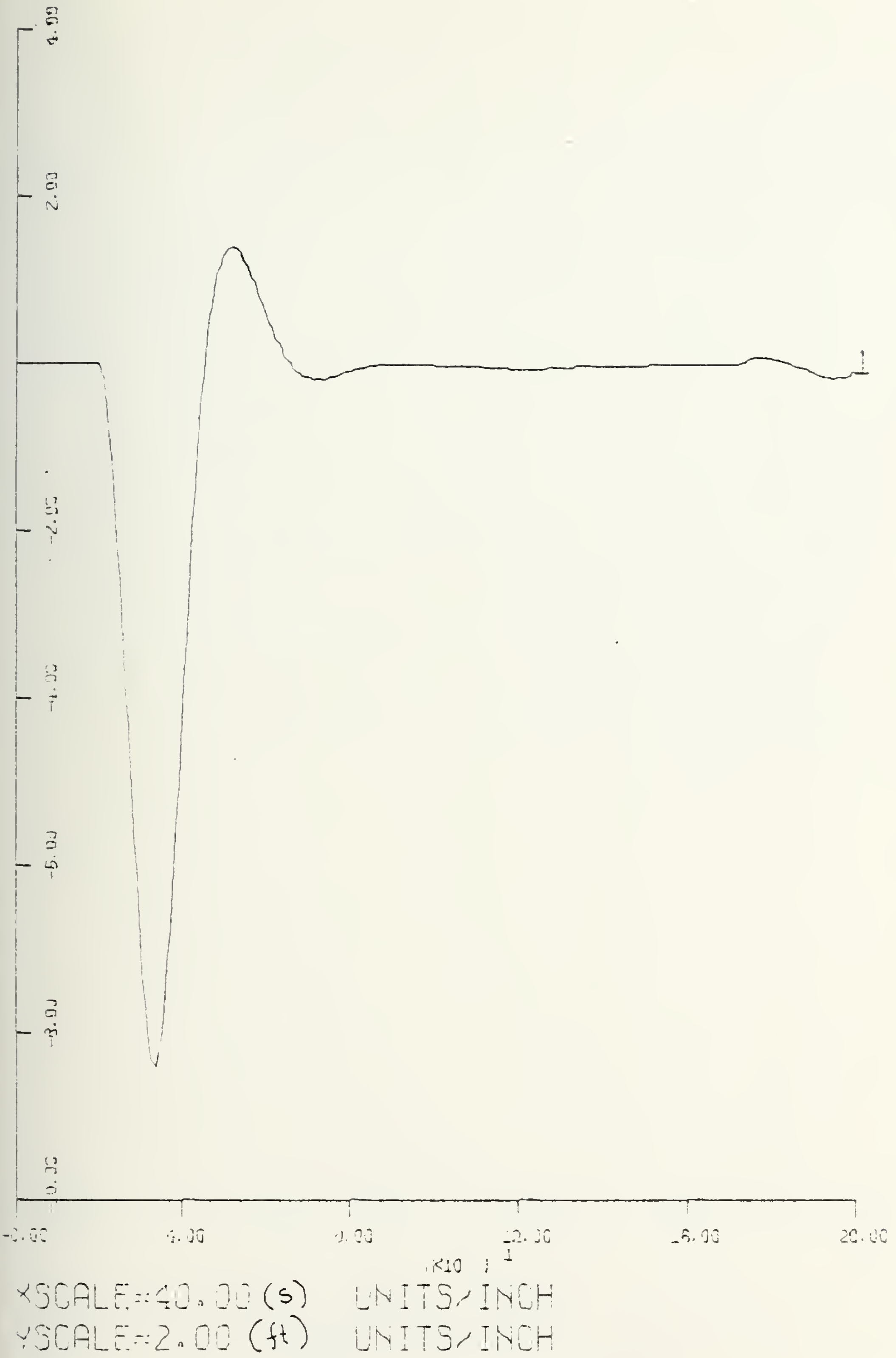


Fig. IV-36a. Depth vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.005

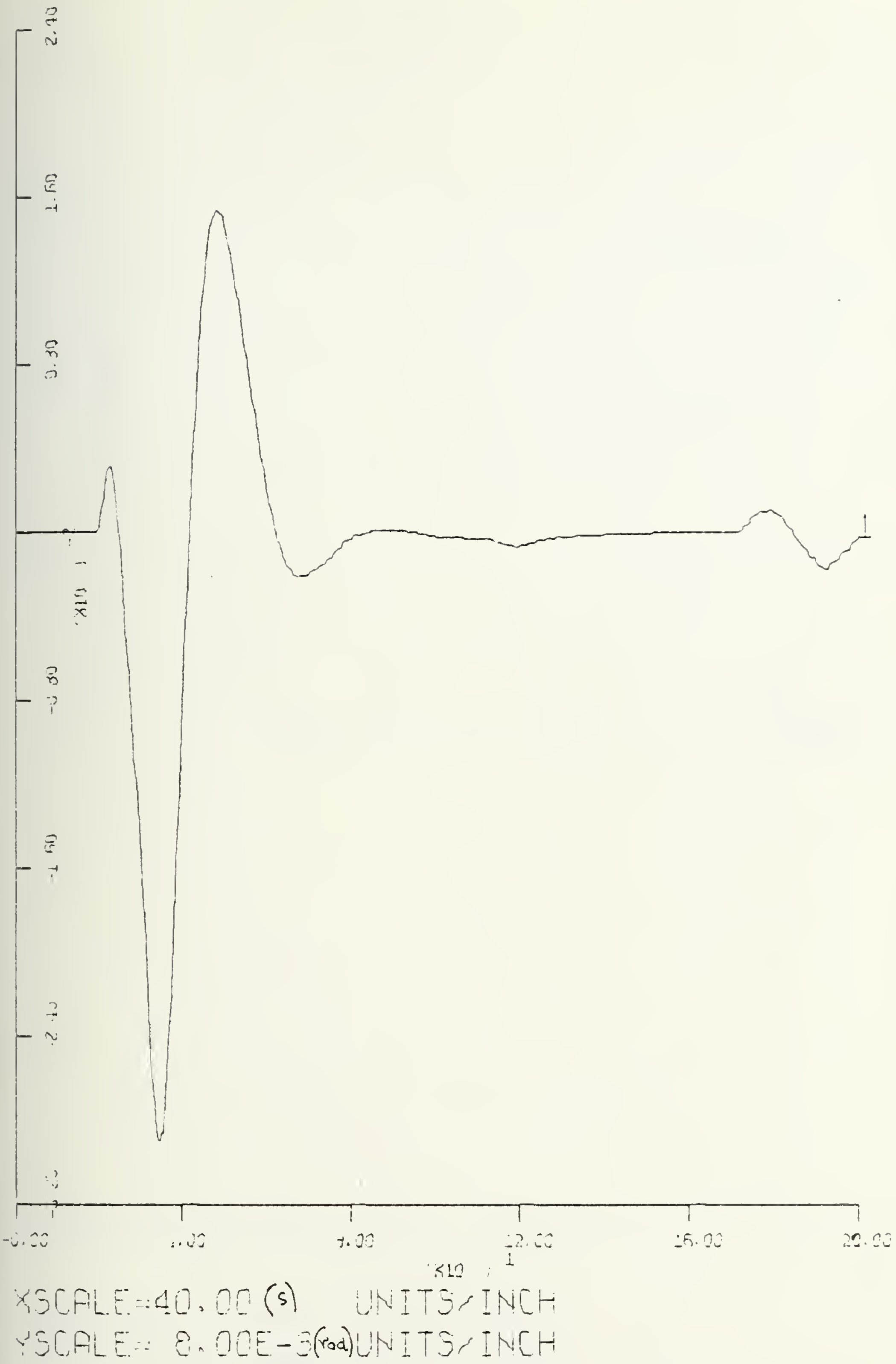


Fig. IV-36b. Pitch vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.005

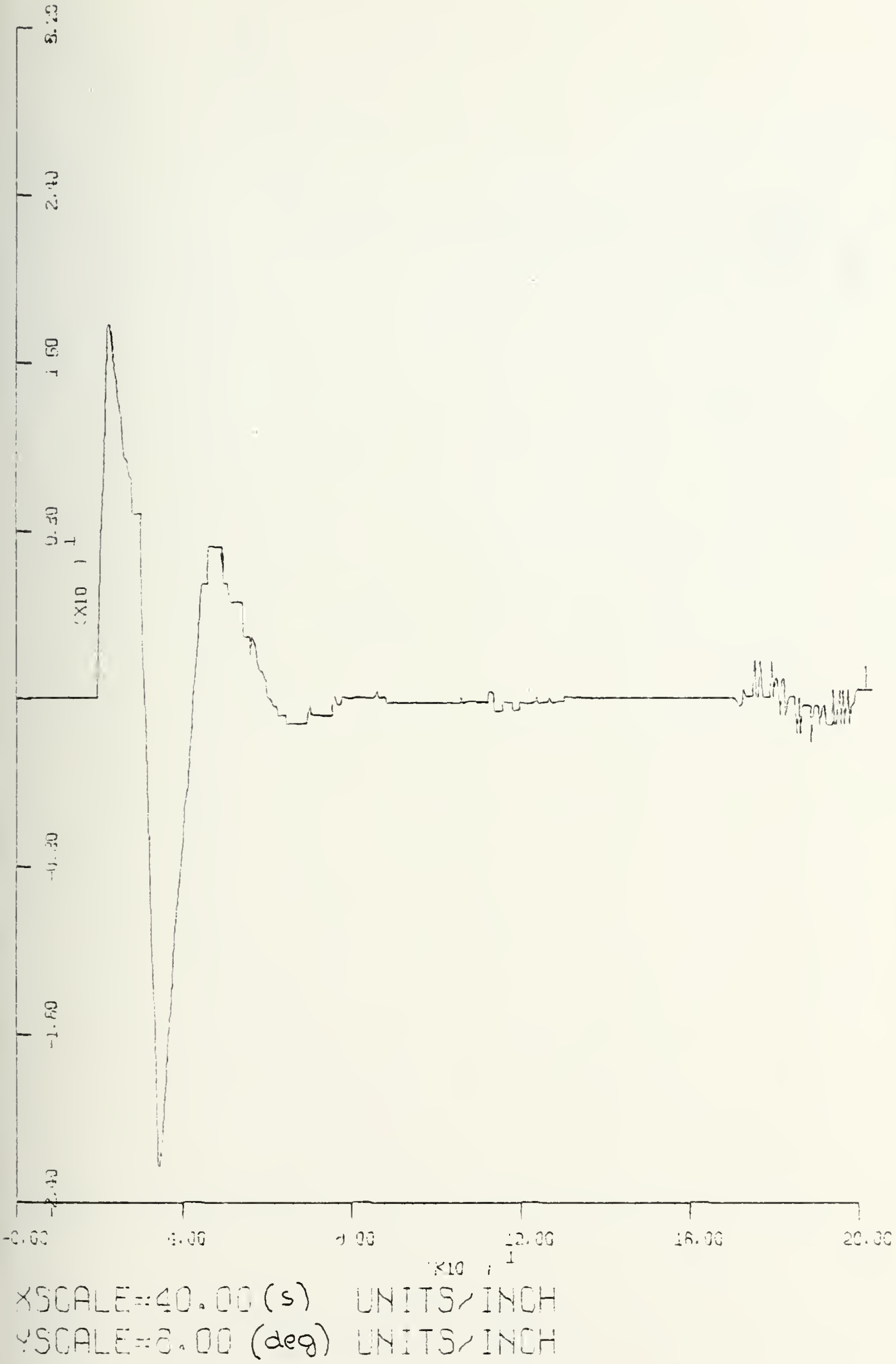


Fig. IV-36c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.005

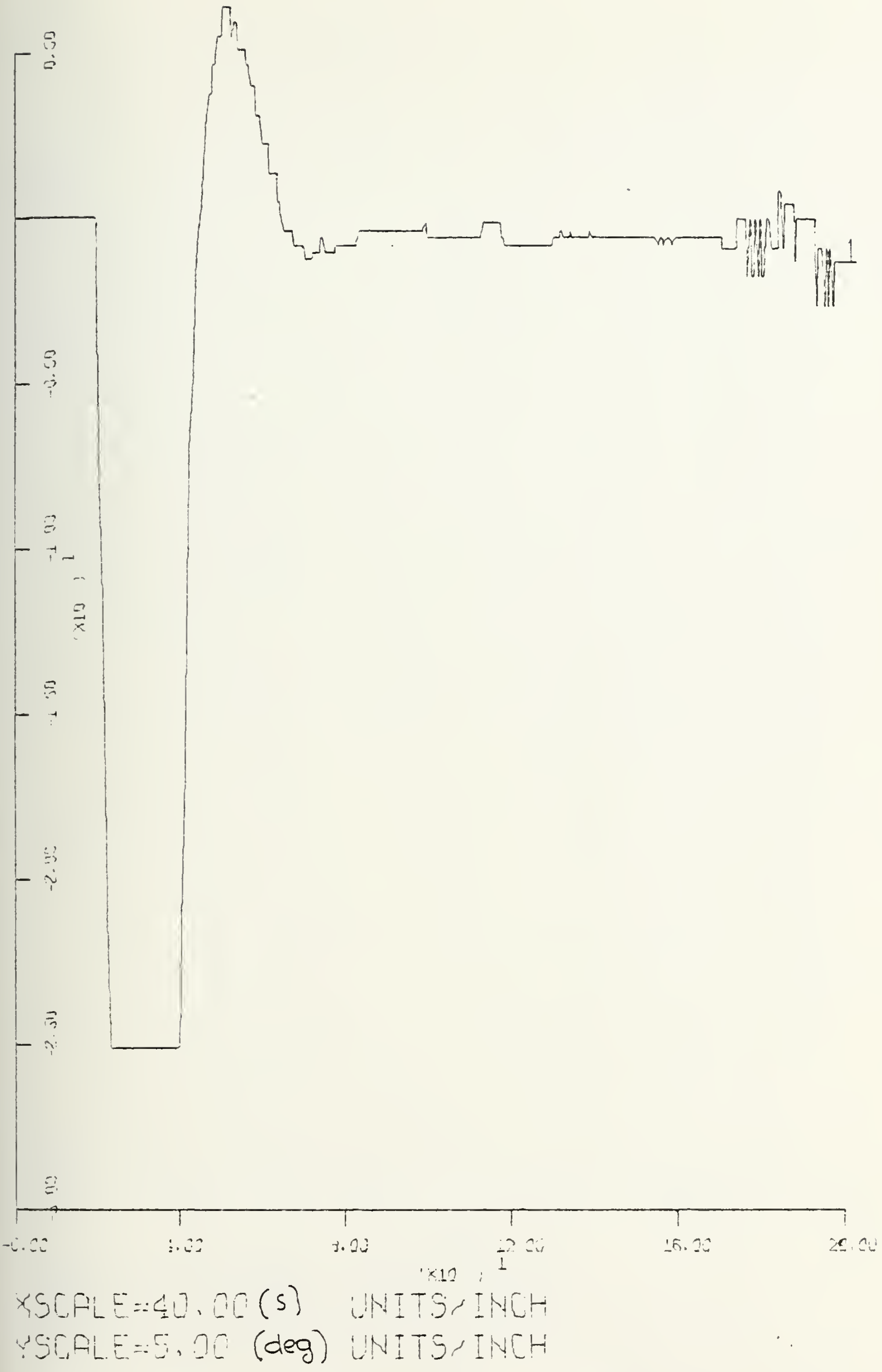


Fig. IV-36d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=800, C=10, E=1. Parameter X=0.005

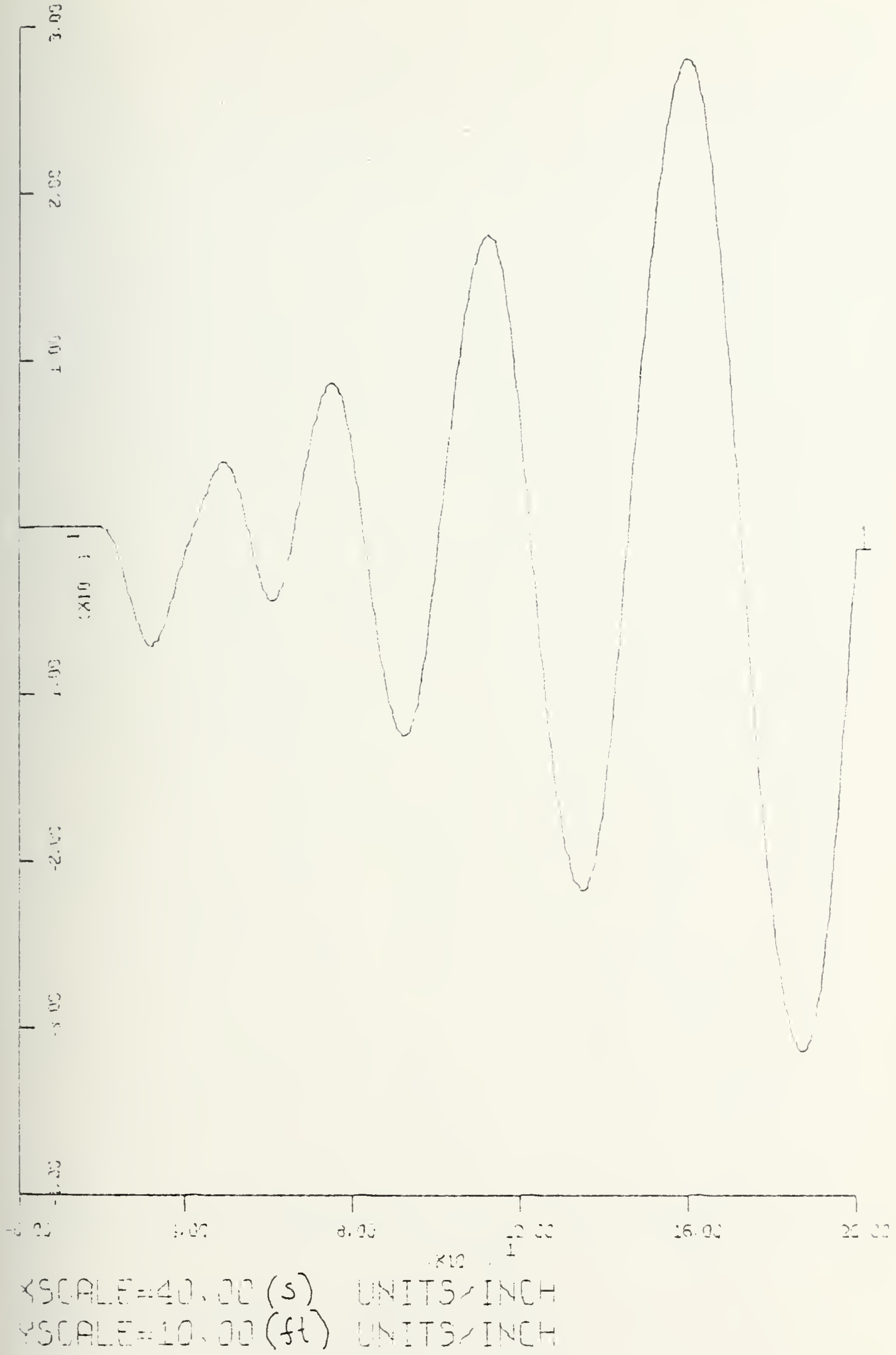


Fig. IV-37a. Depth vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.5

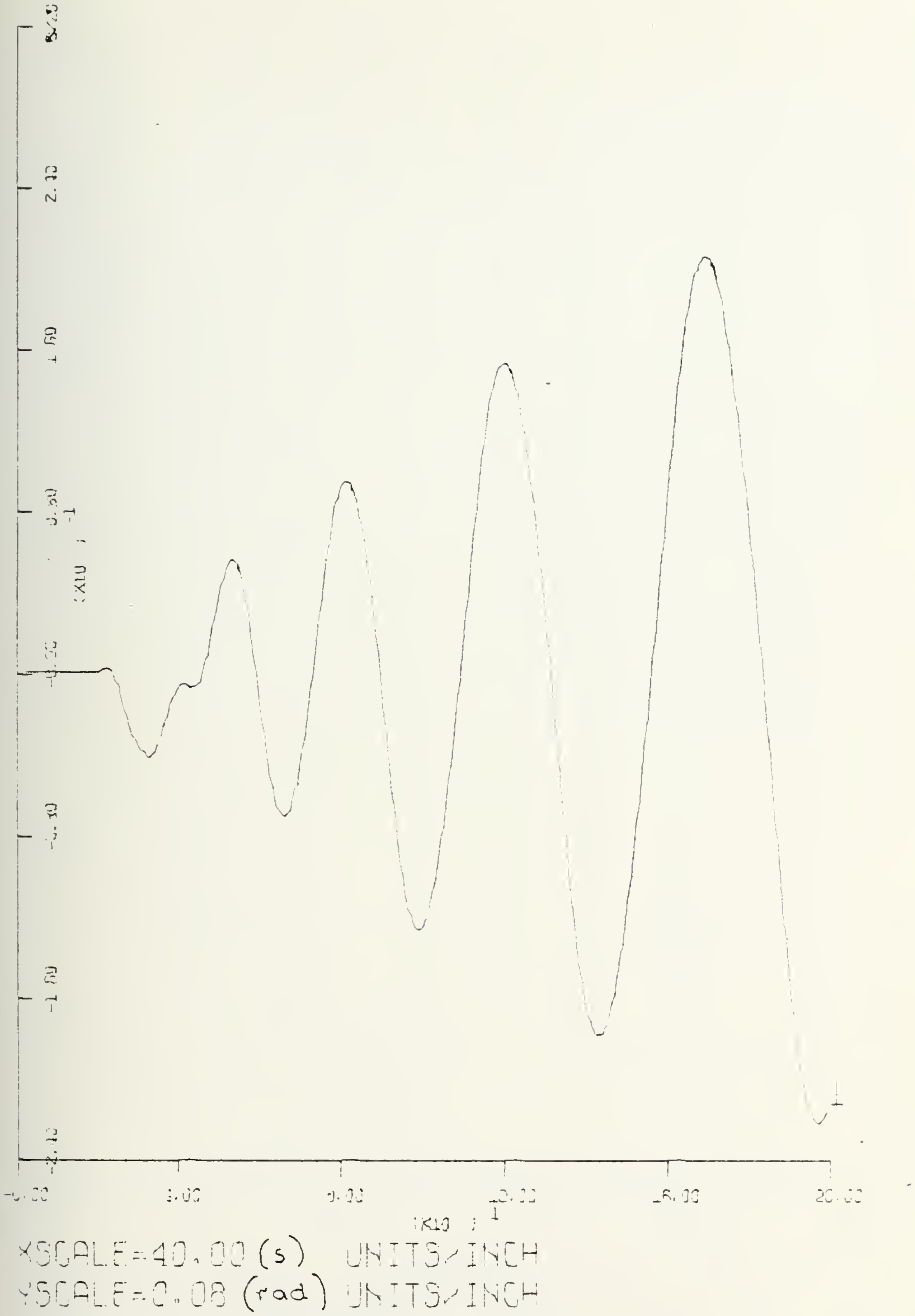
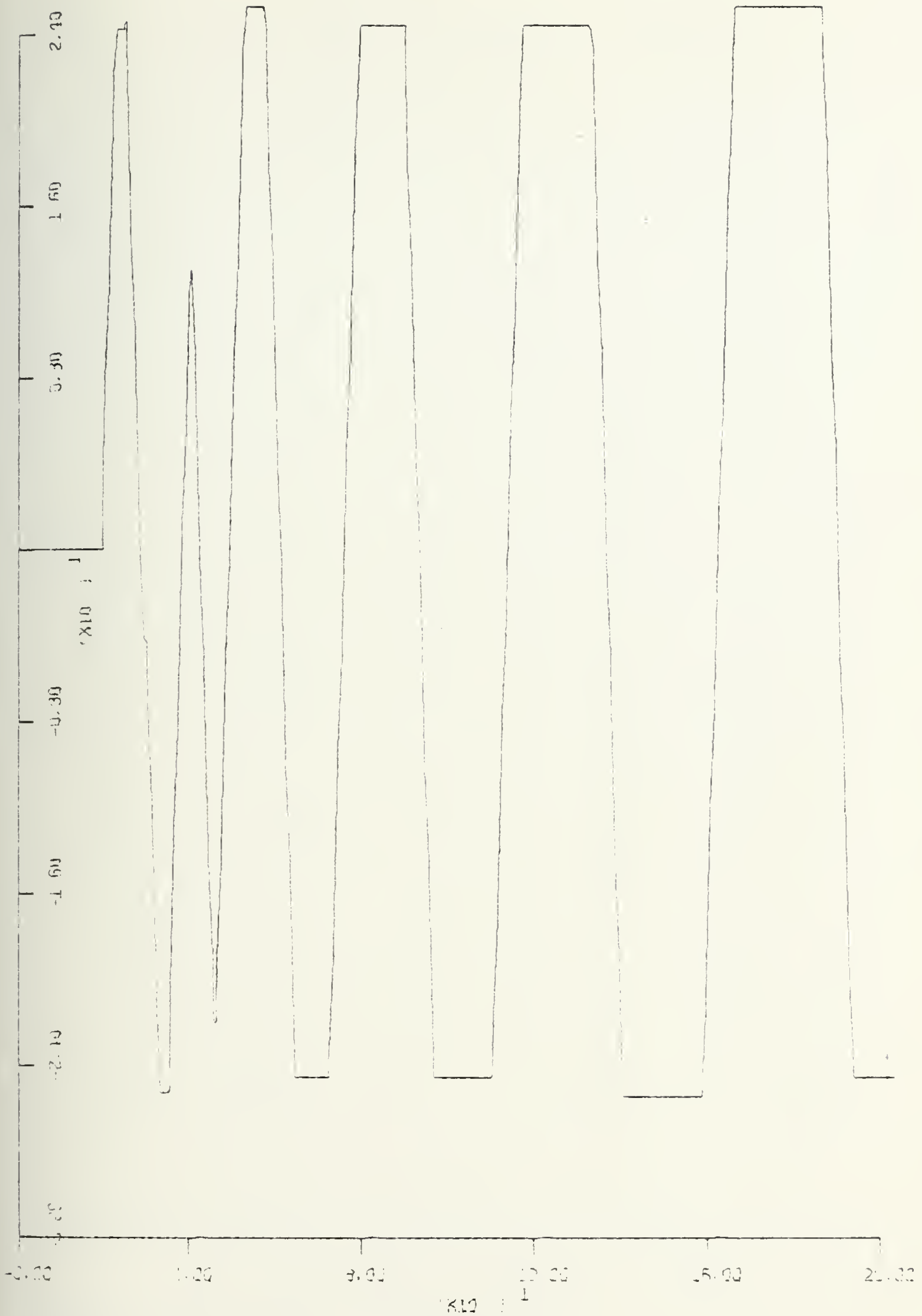
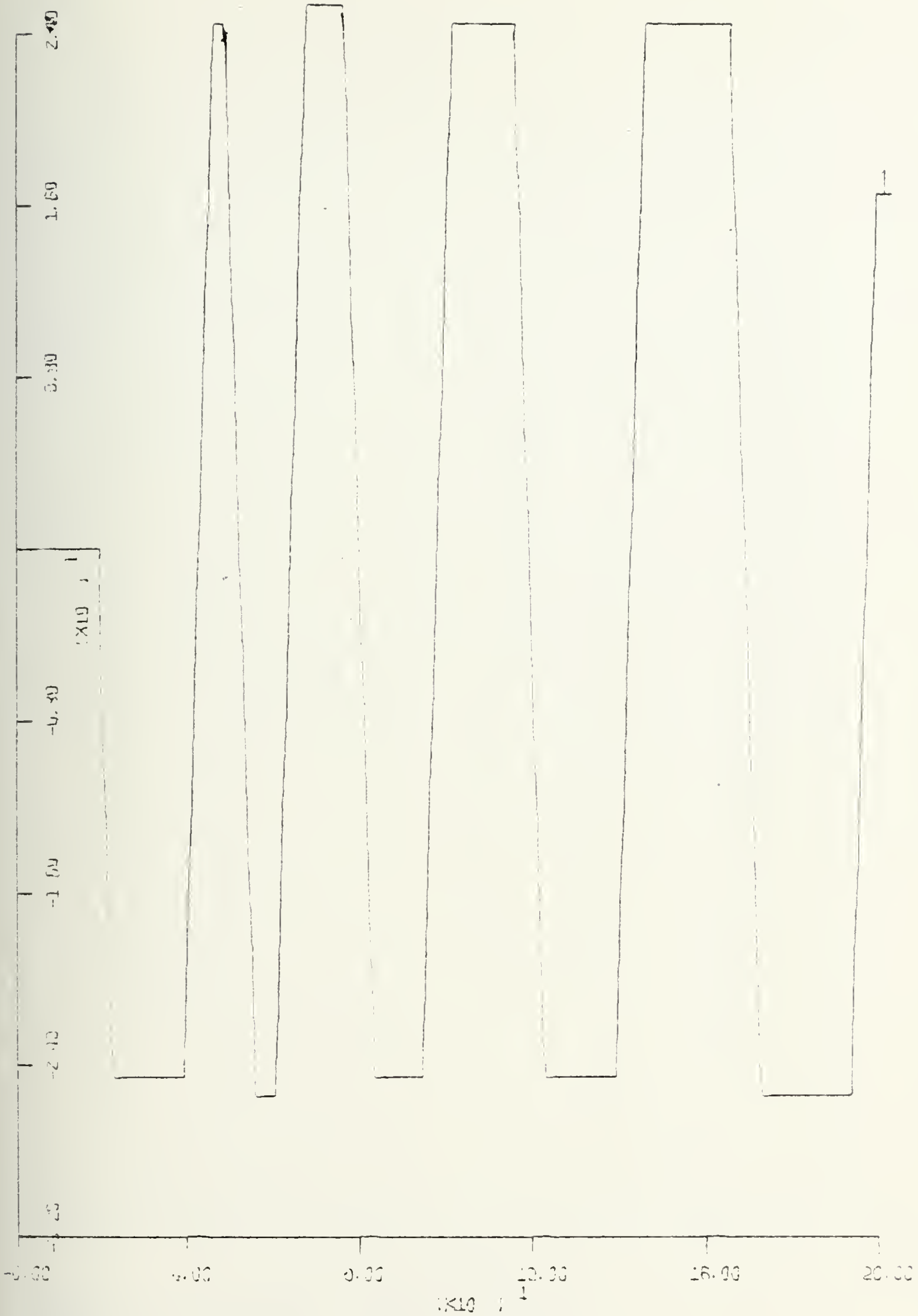


Fig. IV-37b. Pitch vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.5



XSCALE=40.00 (s) UNITS/INCH
YSCALE=8.00 (deg) UNITS/INCH

Fig. IV-37c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.5



XSCALE=40.00 (s) UNITS/INCH
YSCALE=2.00 (deg) UNITS/INCH

Fig. IV-37d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.5

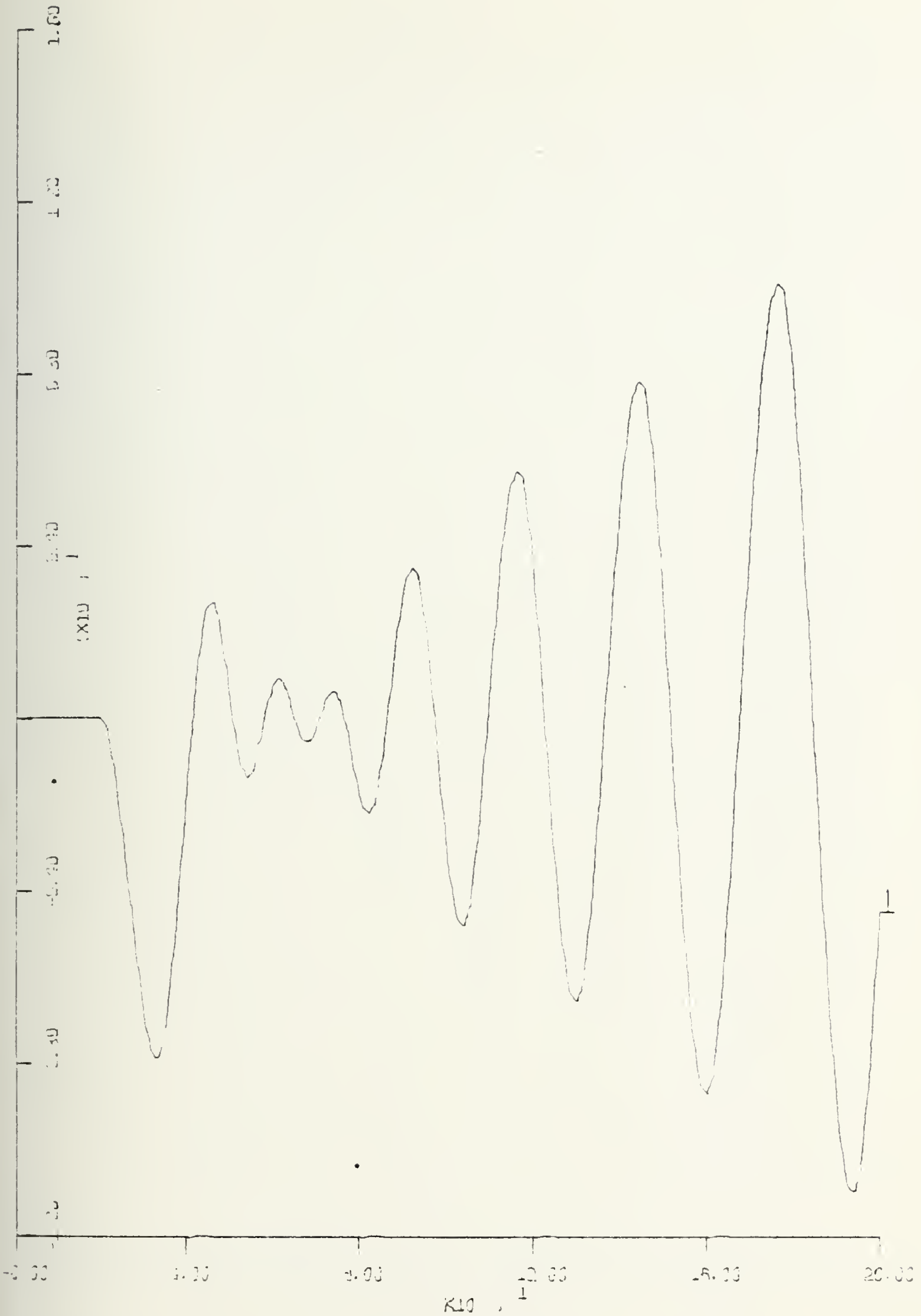


Fig. IV-38a. Depth vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.3

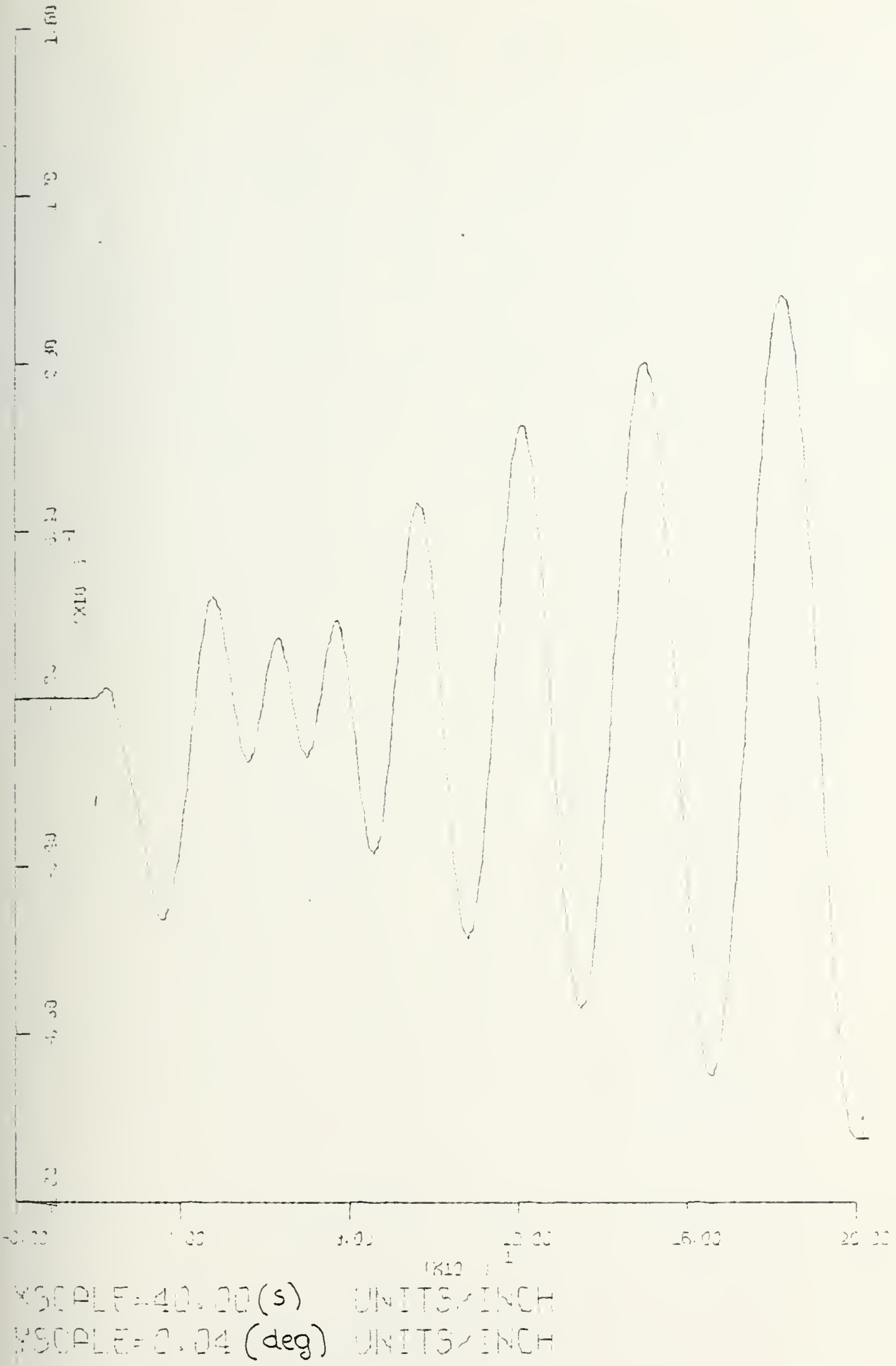
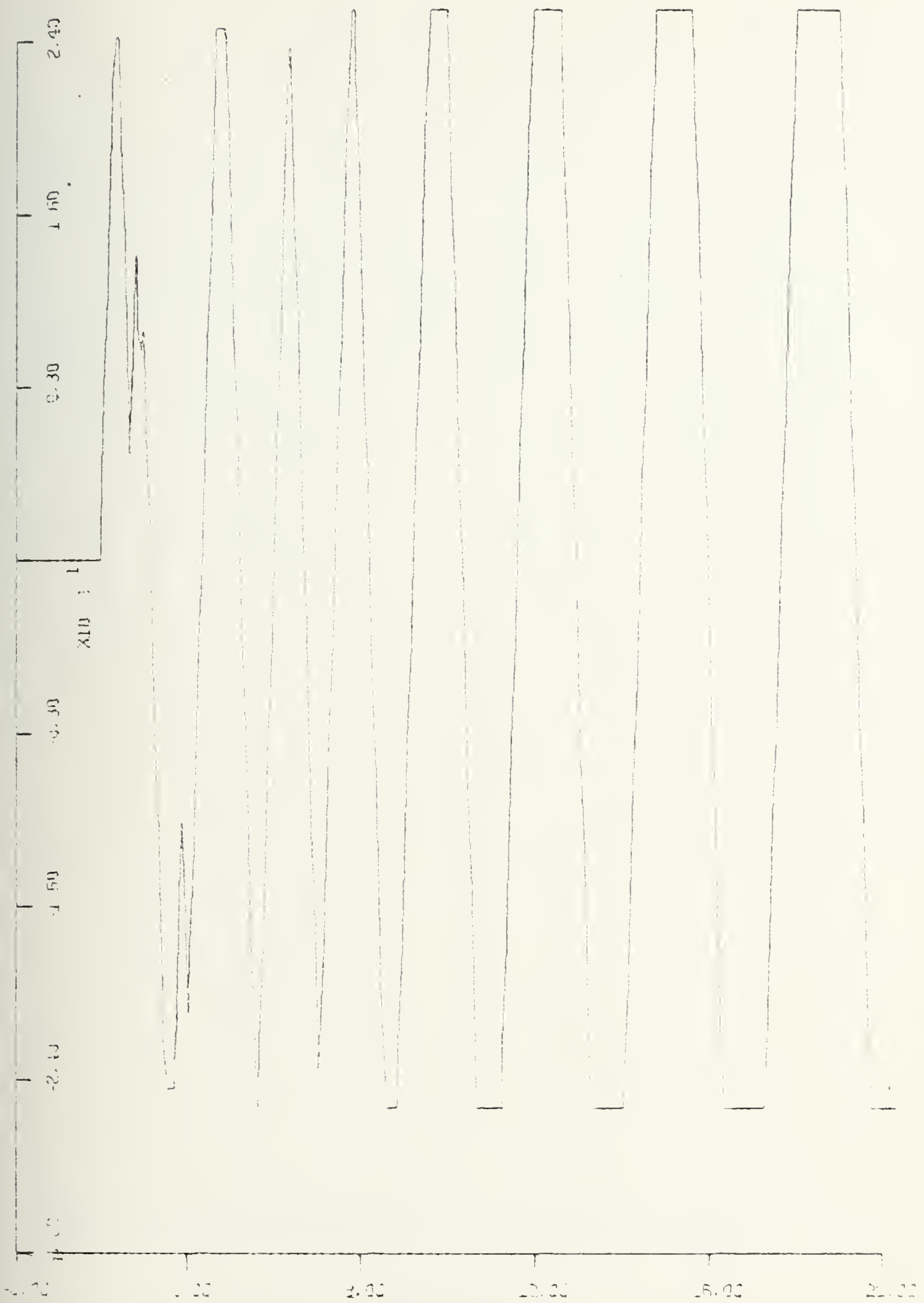


Fig. IV-38b. Pitch vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.3



XSCALE=40.00(s) UNITS/INCH
YSCALE=~~2.00~~ (deg) UNITS/INCH

Fig. IV-38c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.3

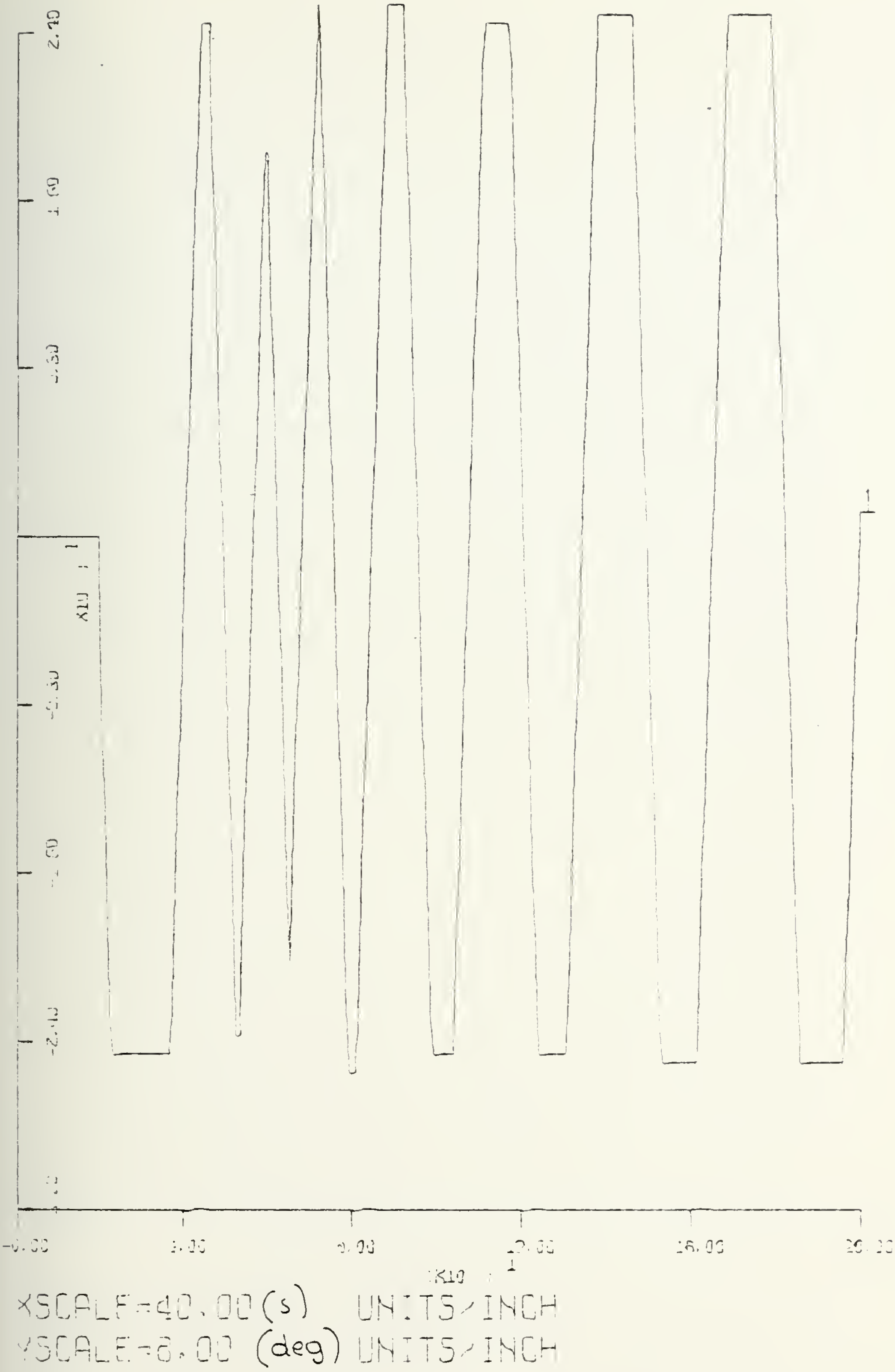


Fig. IV-38d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.3

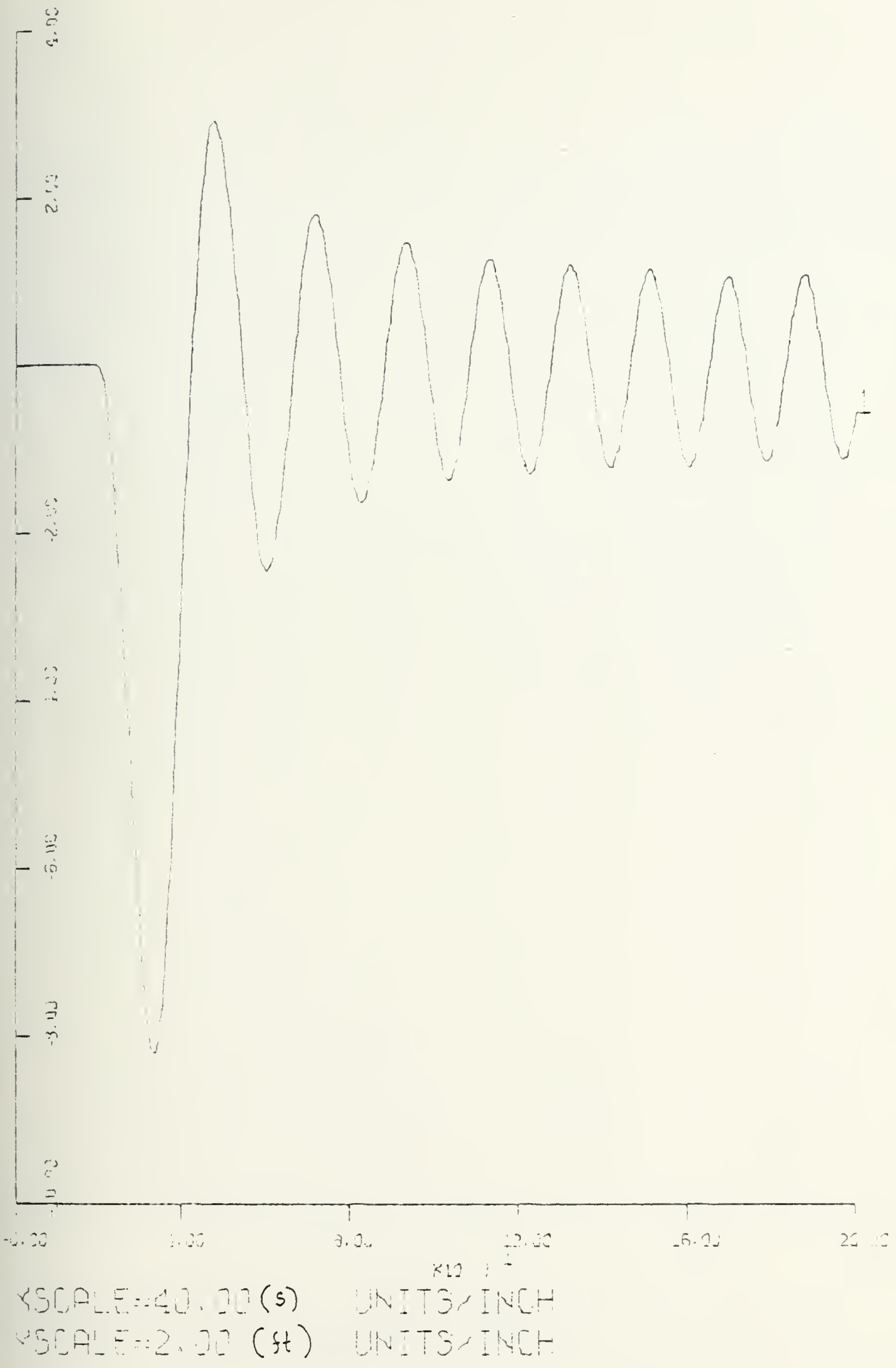
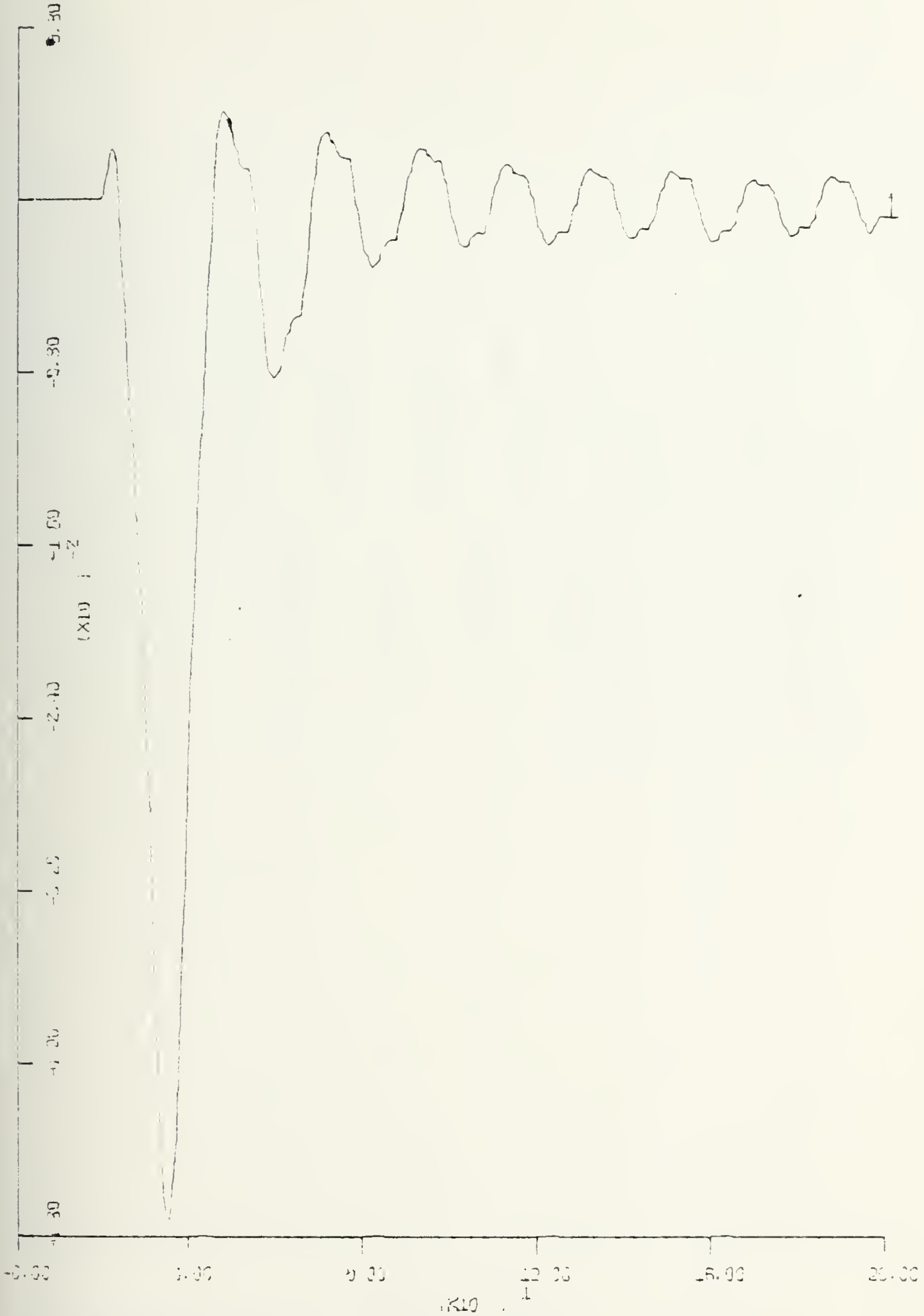
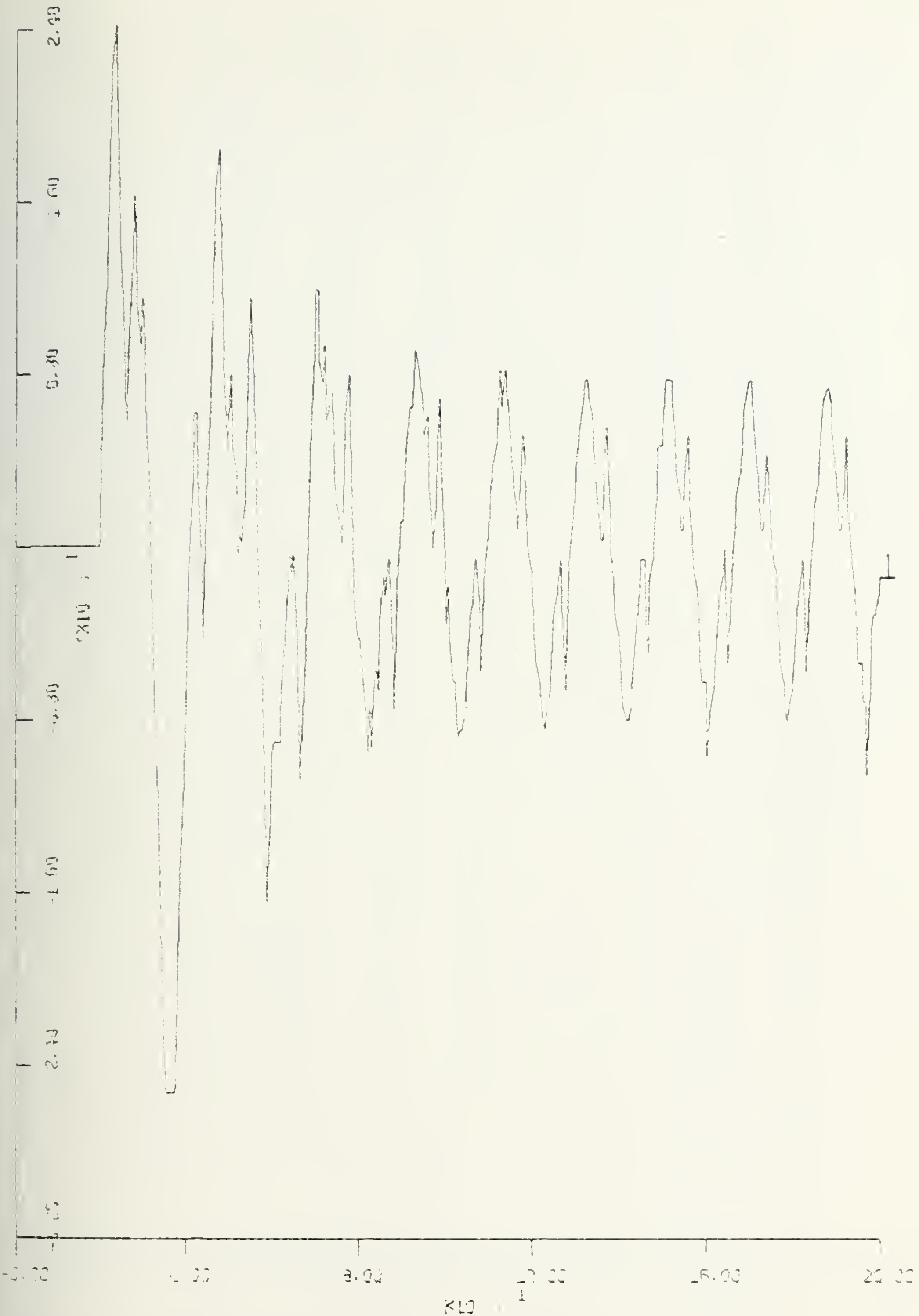


Fig. IV-39a. Depth vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.2



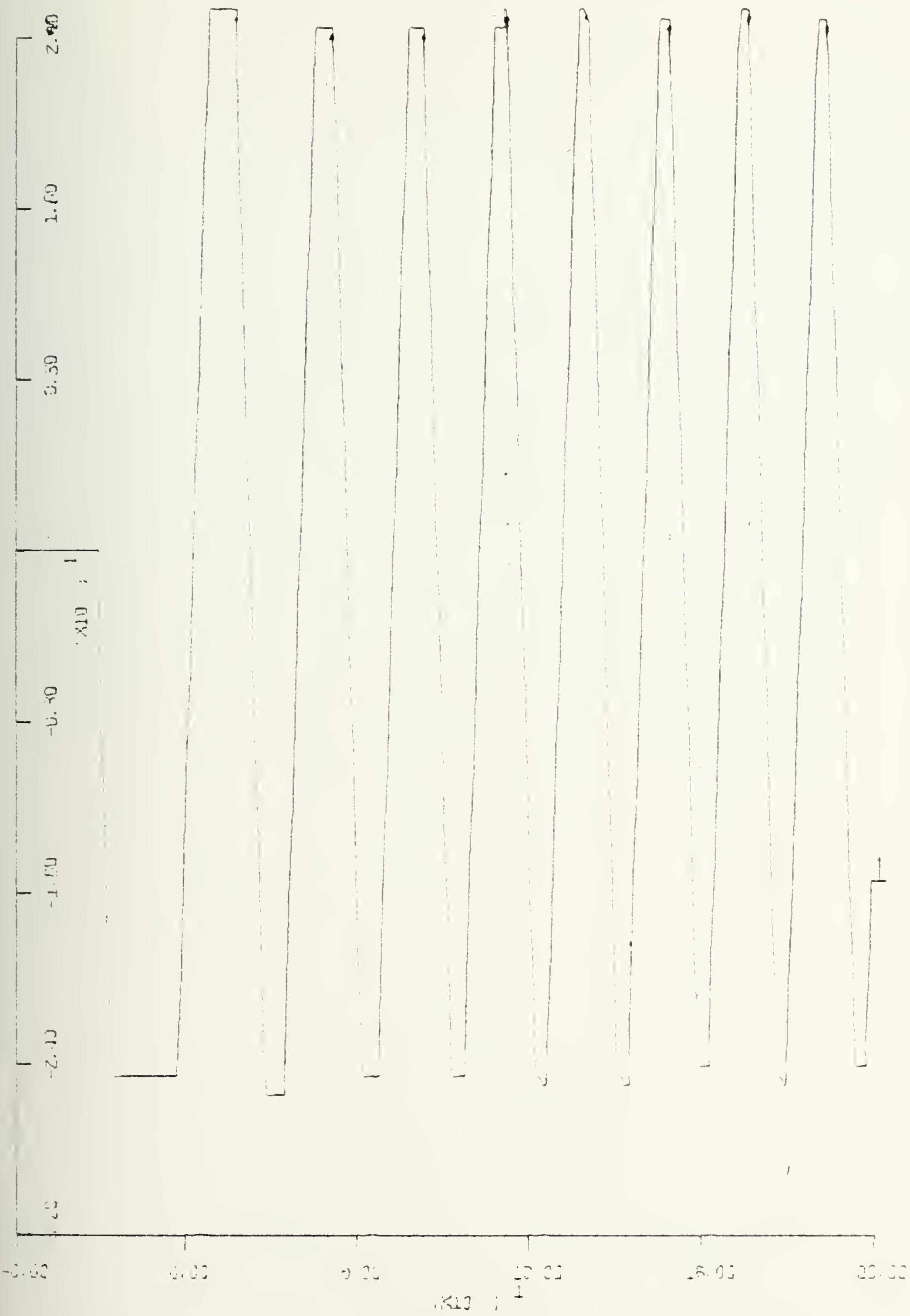
XSCALE=40.00 (s) UNITS/INCH
YSCALE= 8.00E-3(rad)UNITS/INCH

Fig. IV-39b. Pitch vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.2



XSCALE=40.00(s) UNITS/INCH
YSCALE=8.00(deg) UNITS/INCH

Fig. IV-39c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.2



XSCALE=40.00 (s) UNITS/INCH
YSCALE=8.00 (deg) UNITS/INCH

Fig. IV-39d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.2

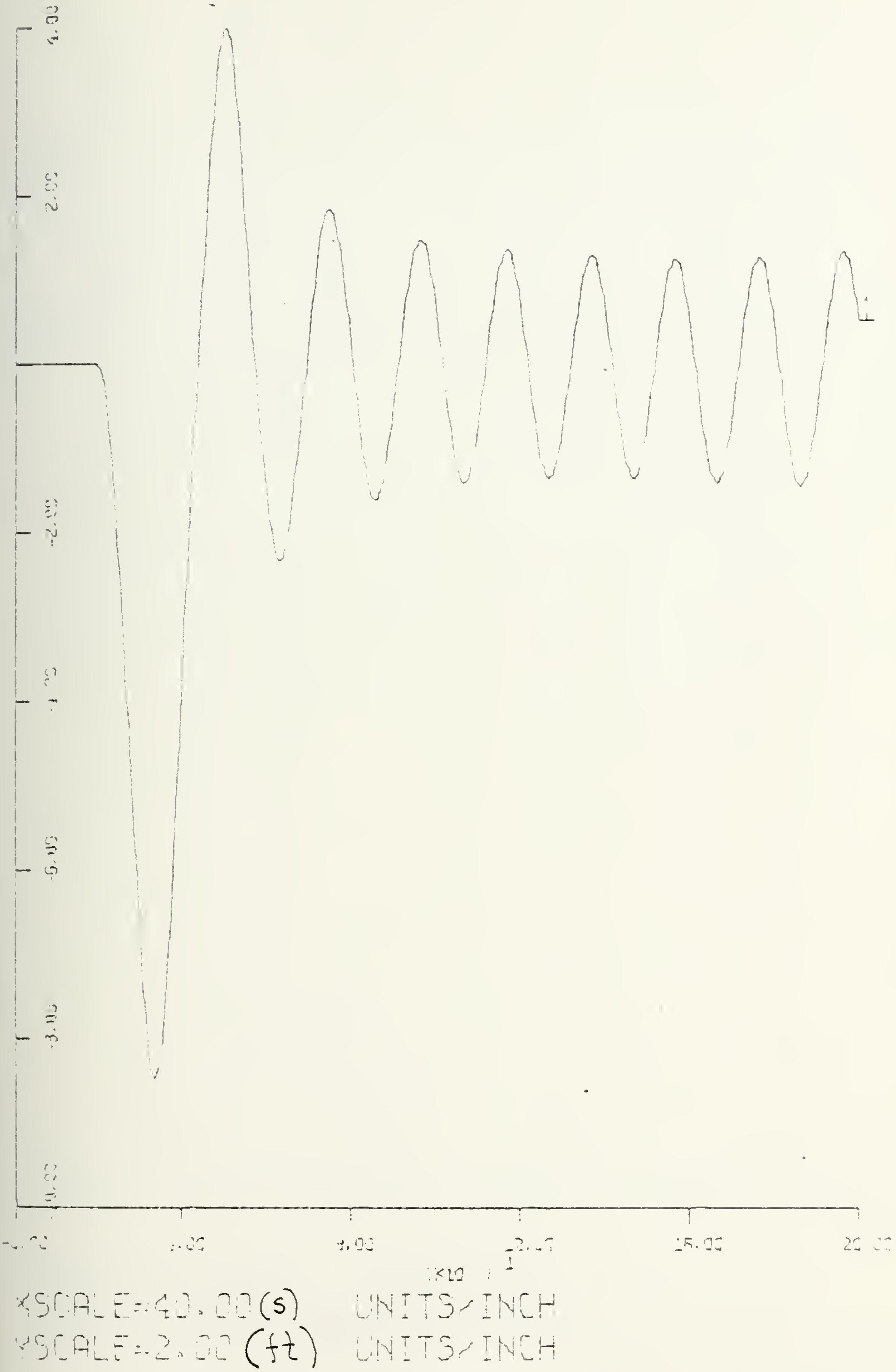
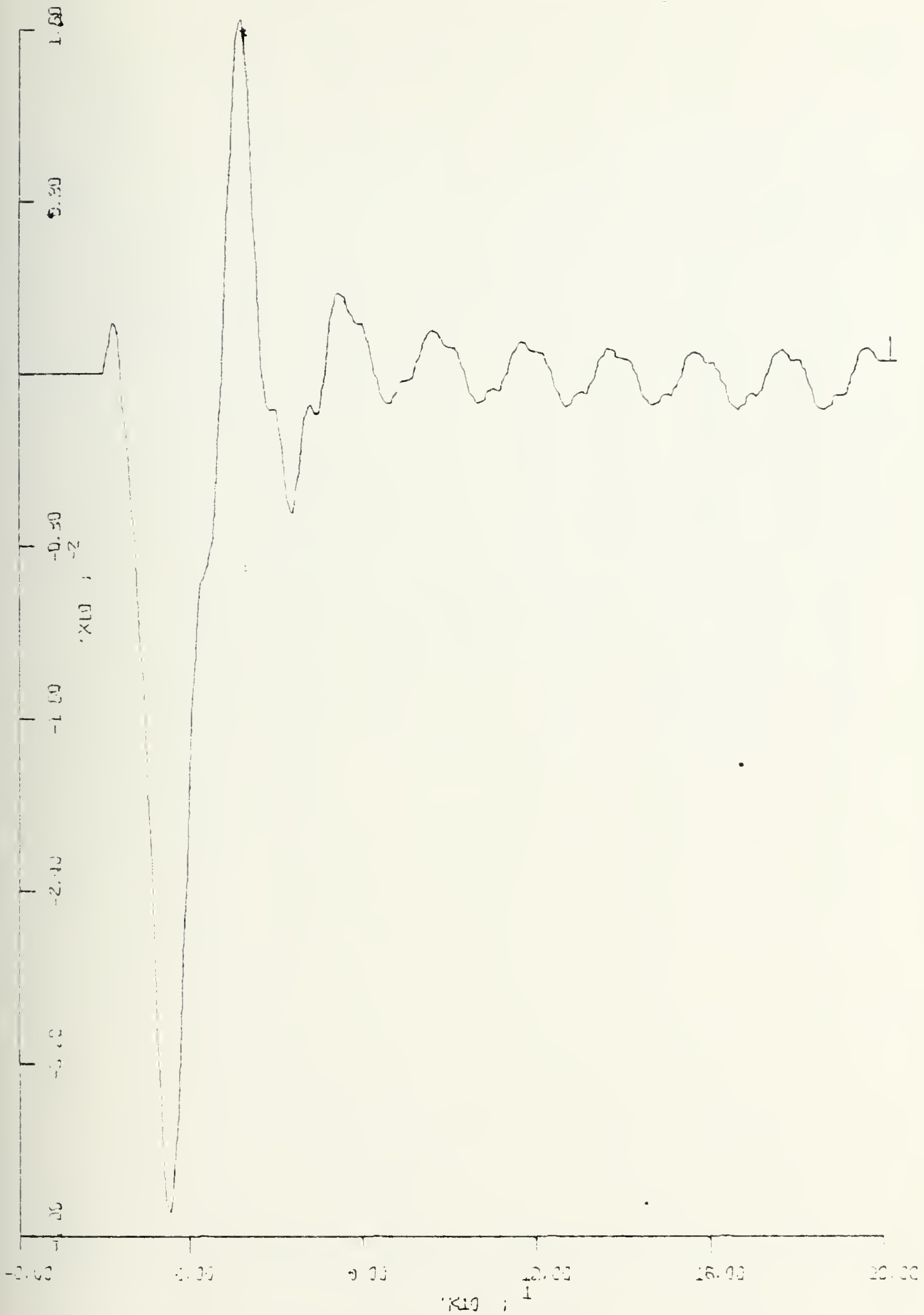


Fig. IV-40a. Depth vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.15



XSCALE=40.00(s) UNITS/INCH
YSCALE= 8.00E-3(rad)UNITS/INCH

Fig. IV-40b. Pitch vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.15

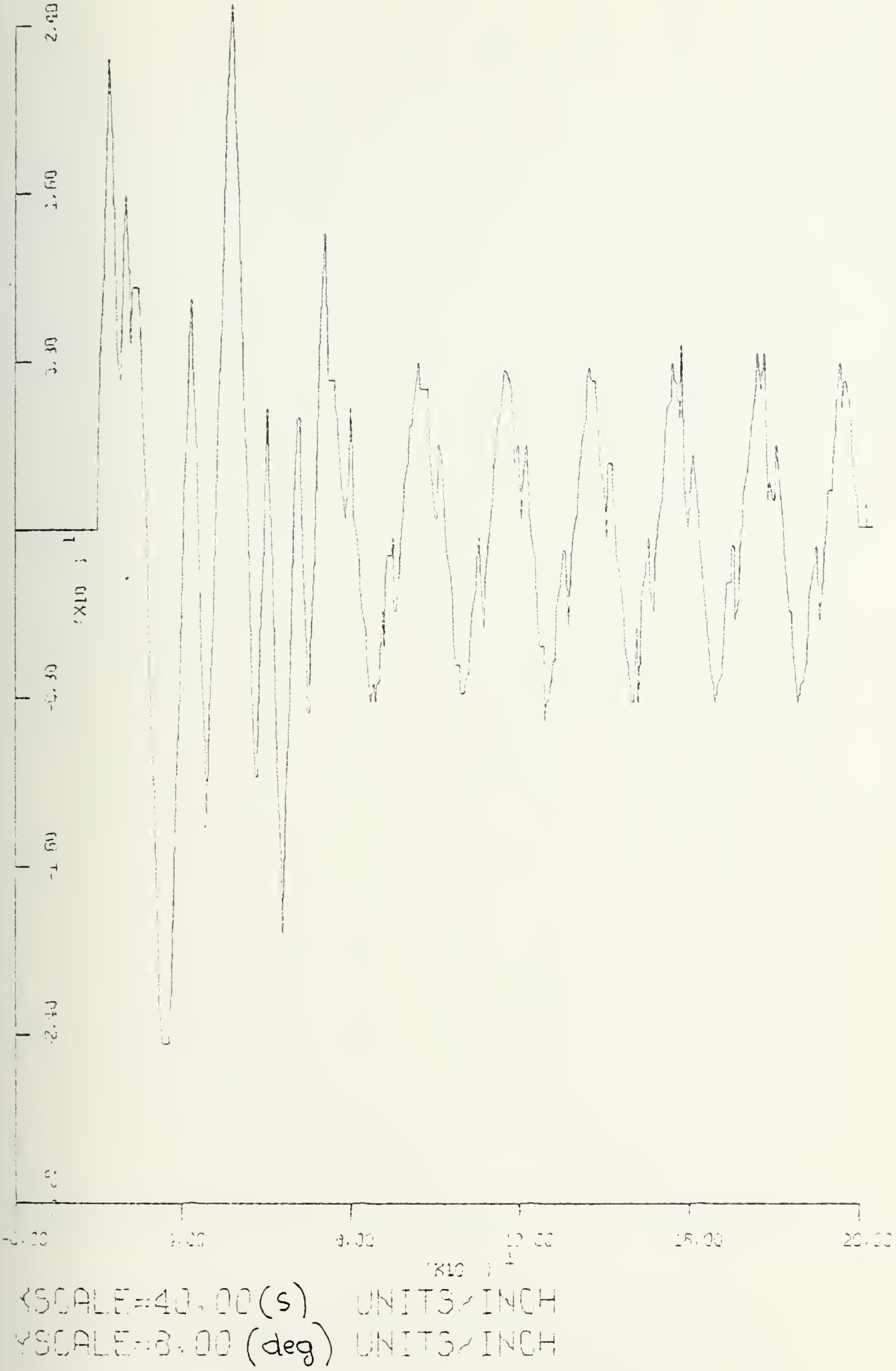


Fig. IV-40c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.15

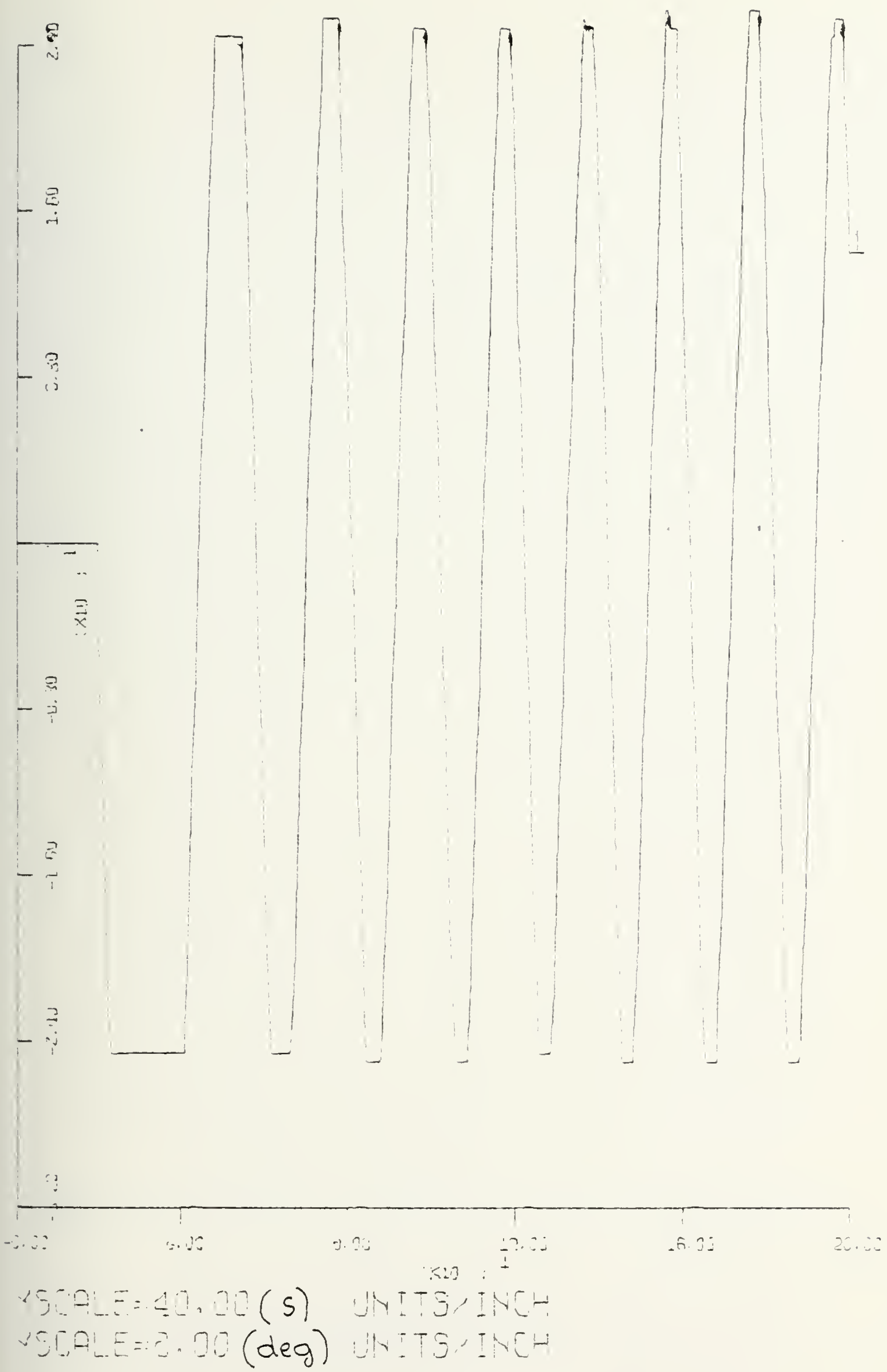


Fig. IV-40d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.15

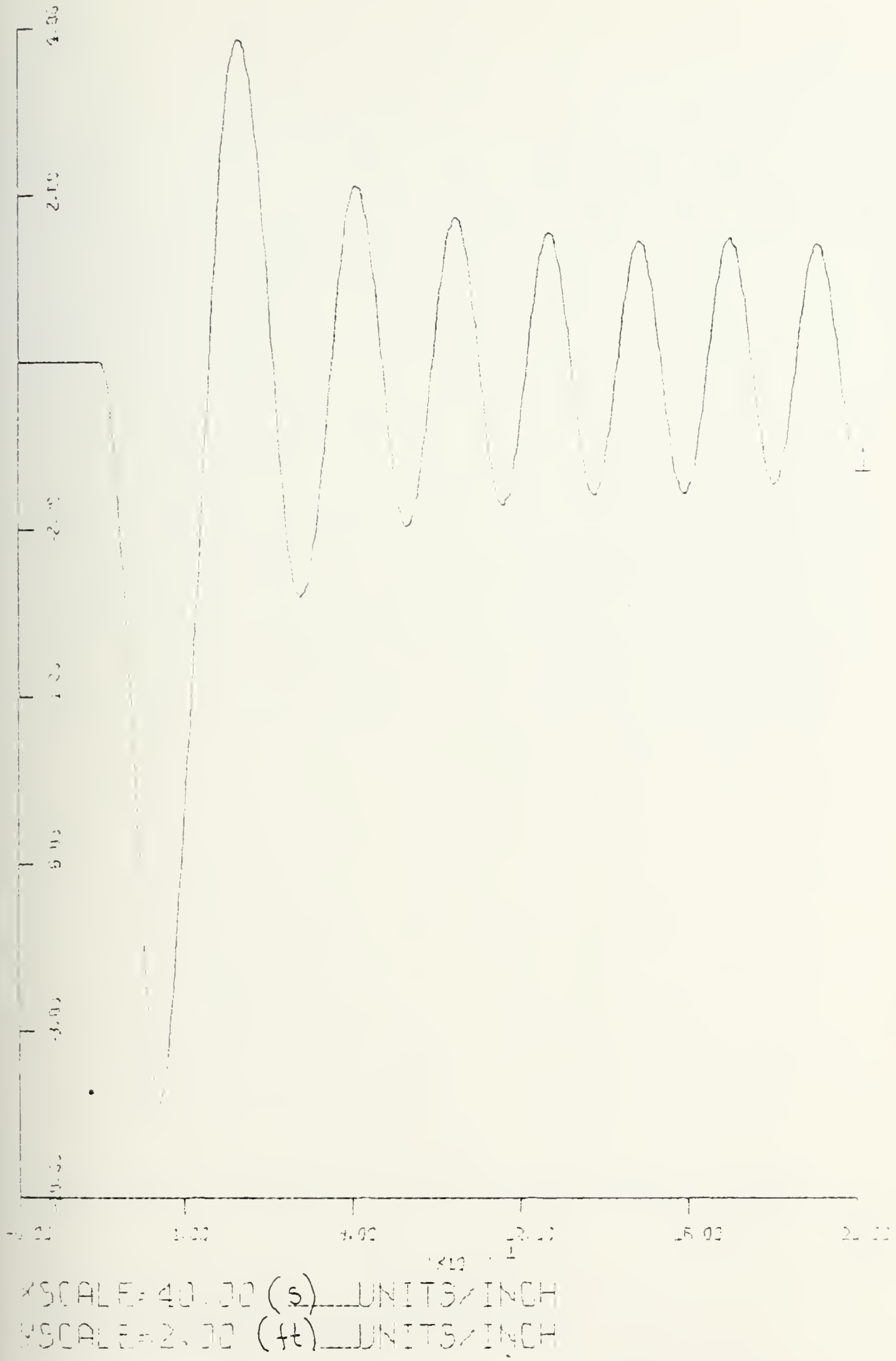


Fig. IV-41a. Depth vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.1

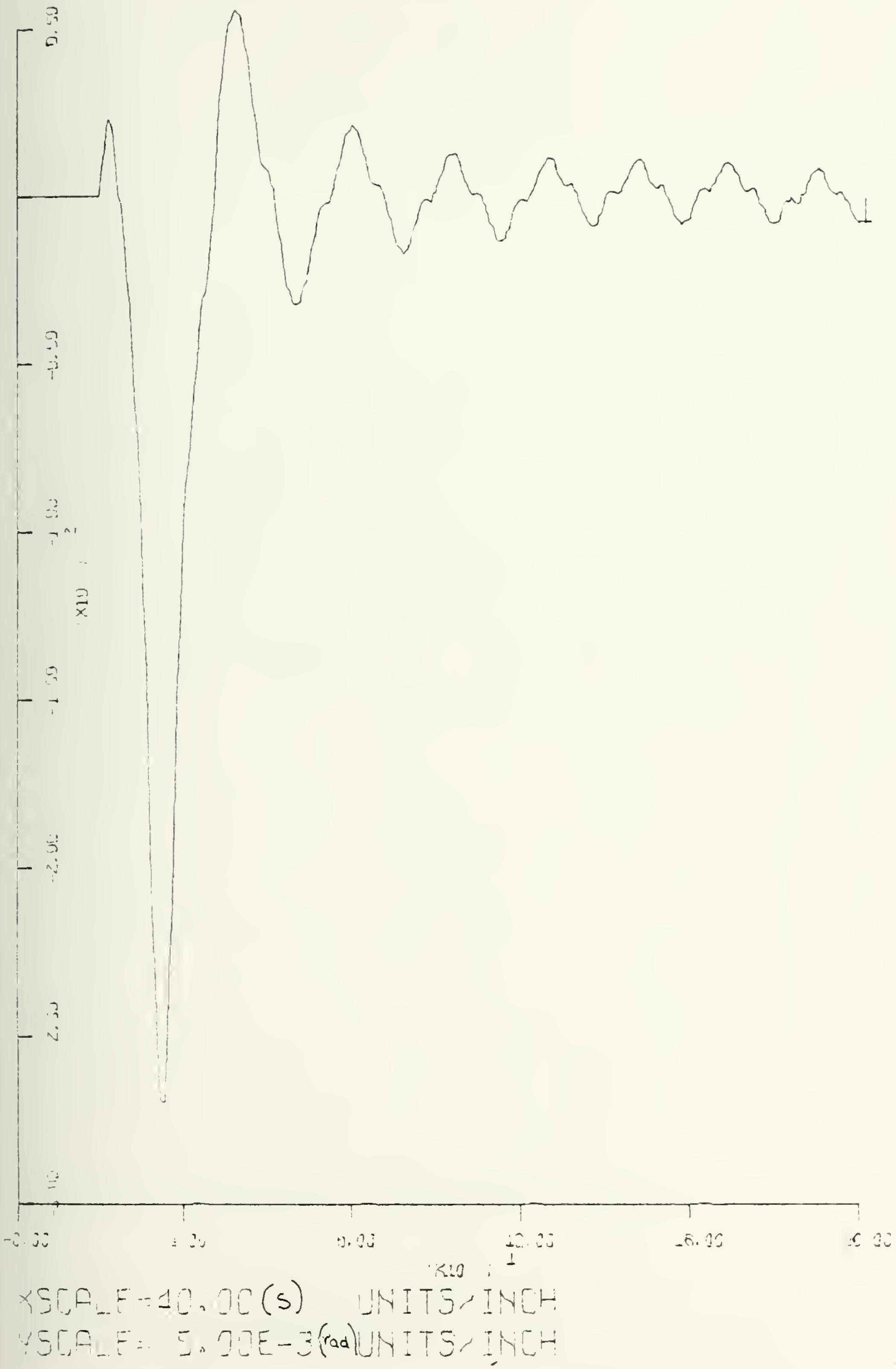


Fig. IV-41b. Pitch vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.1

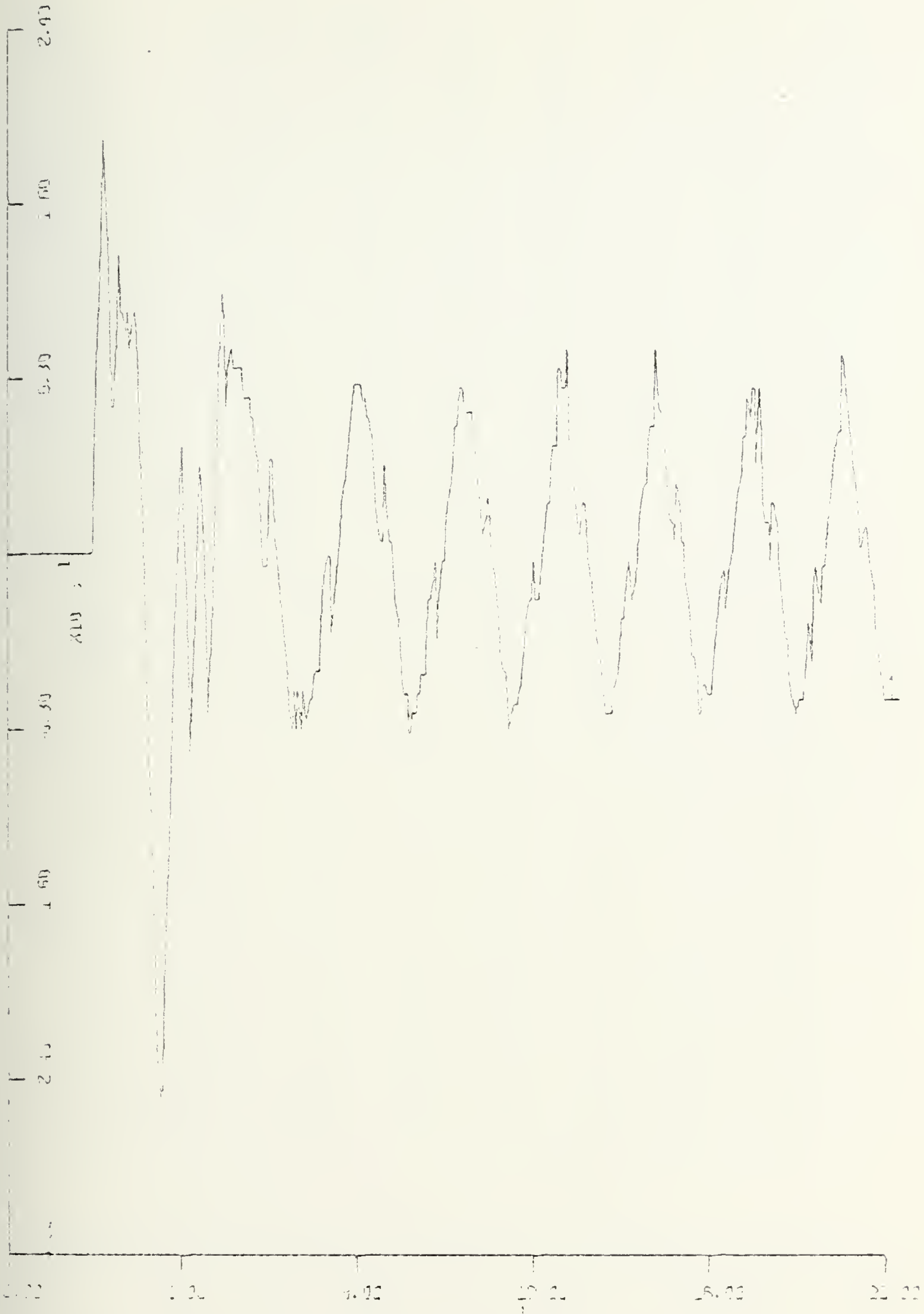


Fig. IV-41c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.1

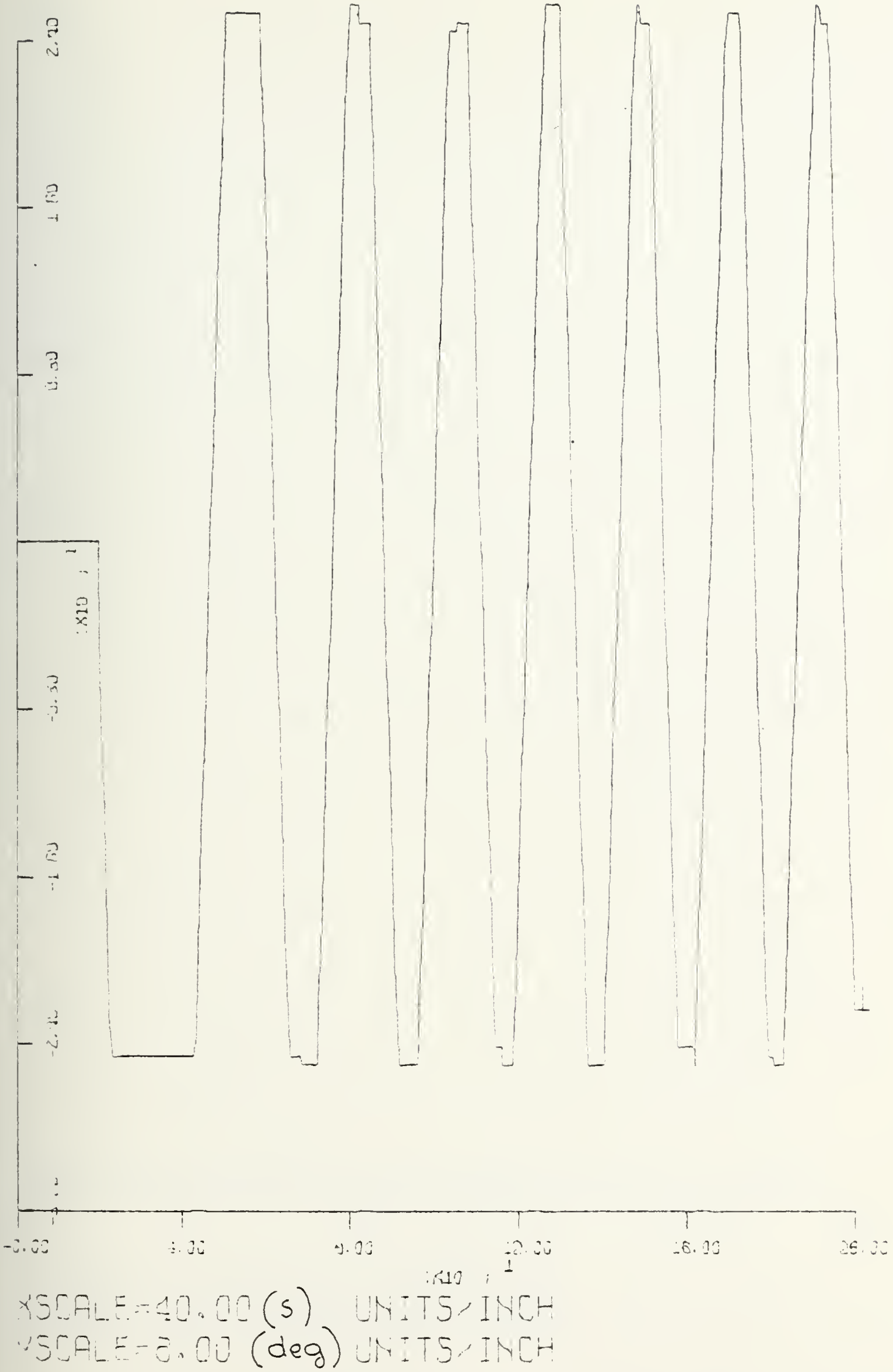
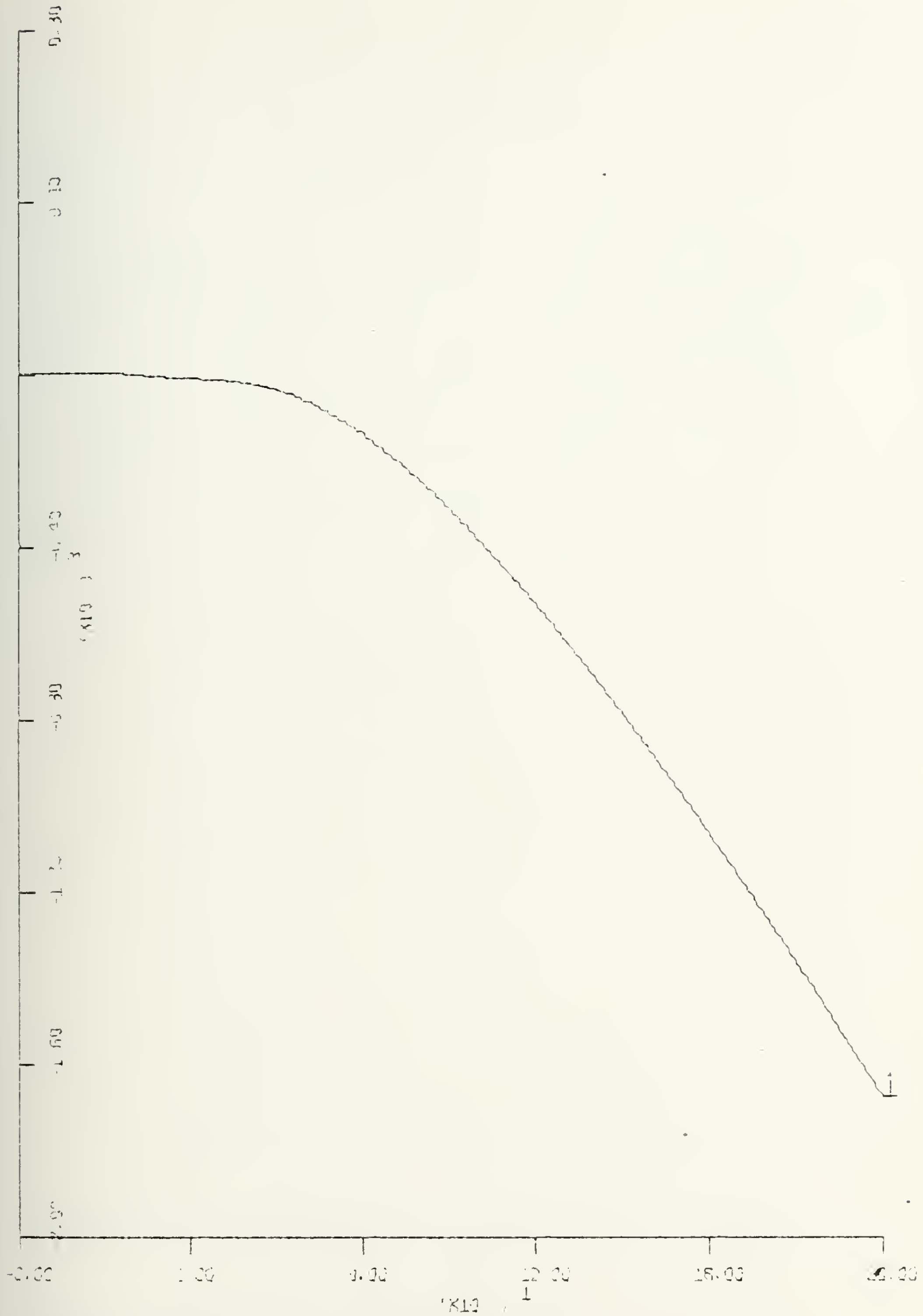


Fig. IV-41d. Fairwater Plane Angle vs. Time. Response to a pulse force of FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.1



XSCALE=40.00(s) UNITS/INCH
YSCALE=400.00(ft) UNITS/INCH

Fig. IV-42a. Depth vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.005

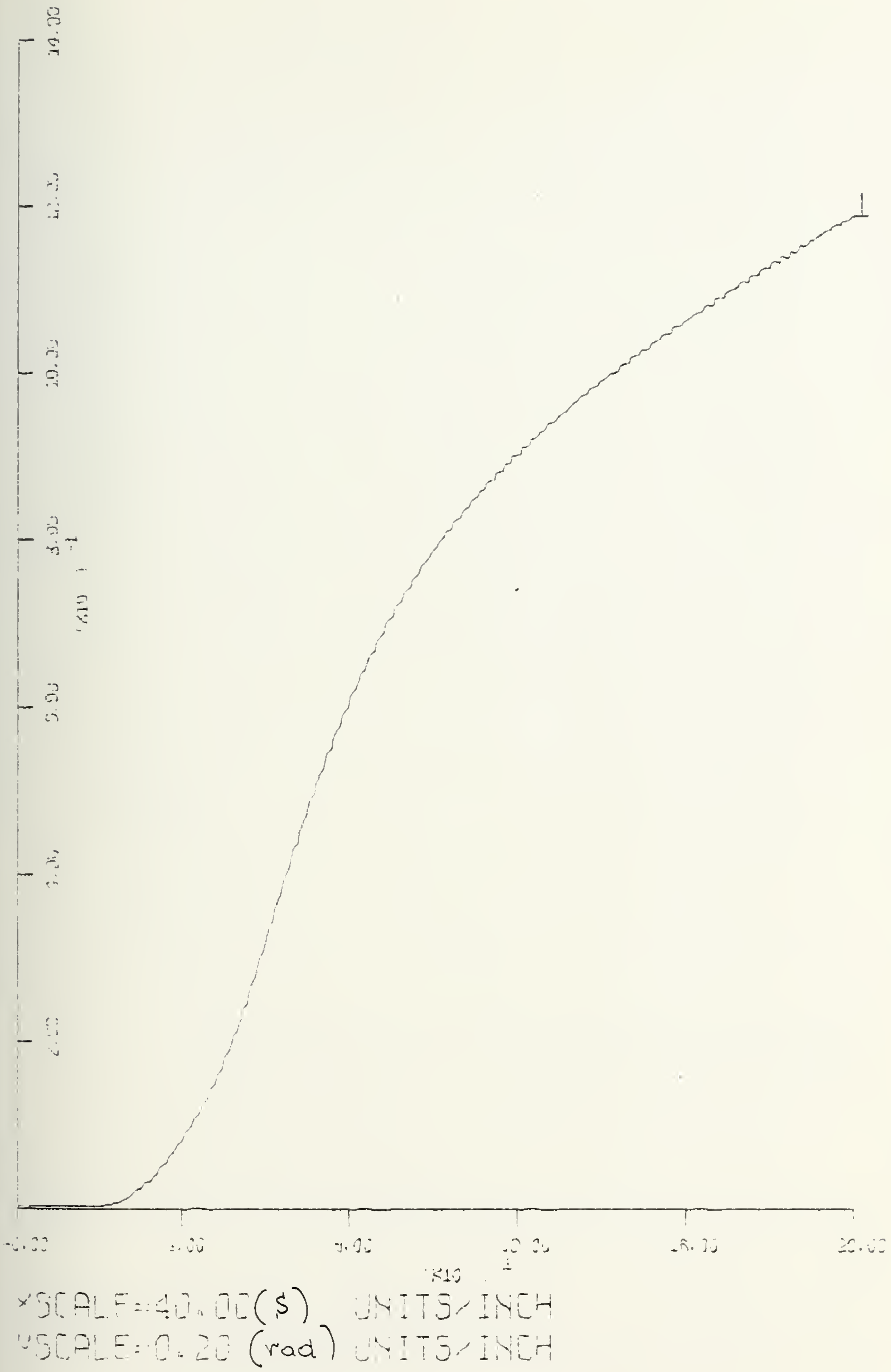


Fig. IV-42b. Pitch vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.005

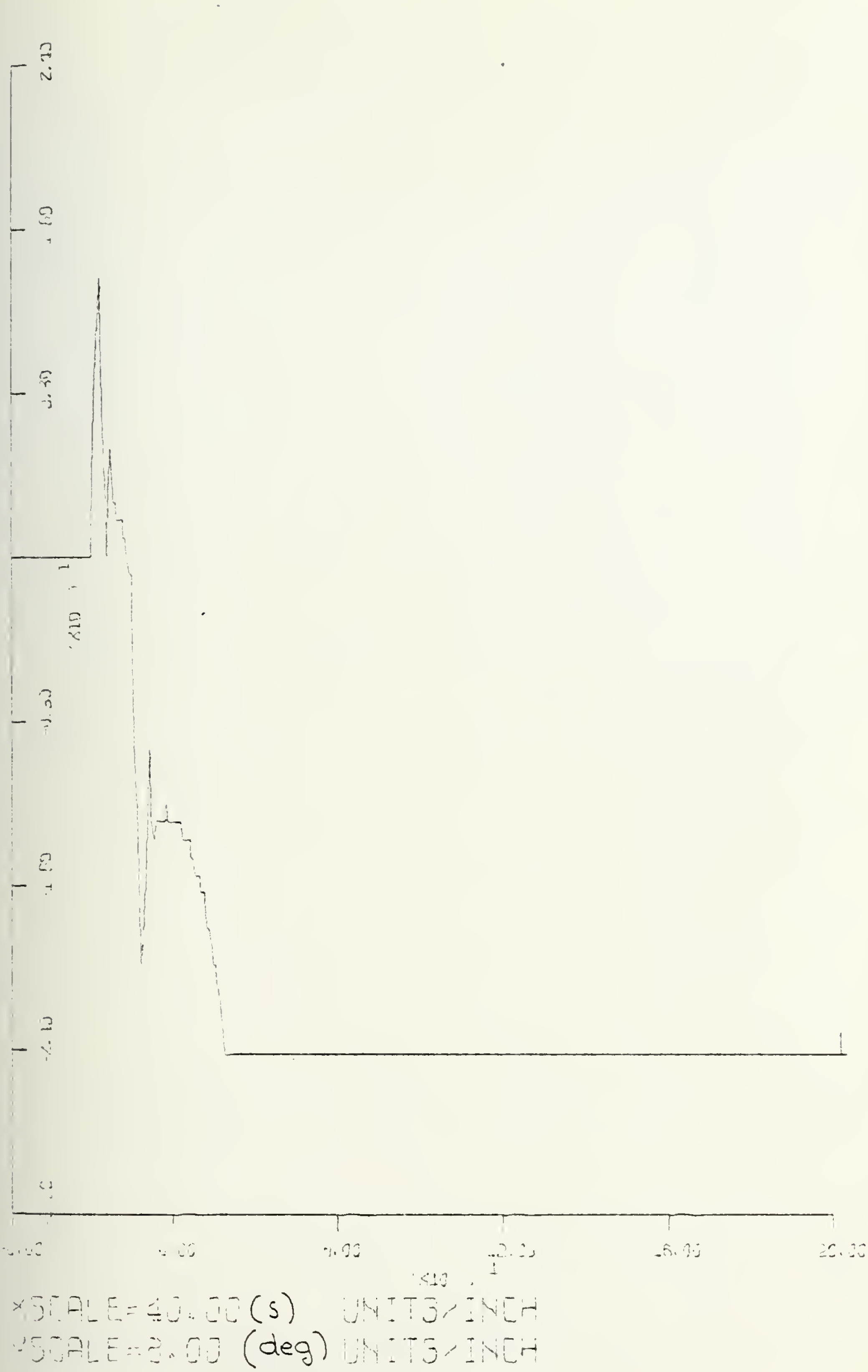


Fig. IV-42c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.005

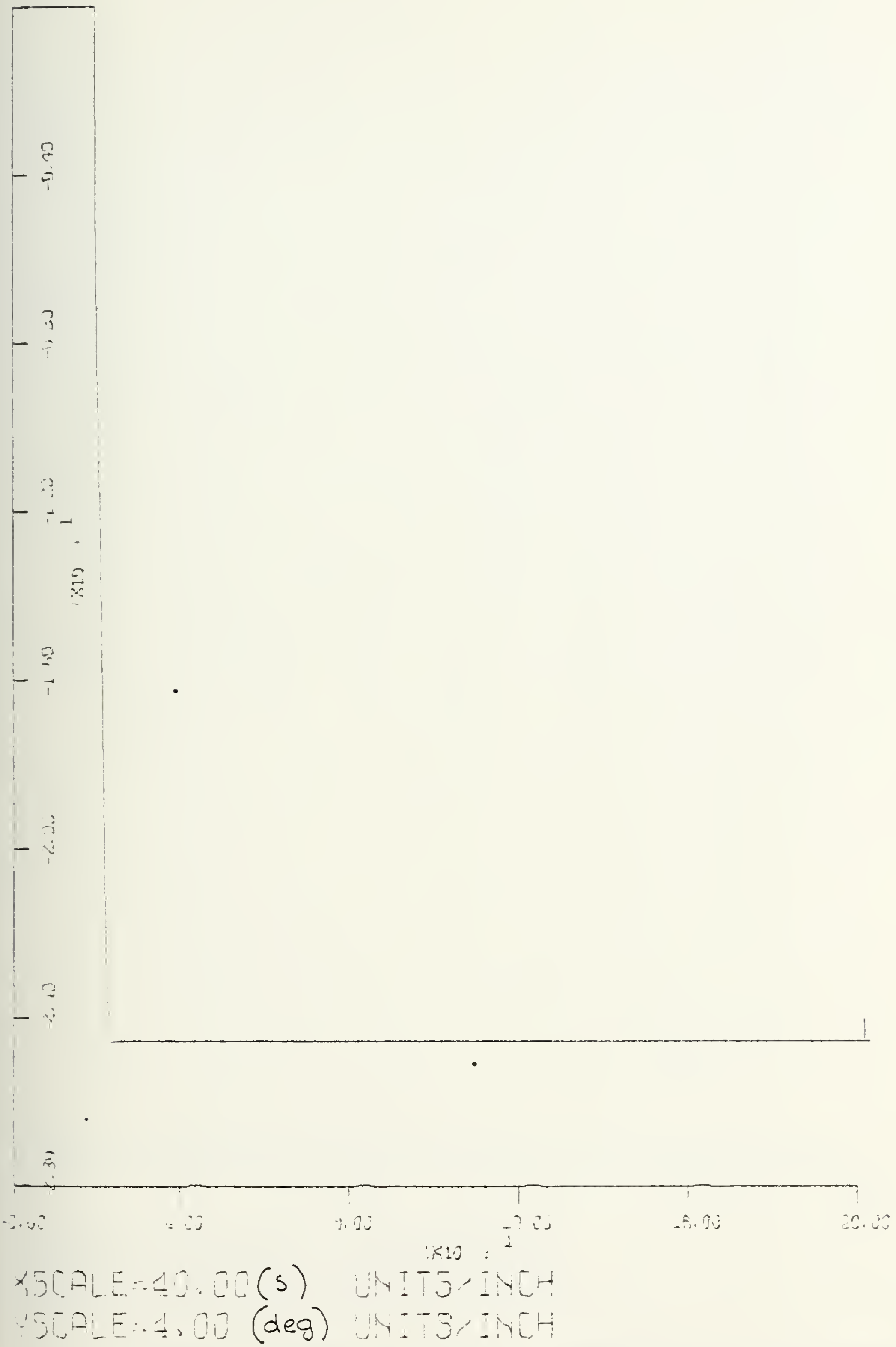


Fig. IV-42d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with B=164, C=0.001, E=0.001. Parameter X=0.005

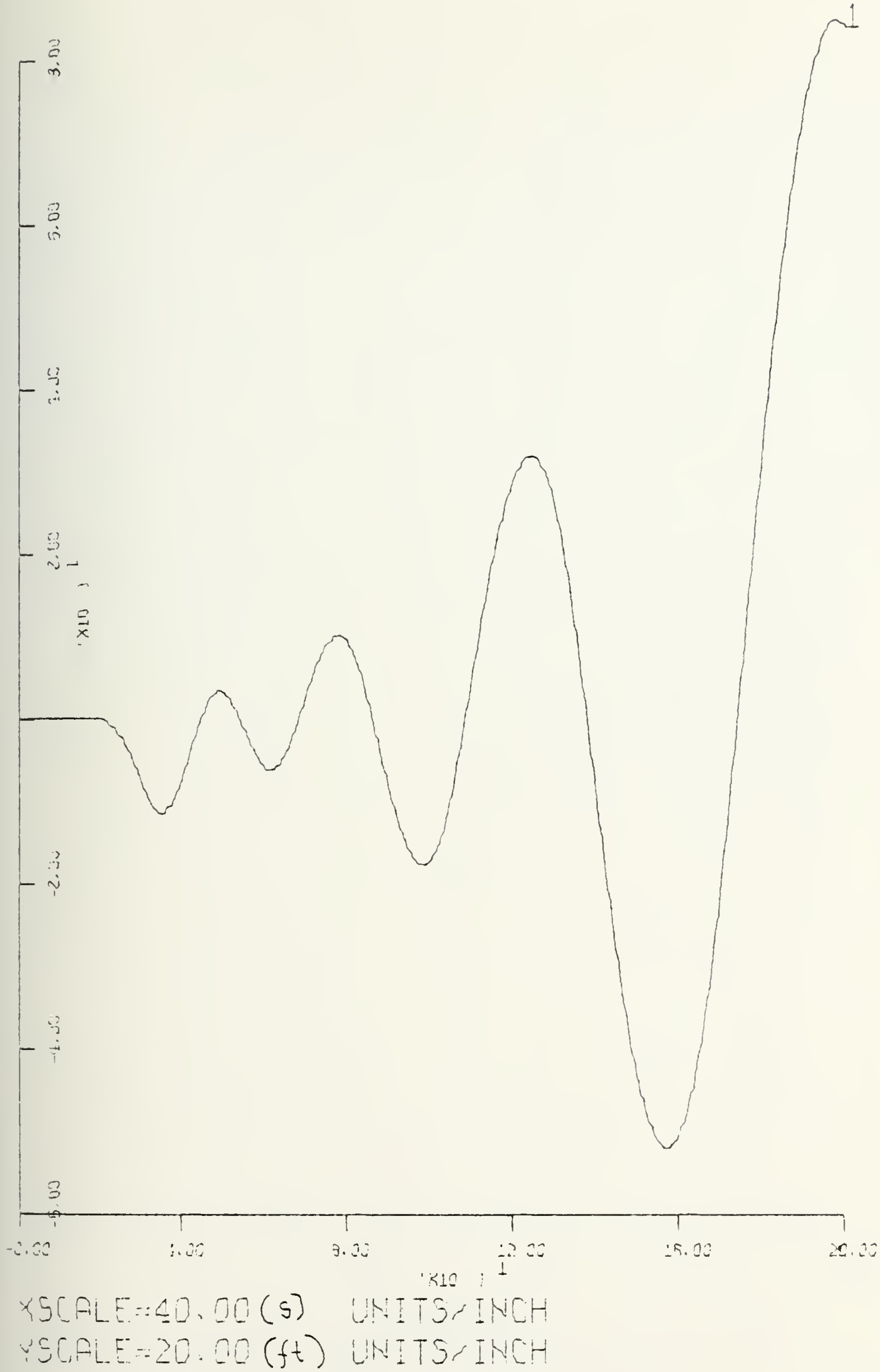


Fig. IV-43a. Depth vs. Time. Response to a pulse force at FT. CMC uses BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.7

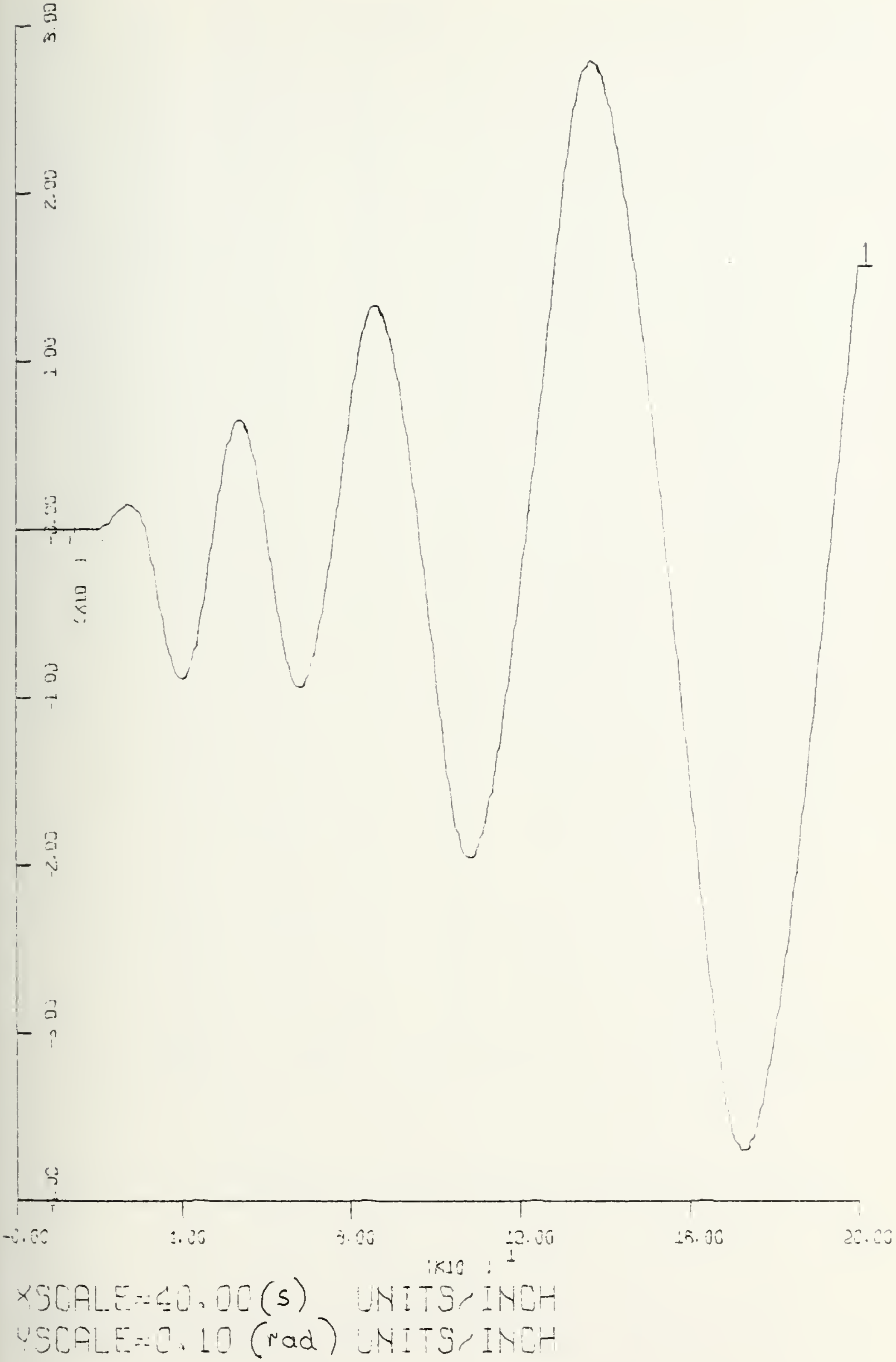


Fig. IV-43b. Pitch vs. Time. Response to a pulse force at FT. CMC uses BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.7

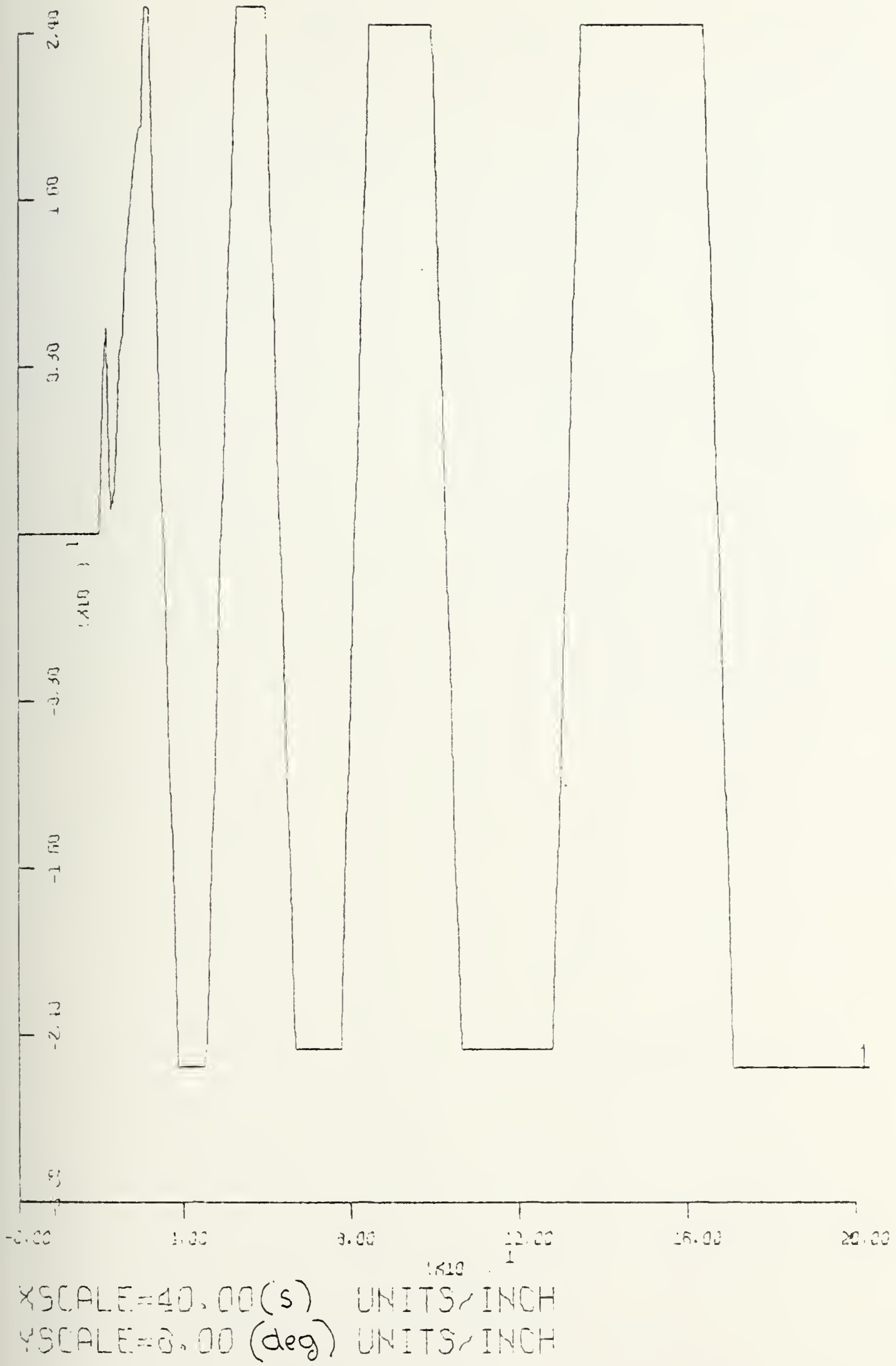


Fig. IV-43c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.7

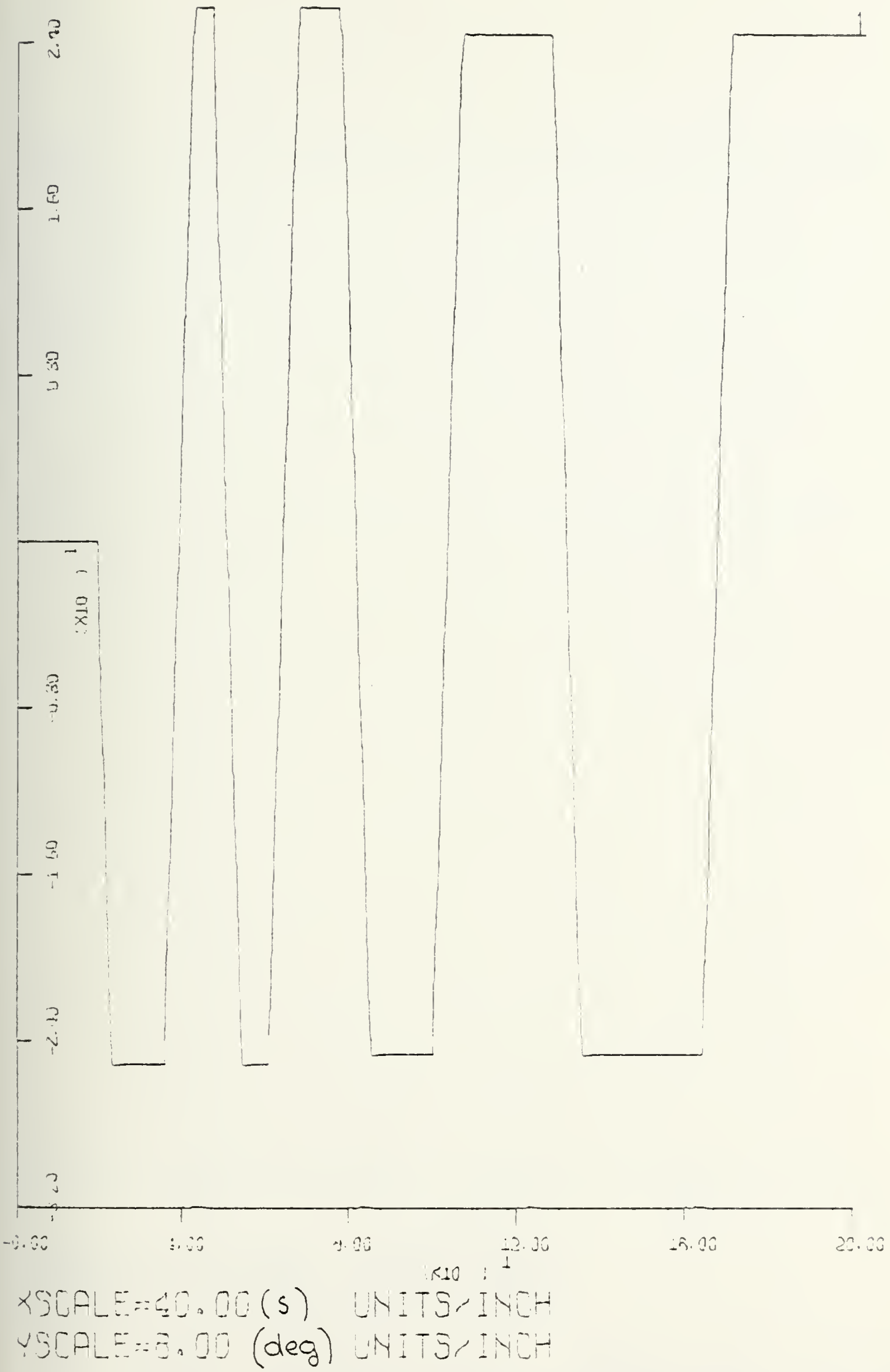


Fig. IV-43d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.7

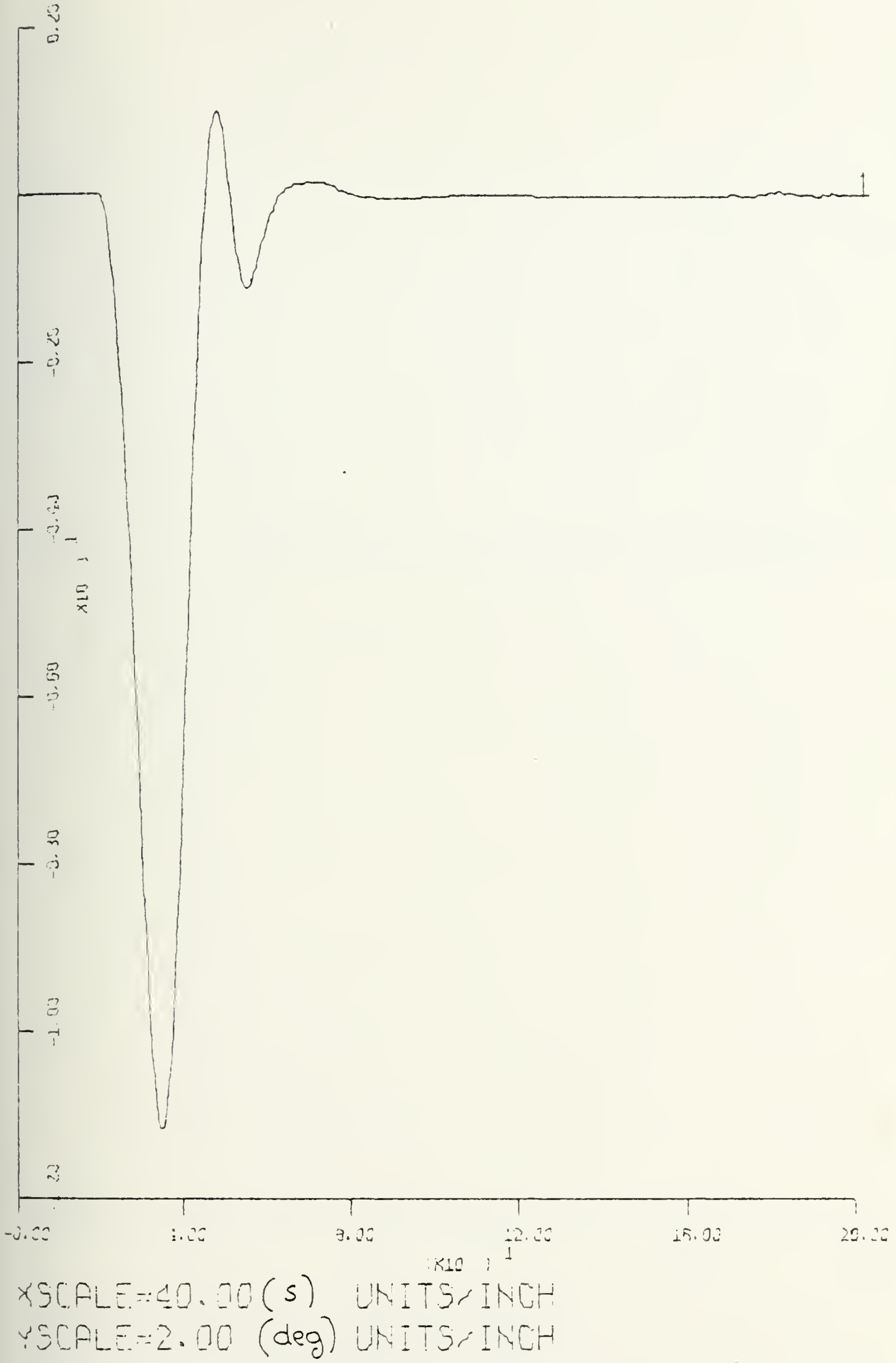


Fig. IV-44a. Depth vs. Time. Response to a pulse force at FT. CMC uses BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.5

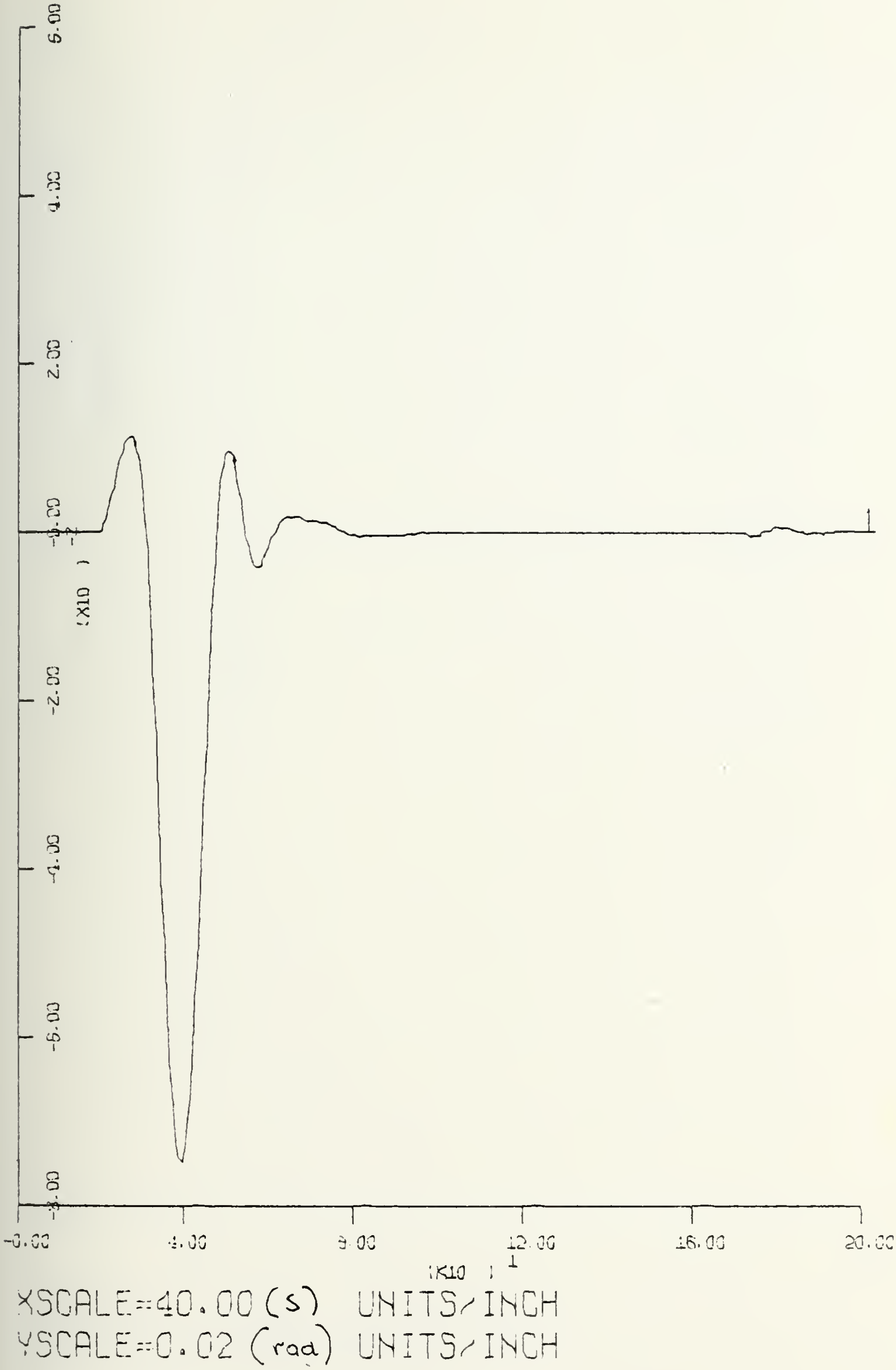
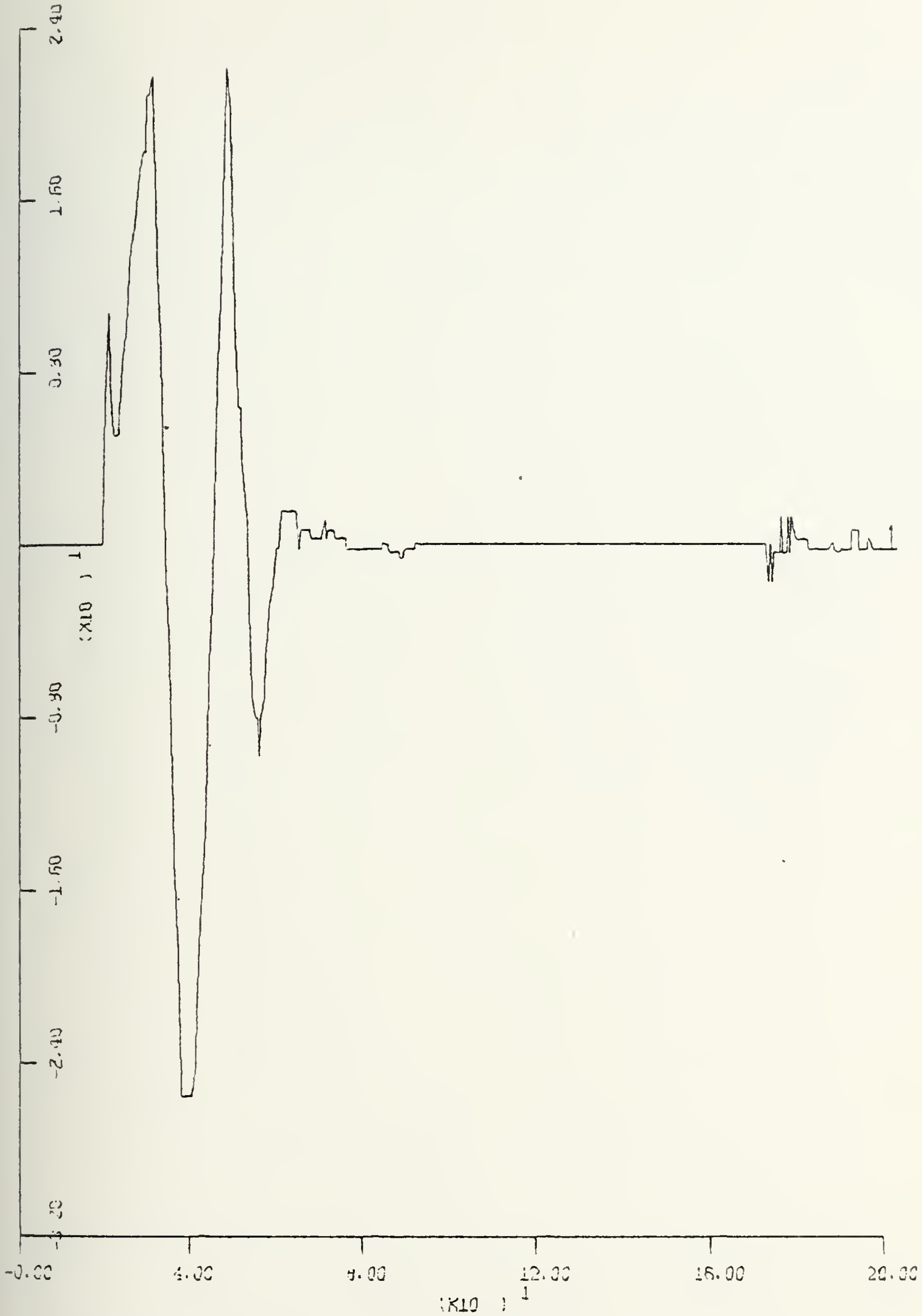


Fig. IV-44b. Pitch vs. Time. Response to a pulse force at FT. CMC uses BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.5



XSCALE=40.00(s) UNITS/INCH
YSCALE=8.00 (deg) UNITS/INCH

Fig. IV-44c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.5

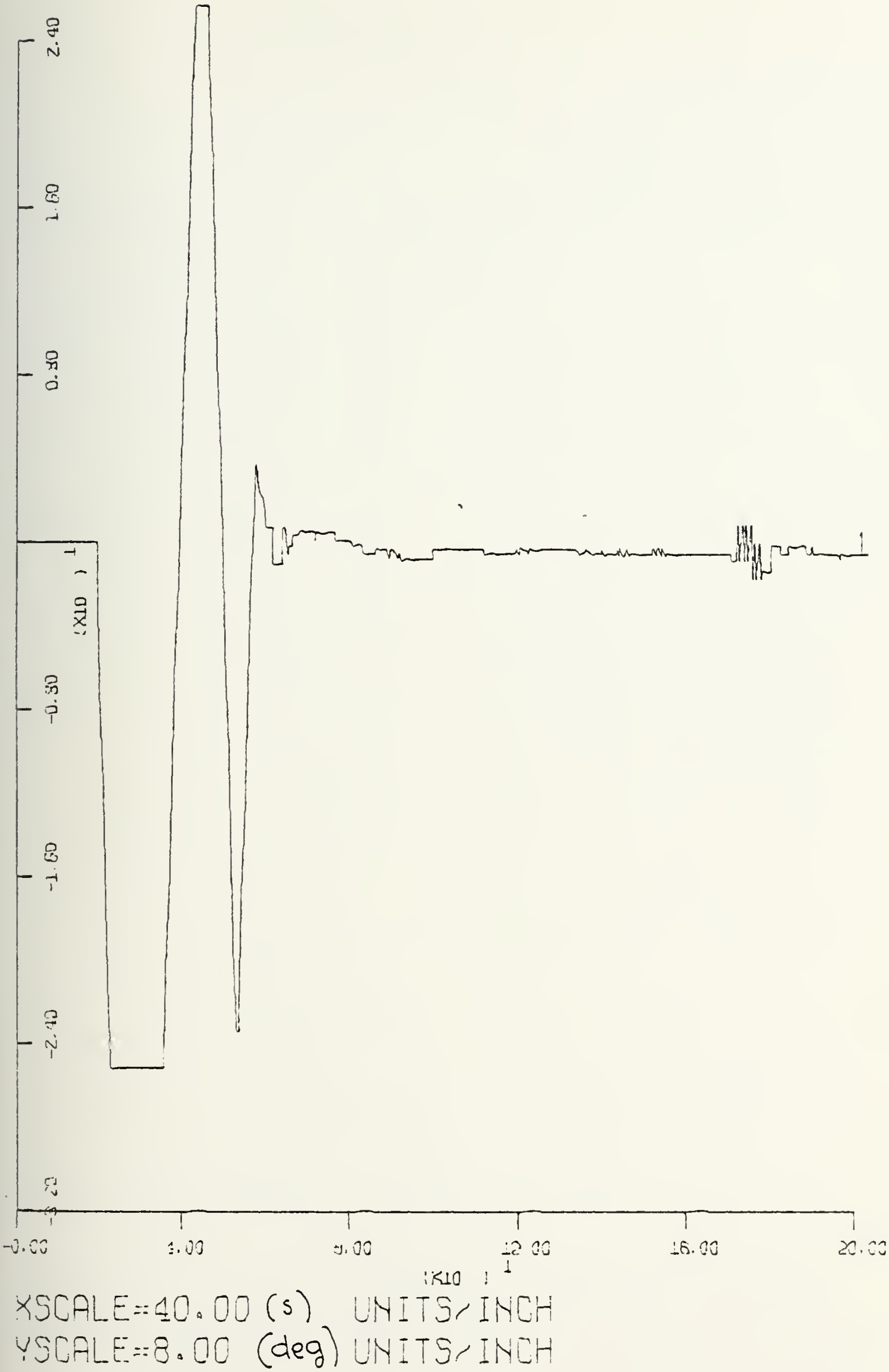


Fig. IV-44d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.5

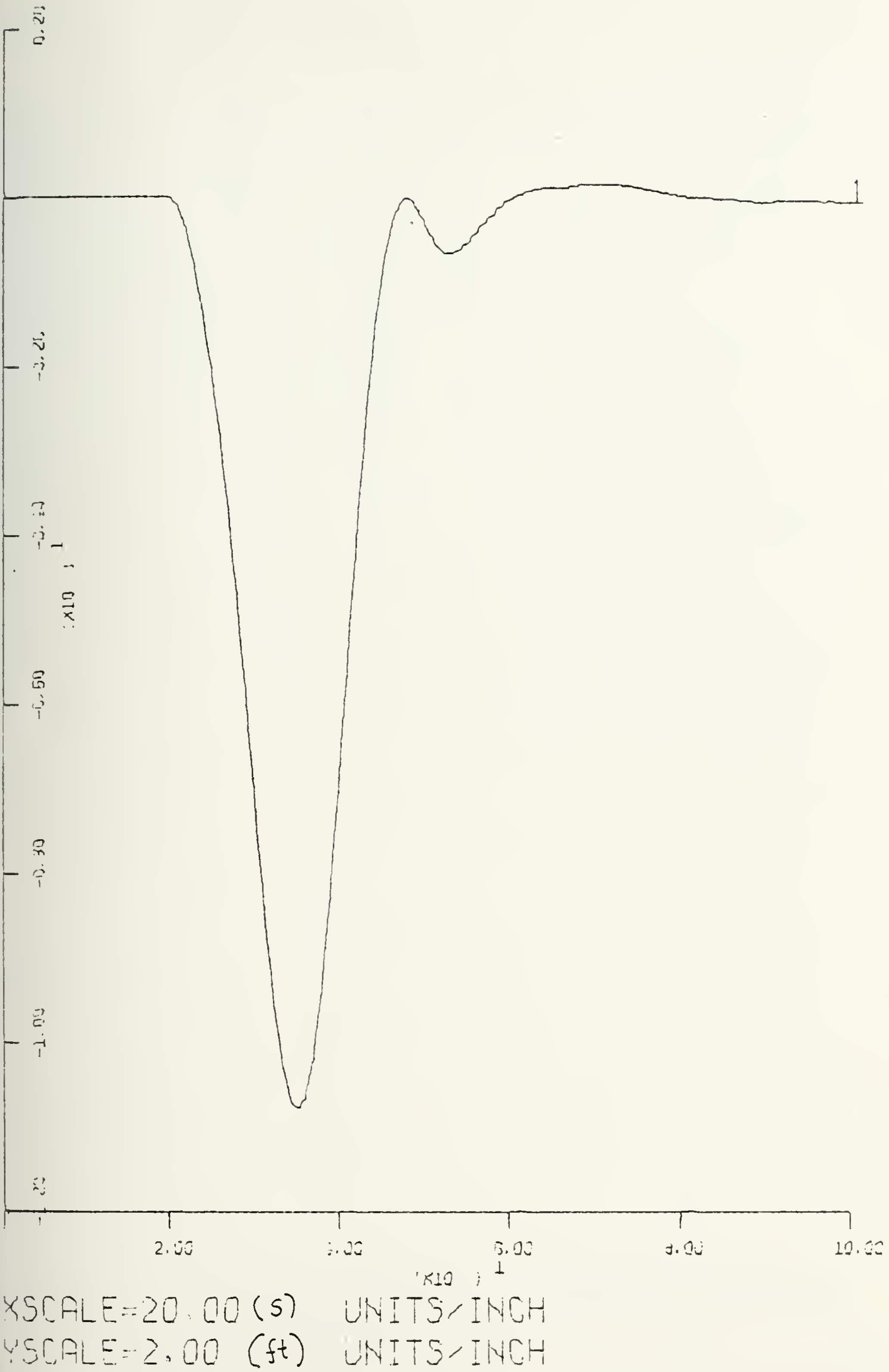


Fig. IV-45a. Depth vs. Time. Response to a pulse force at FT. CMC uses BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.4

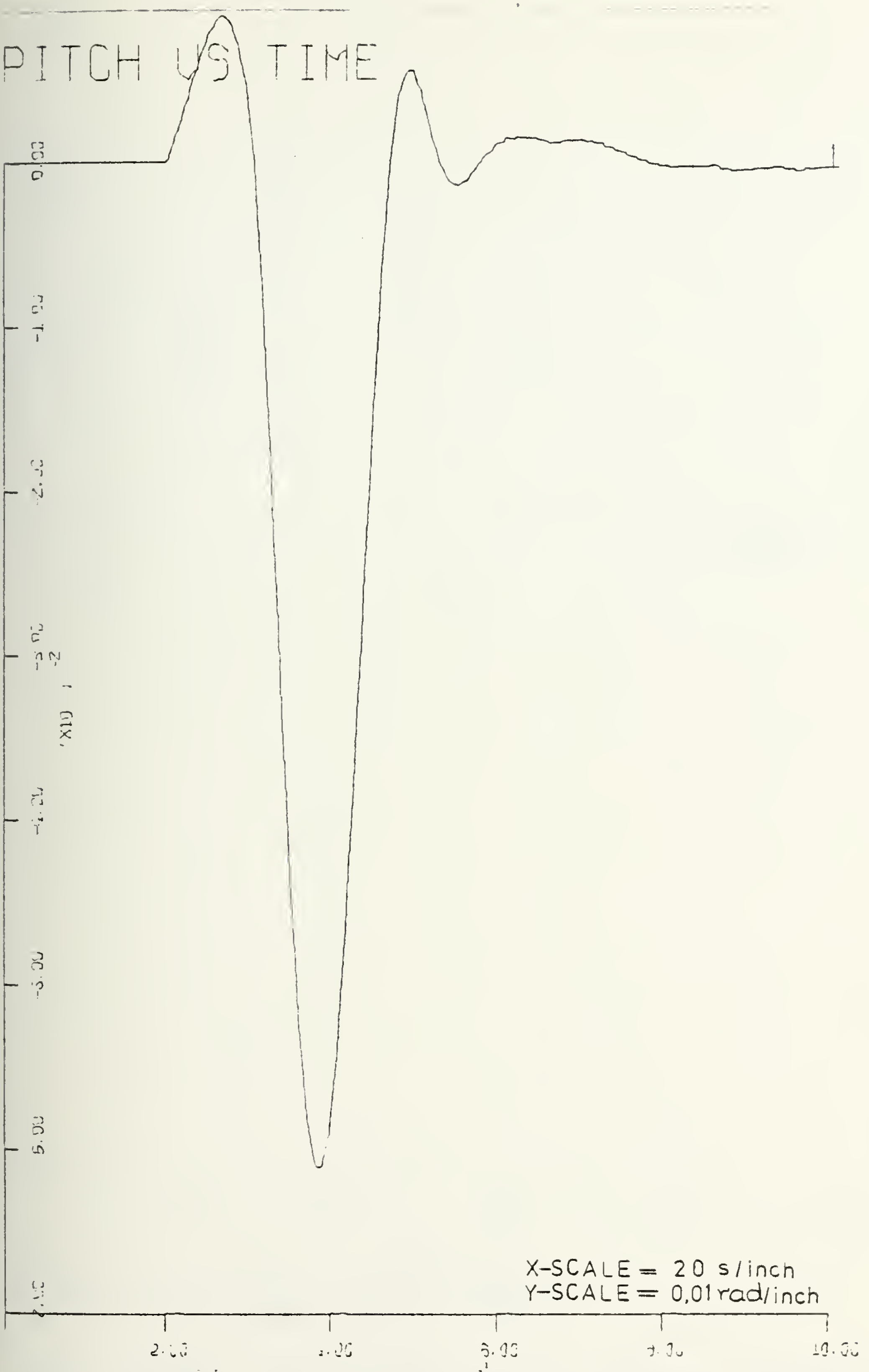


Fig. IV-45b. Pitch vs. Time. Response to a pulse force at FT. CMC uses BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.4

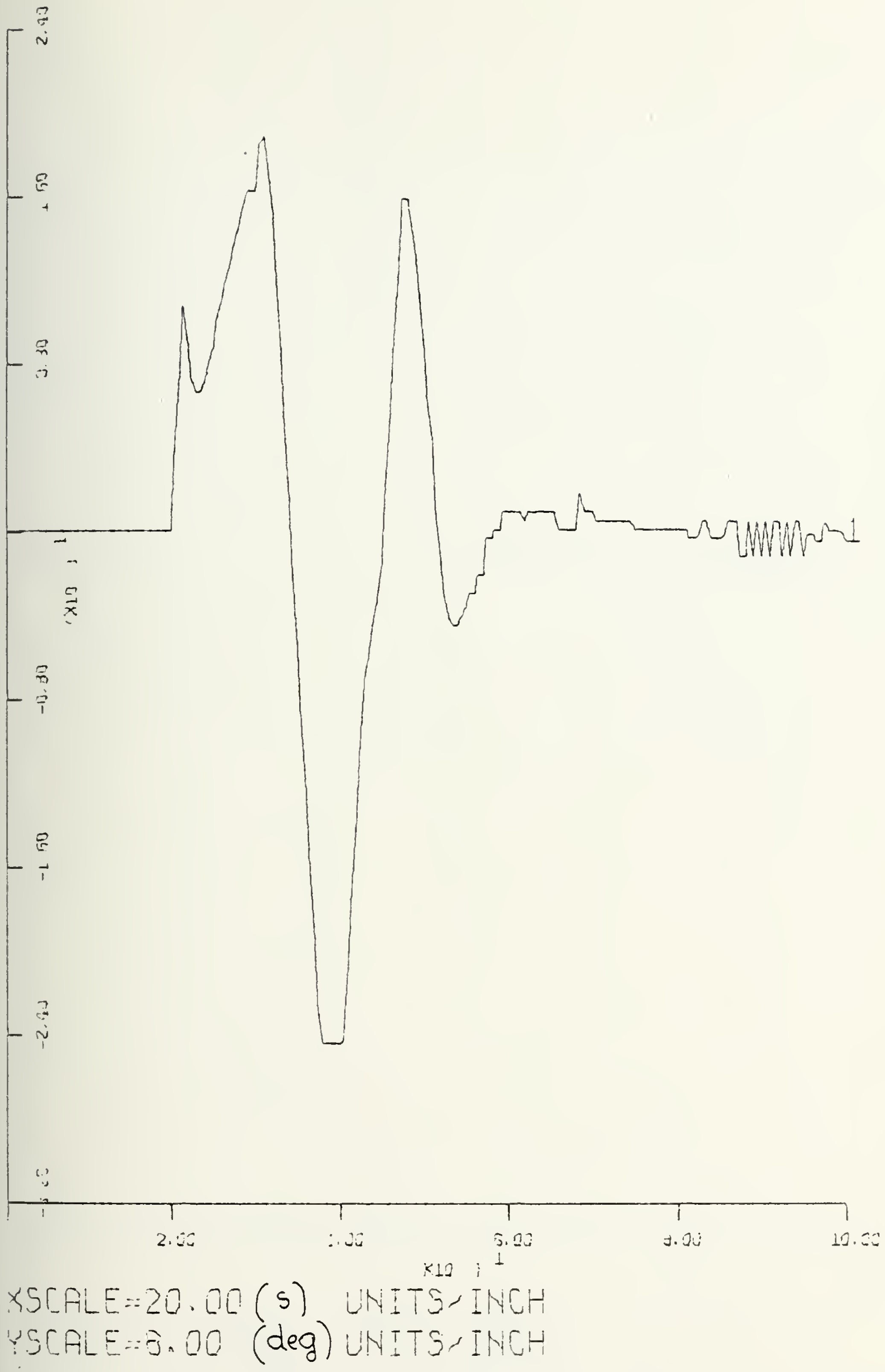
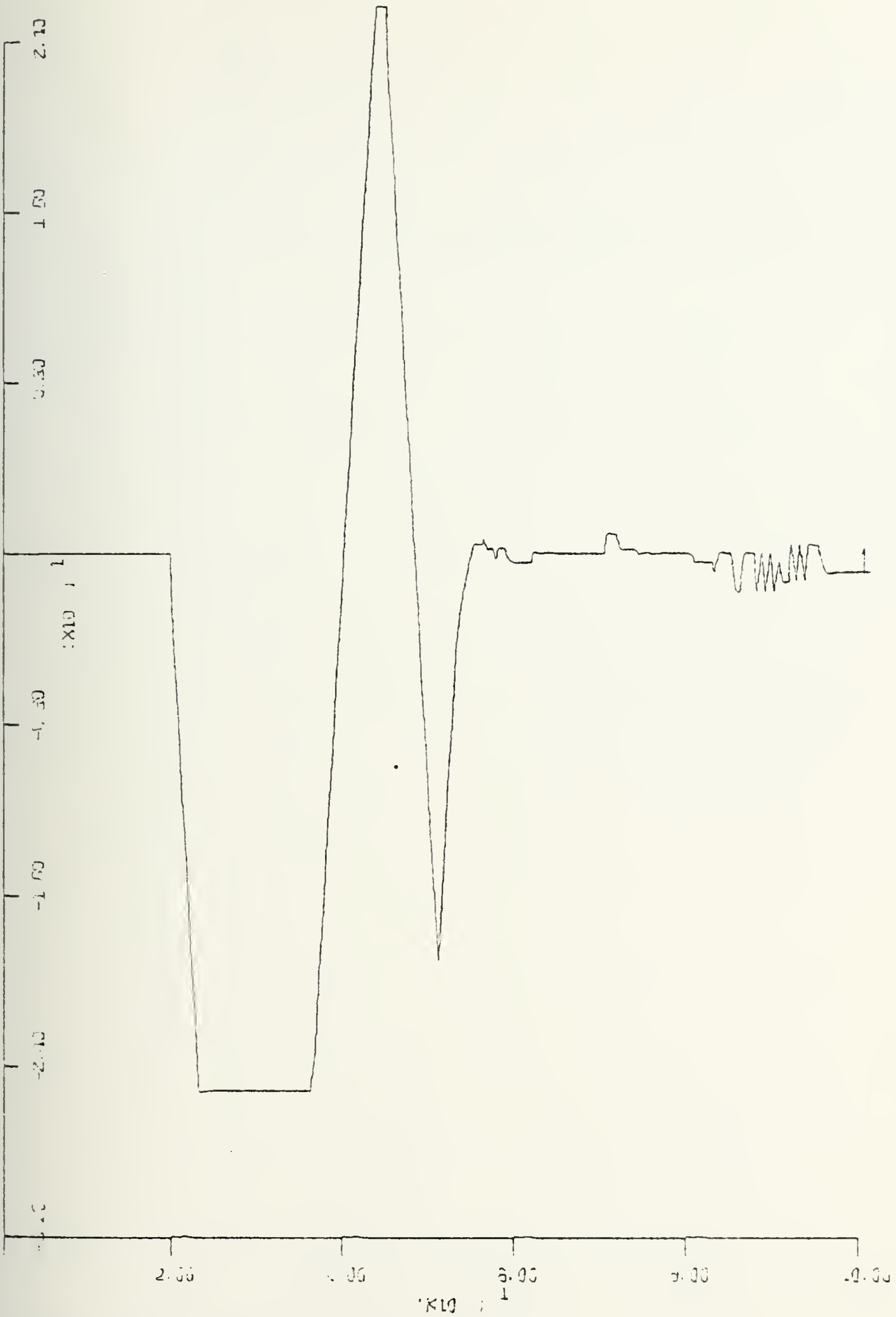
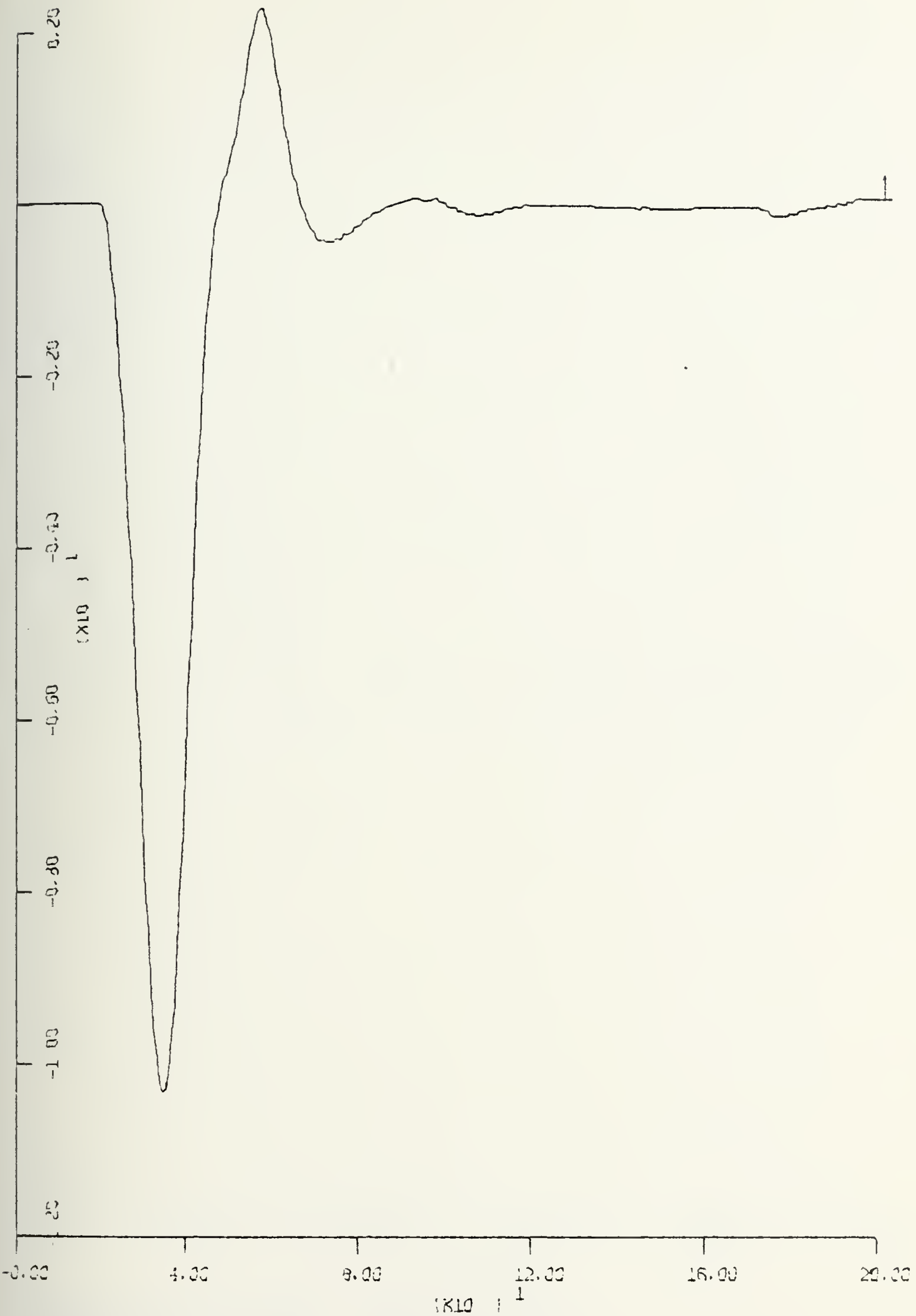


Fig. IV-45c. Stern Plane Angle vs. Time. Response to a pulse firce at FT. CMC uses BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.4



XSCALE=20.00 (s) UNITS/INCH
YSCALE=8.00 (deg) UNITS/INCH

Fig. IV-45d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC used BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.4



XSCALE=40.00 (s) UNITS/INCH
YSCALE=2.00 (ft) UNITS/INCH

Fig. IV-46a. Depth vs. Time. Response to a pulse force at FT. CMC uses BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.3

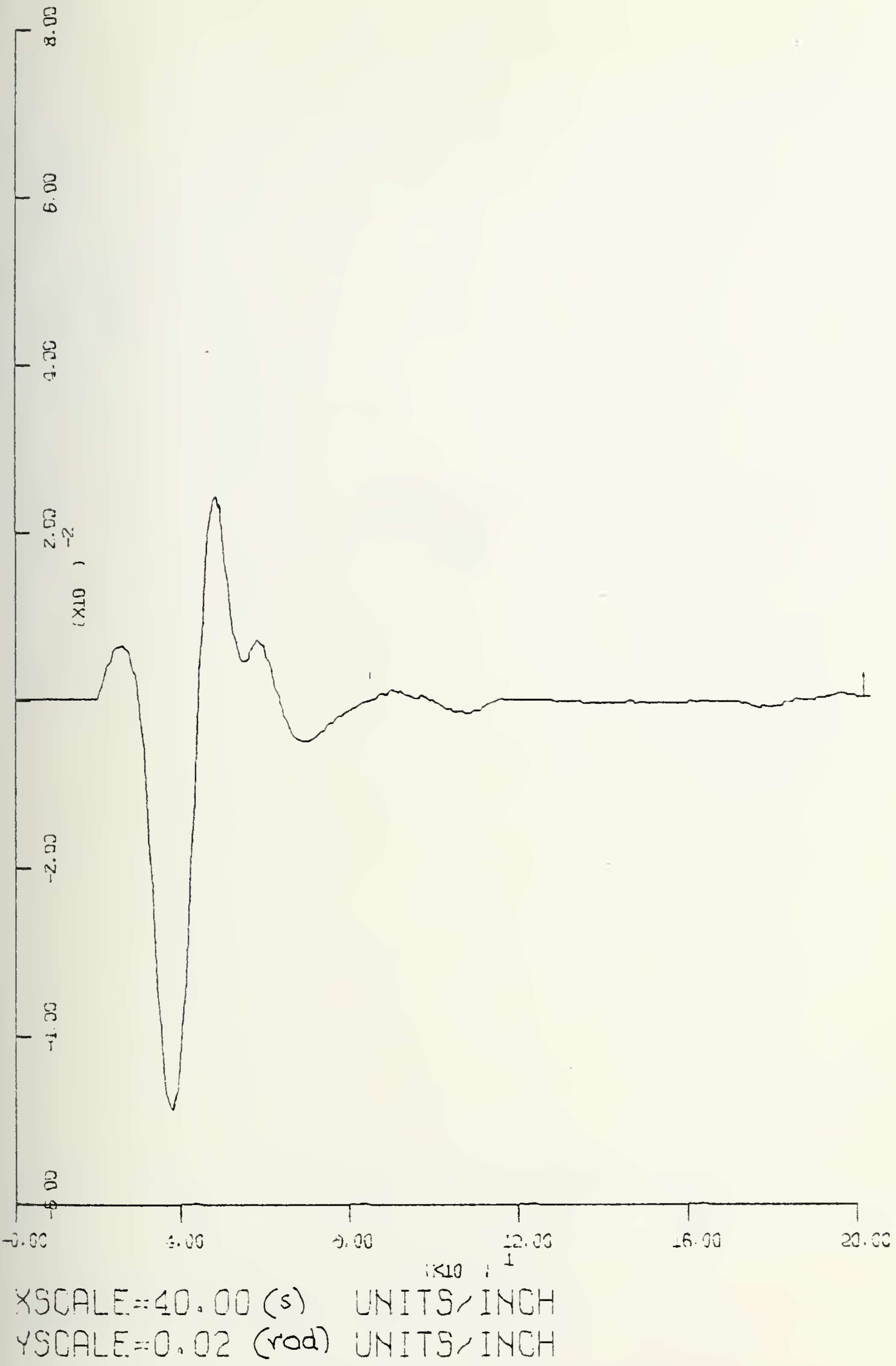
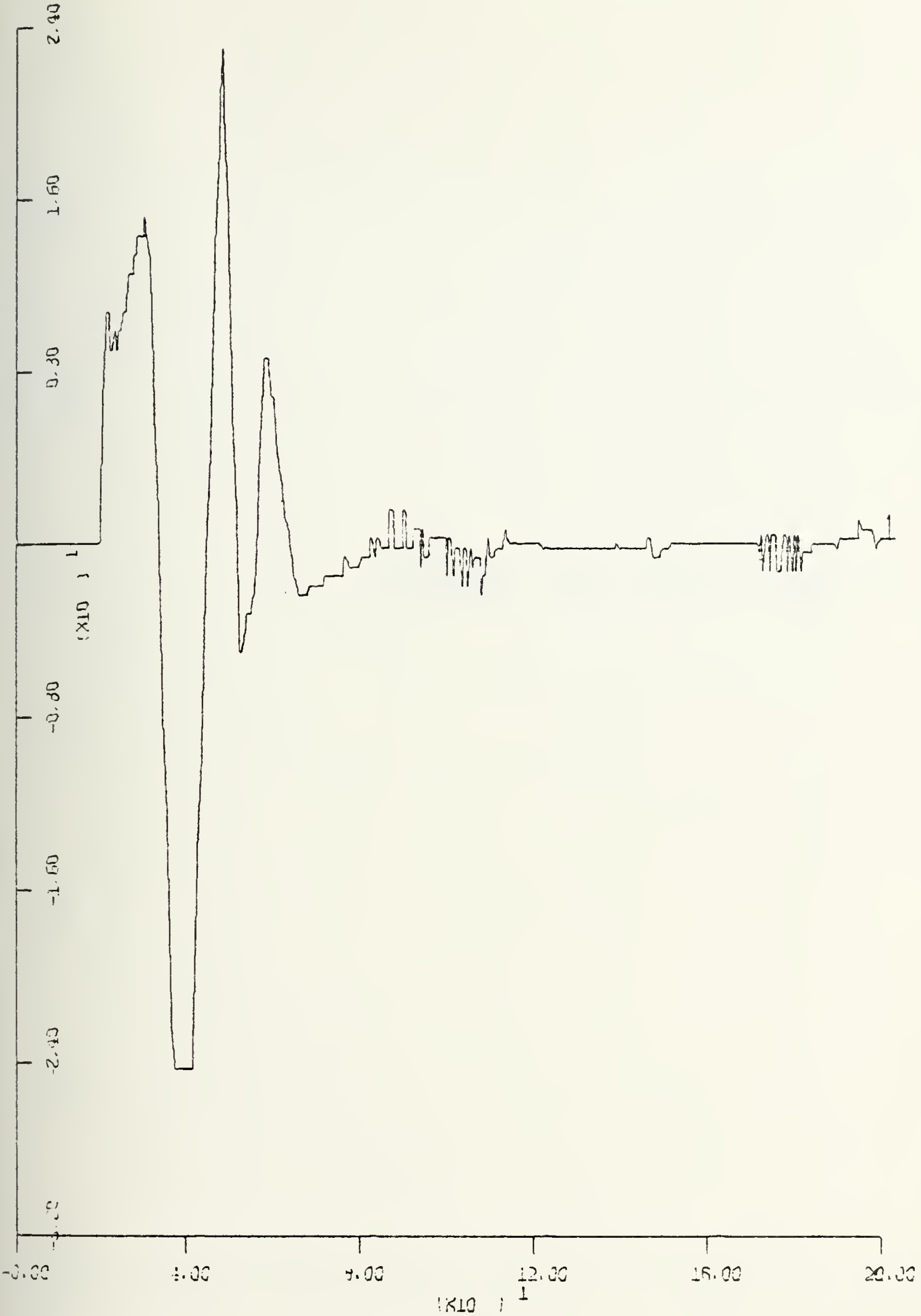


Fig. IV-46b. Pitch vs. Time. Response to a pulse force at FT. CMC uses BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.3



XSCALE=40.00 (s) UNITS/INCH
YSCALE=8.00 (deg) UNITS/INCH

Fig. IV-46c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.3

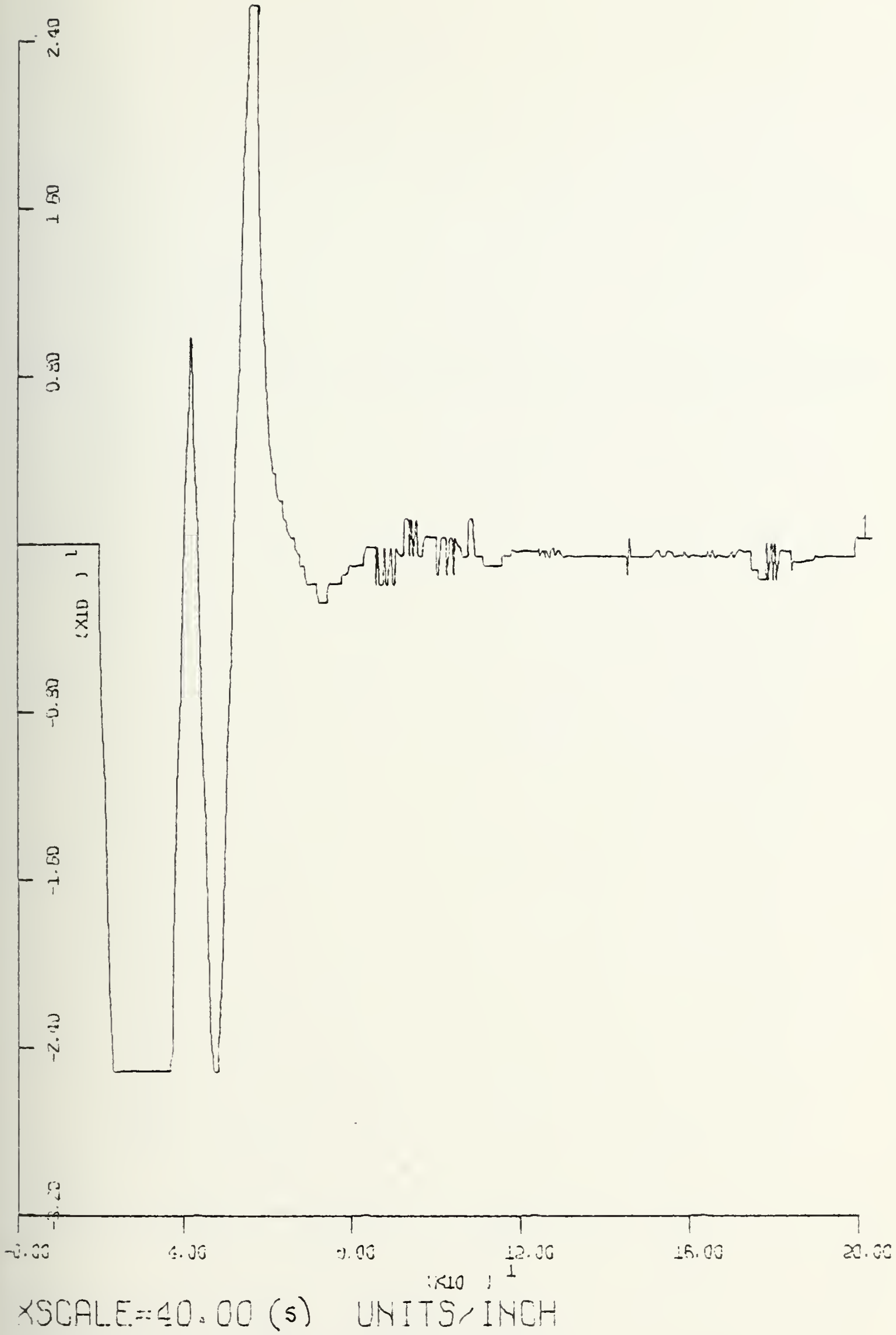


Fig. IV-46d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.3

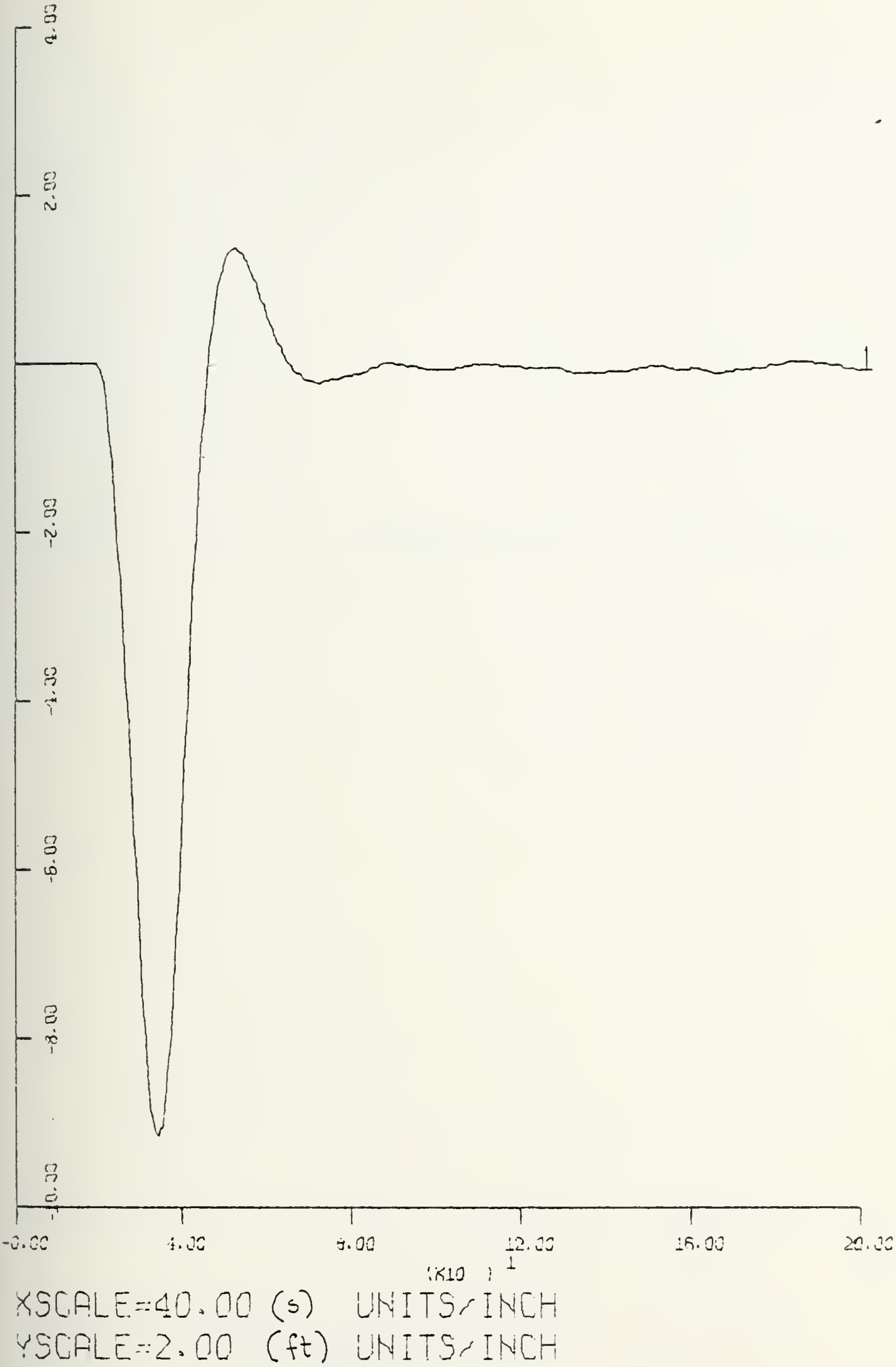


Fig. IV 47a. Depth vs. Time. Response to a pulse force at FT. CMC uses BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.1

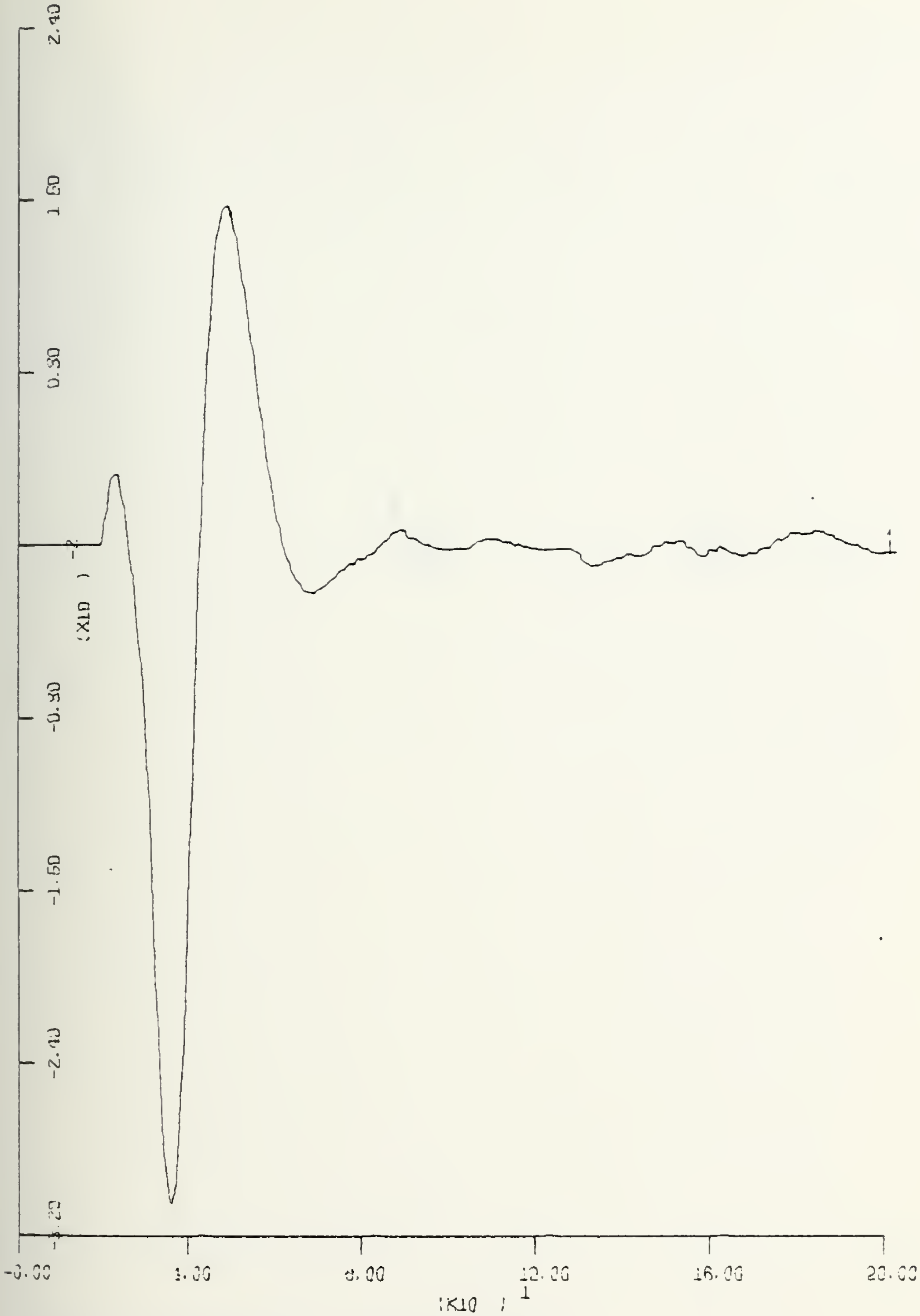
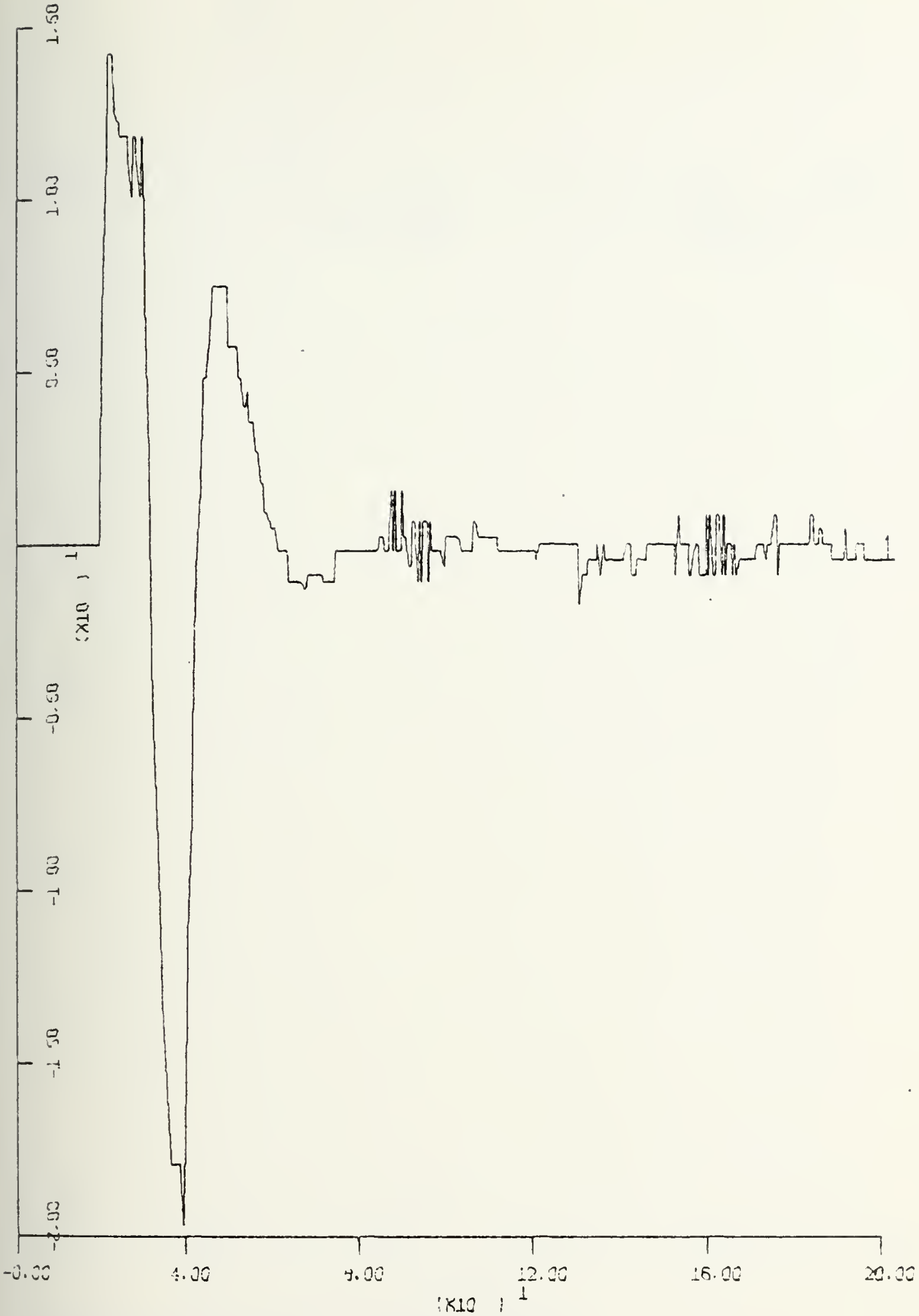
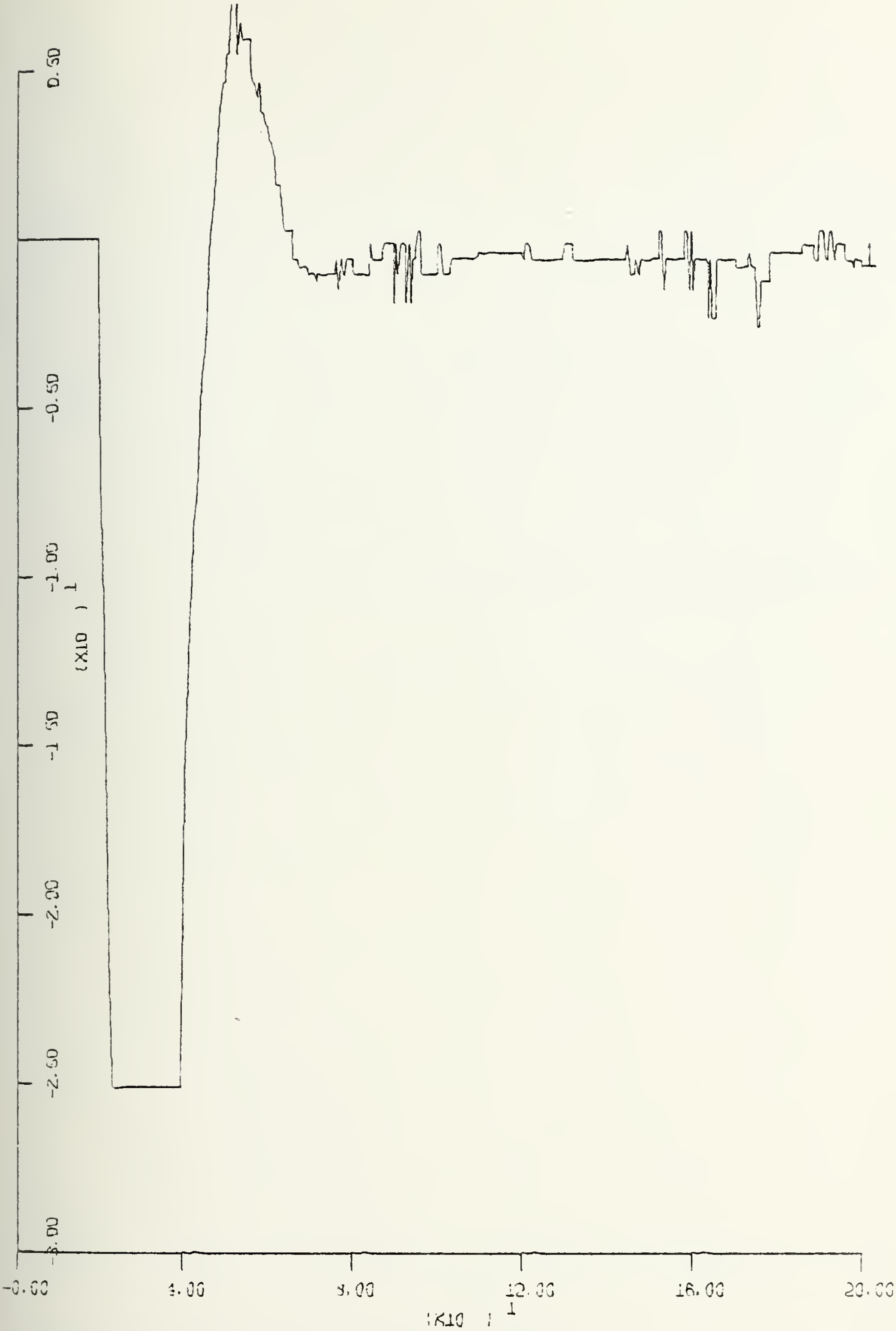


Fig. IV-47b. Pitch vs. Time. Response to a pulse force at FT. CMC uses BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.1



XSCALE=40.00 (s) UNITS/INCH
YSCALE=5.00 (deg) UNITS/INCH

Fig. IV-47c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.1



XSCALE=40.00 (s) UNITS/INCH
YSCALE=5.00 (deg) UNITS/INCH

Fig. IV-47d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses BPOC with B=800, C=10, E=1, and SOPC with the same closed loop C.E. Parameter X=0.1

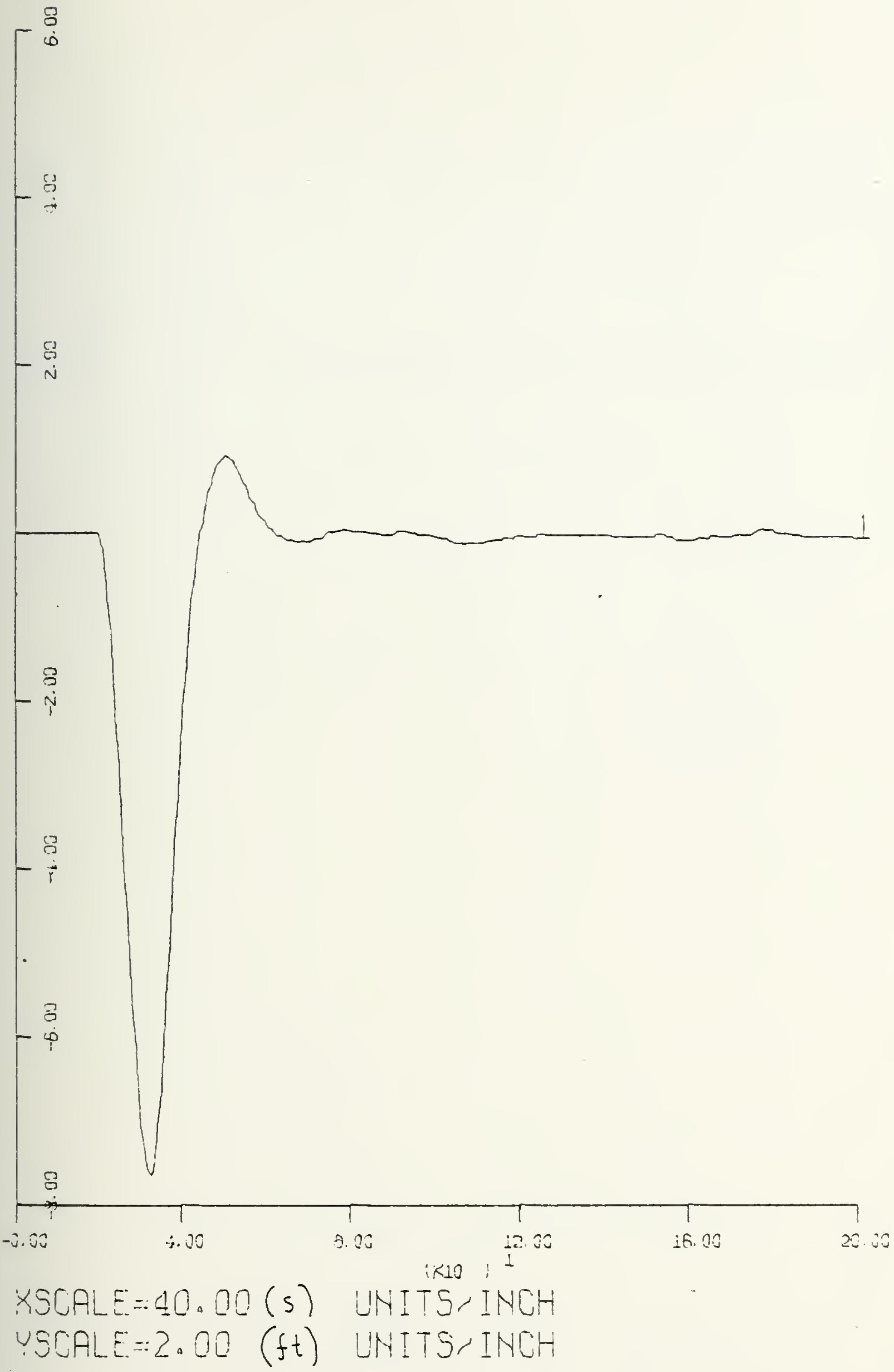
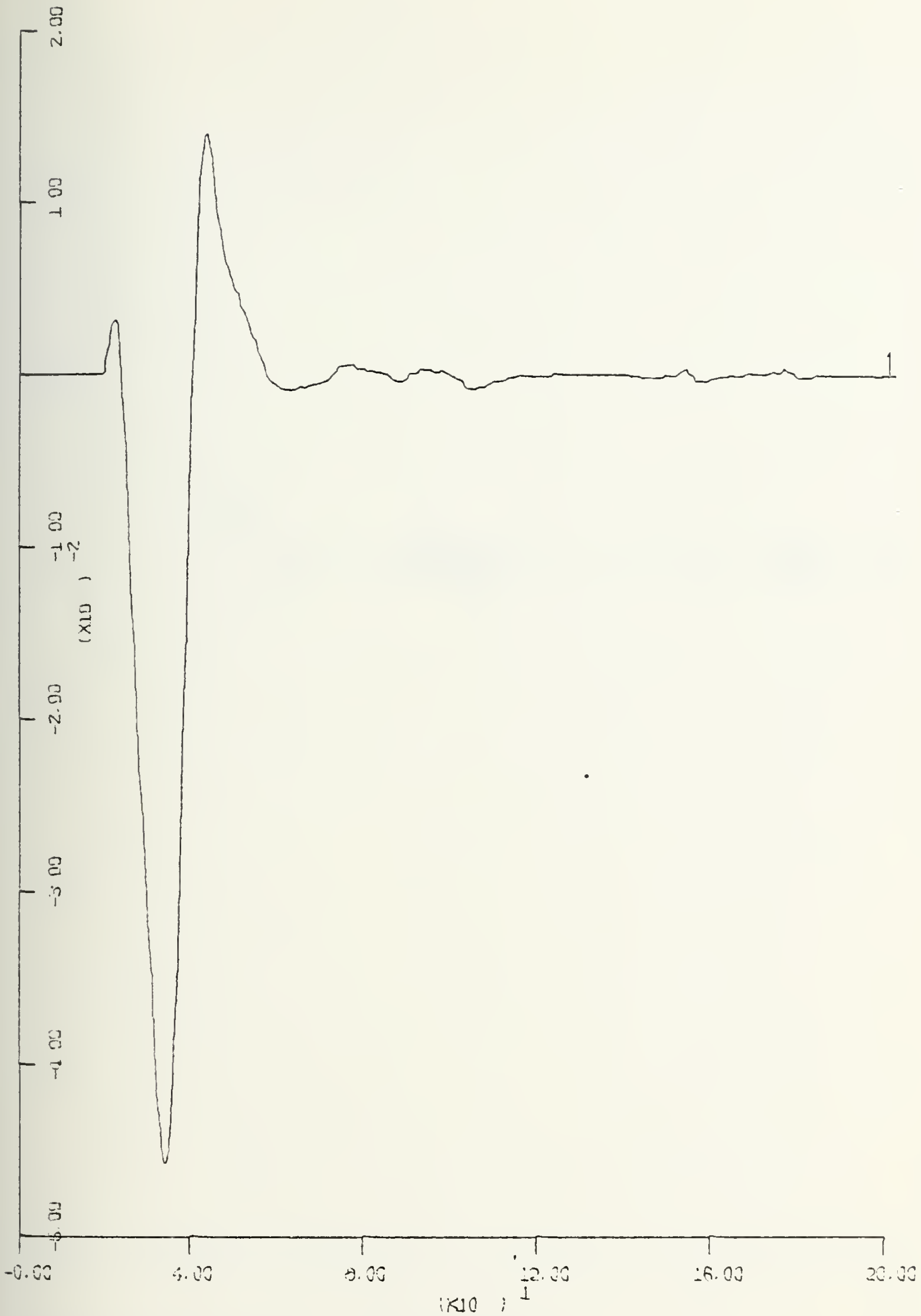


Fig. IV-48a. Depth vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers with C=10, D=3000, E=1. Parameter X=0.4



XSCALE=40.00 (s) UNITS/INCH
YSCALE=0.01 (rad) UNITS/INCH

Fig. IV48b. Pitch vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers with C=10, D=3000, E=1. Parameter X=0.4

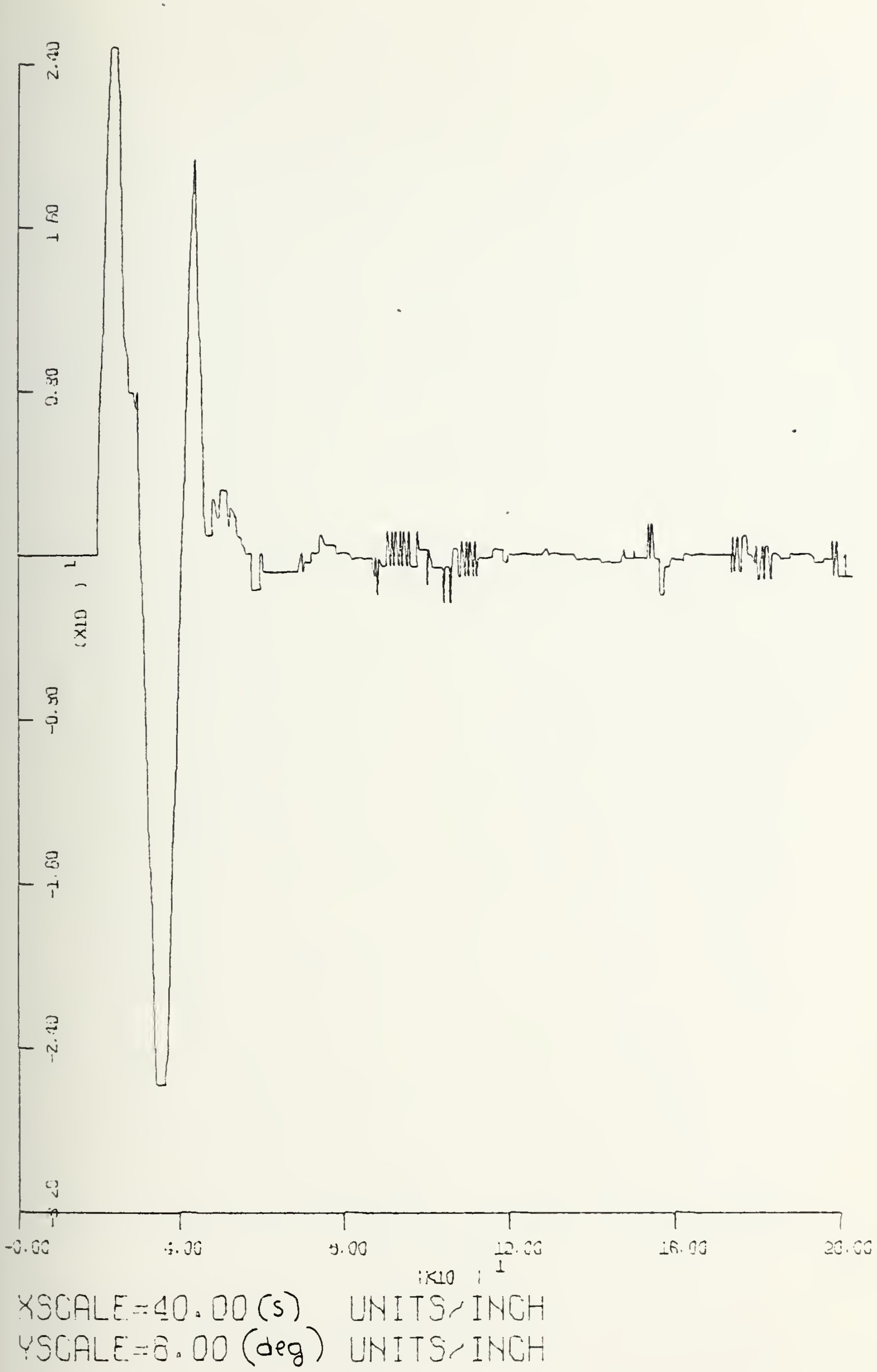


Fig. IV-48c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers with C=10, D=3000, E=1. Parameter X=0.4

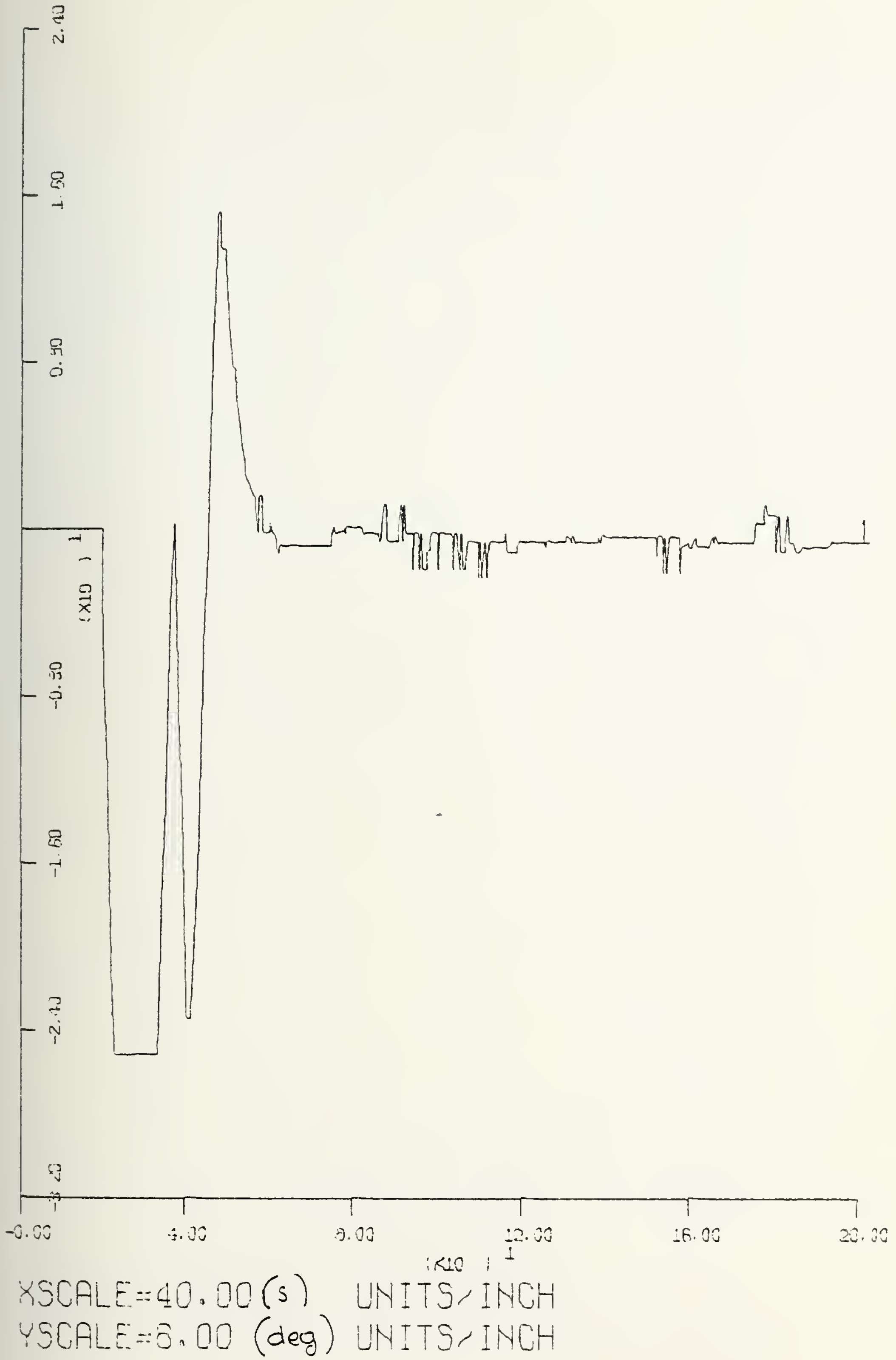


Fig. IV-48d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers with C=10, D=3000, E=1. Parameter X=0.4

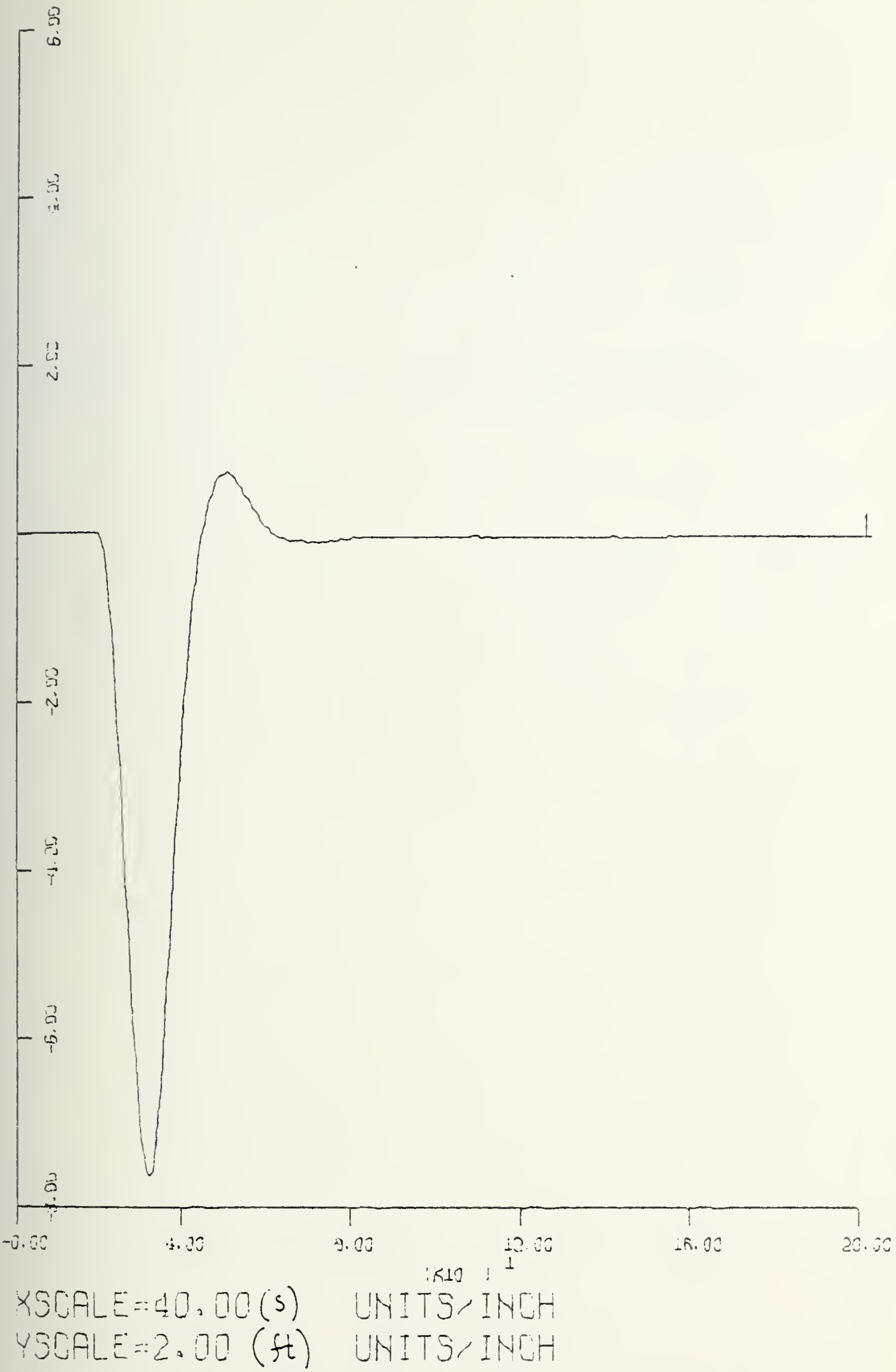


Fig. IV-49a. Depth vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with C=10, D=3000, E=1. Parameter X=0.5

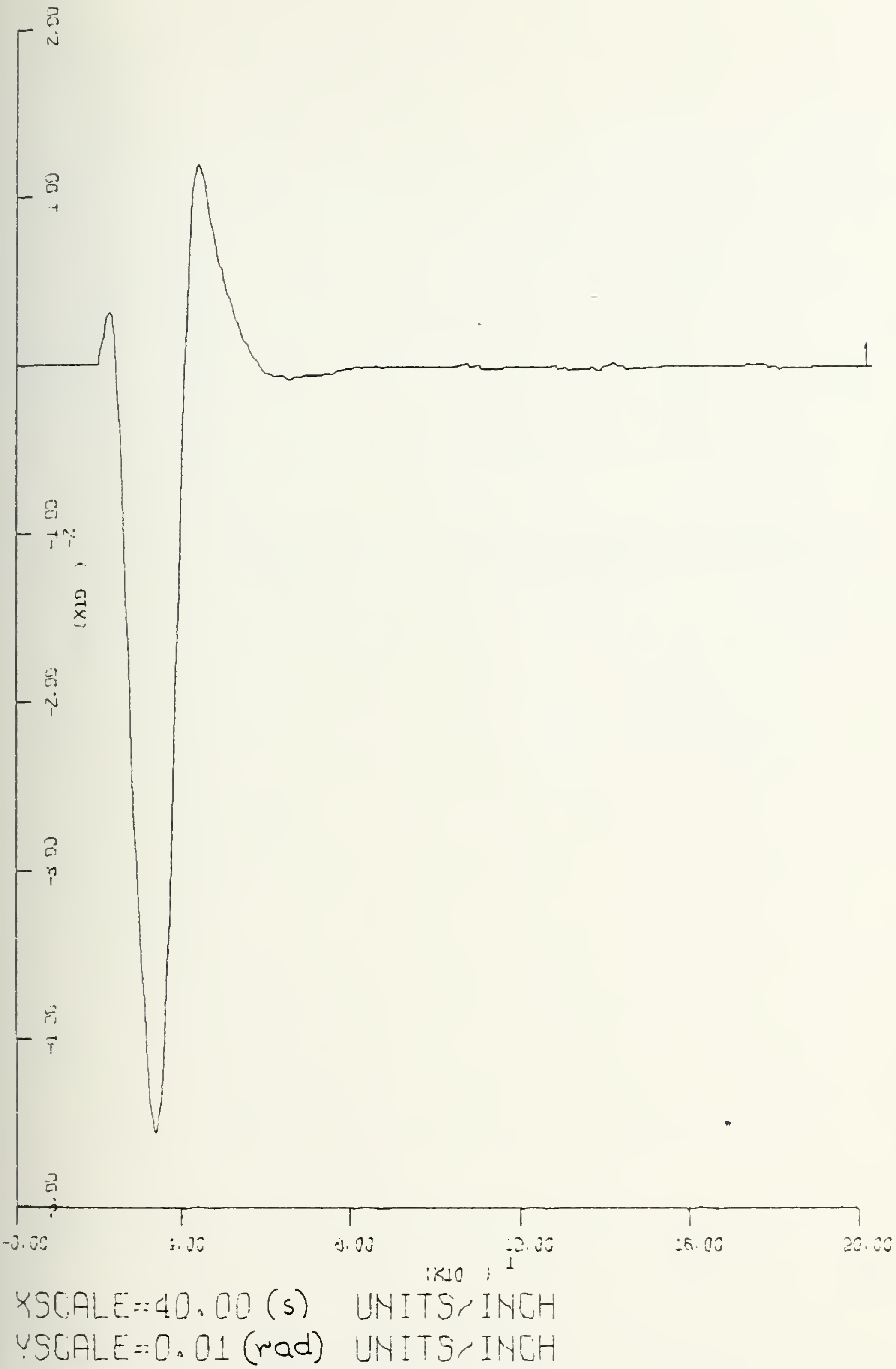


Fig. IV-49b. Pitch vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with C=10, D=3000, E=1. Parameter X=0.5

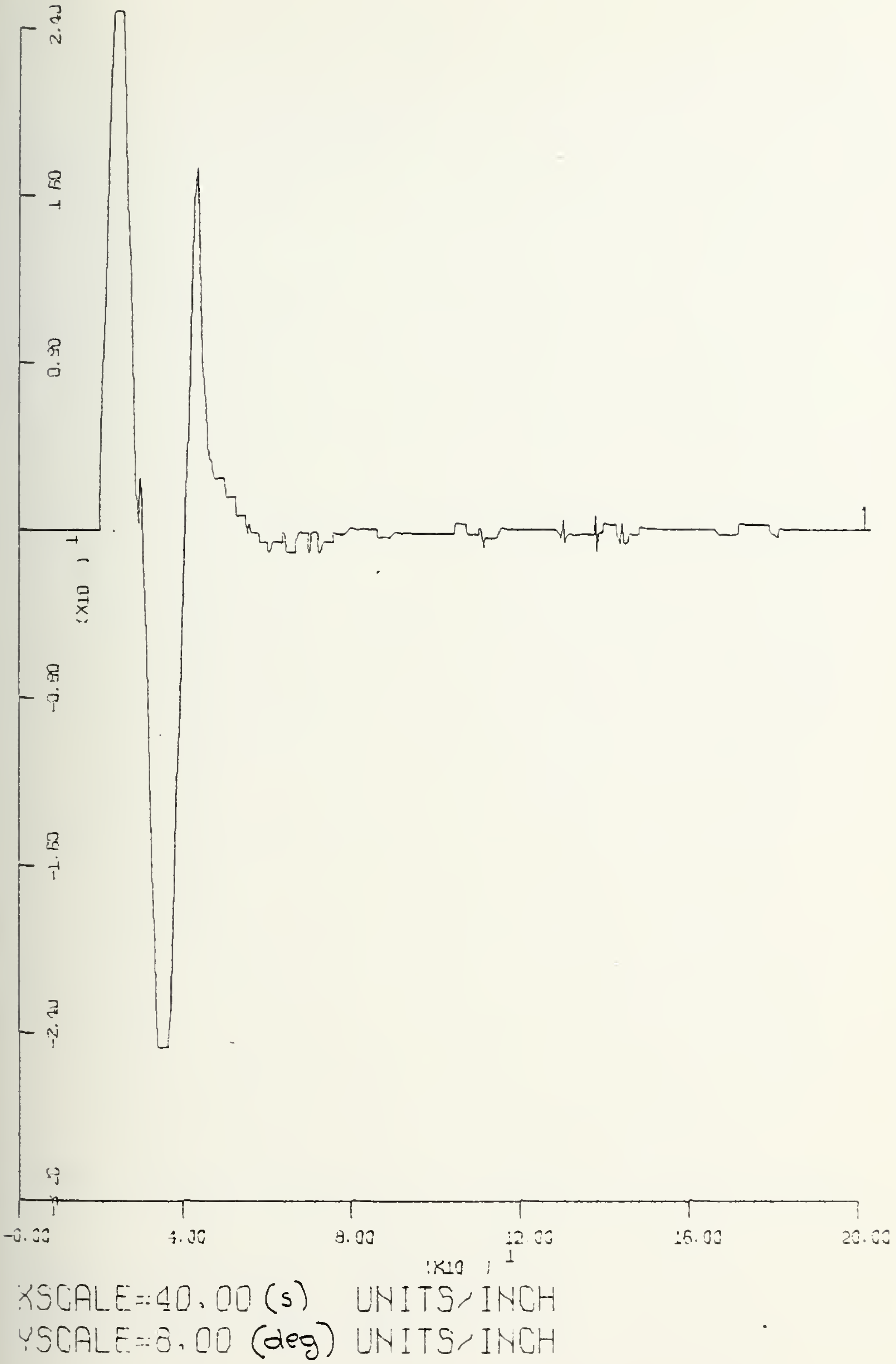


Fig. IV-49c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with C=10, D=3000, E=1. Parameter X=0.5

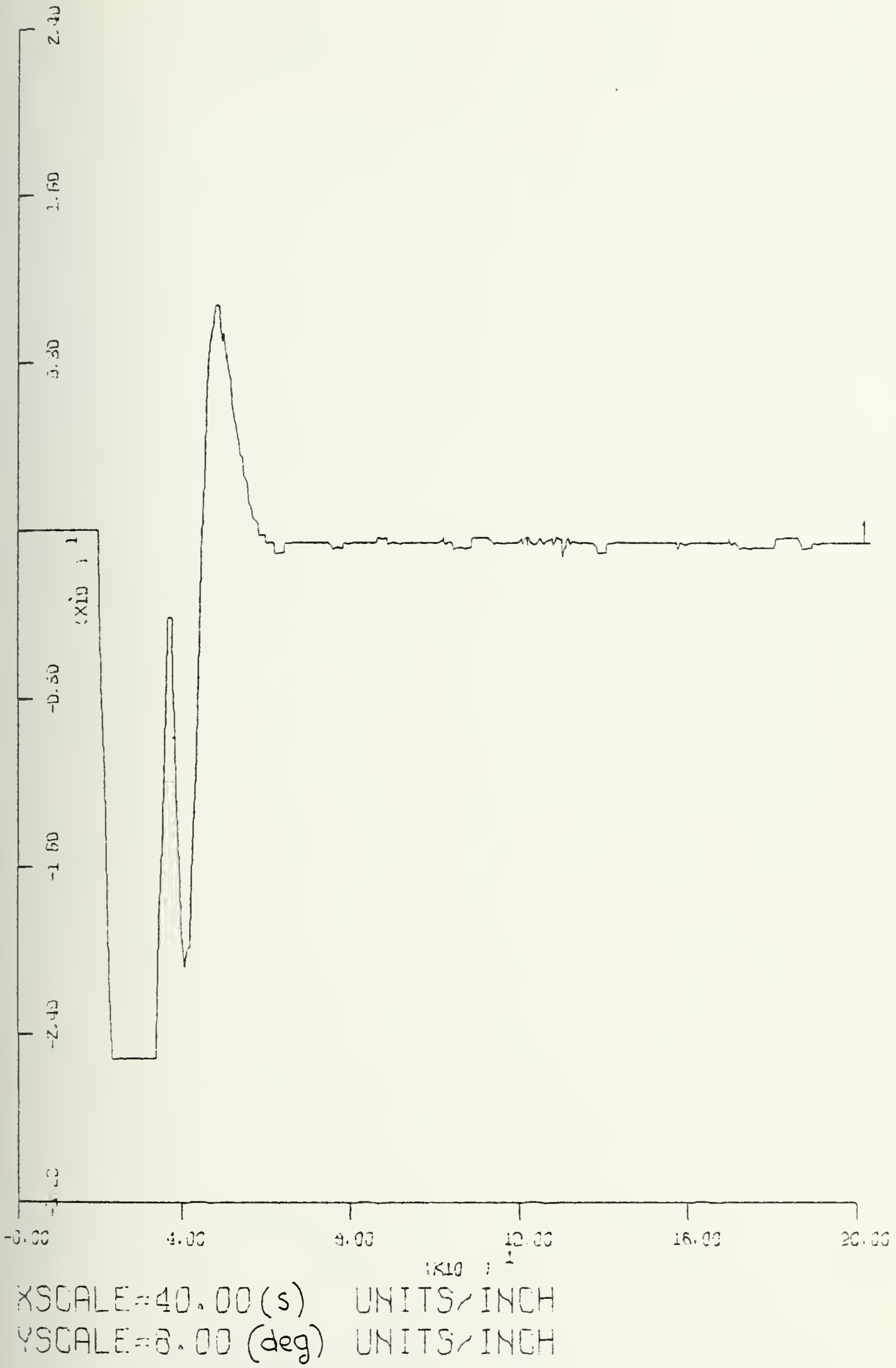


Fig. IV-49d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers with C=10, D=3000, E=1. Parameter X=0.5

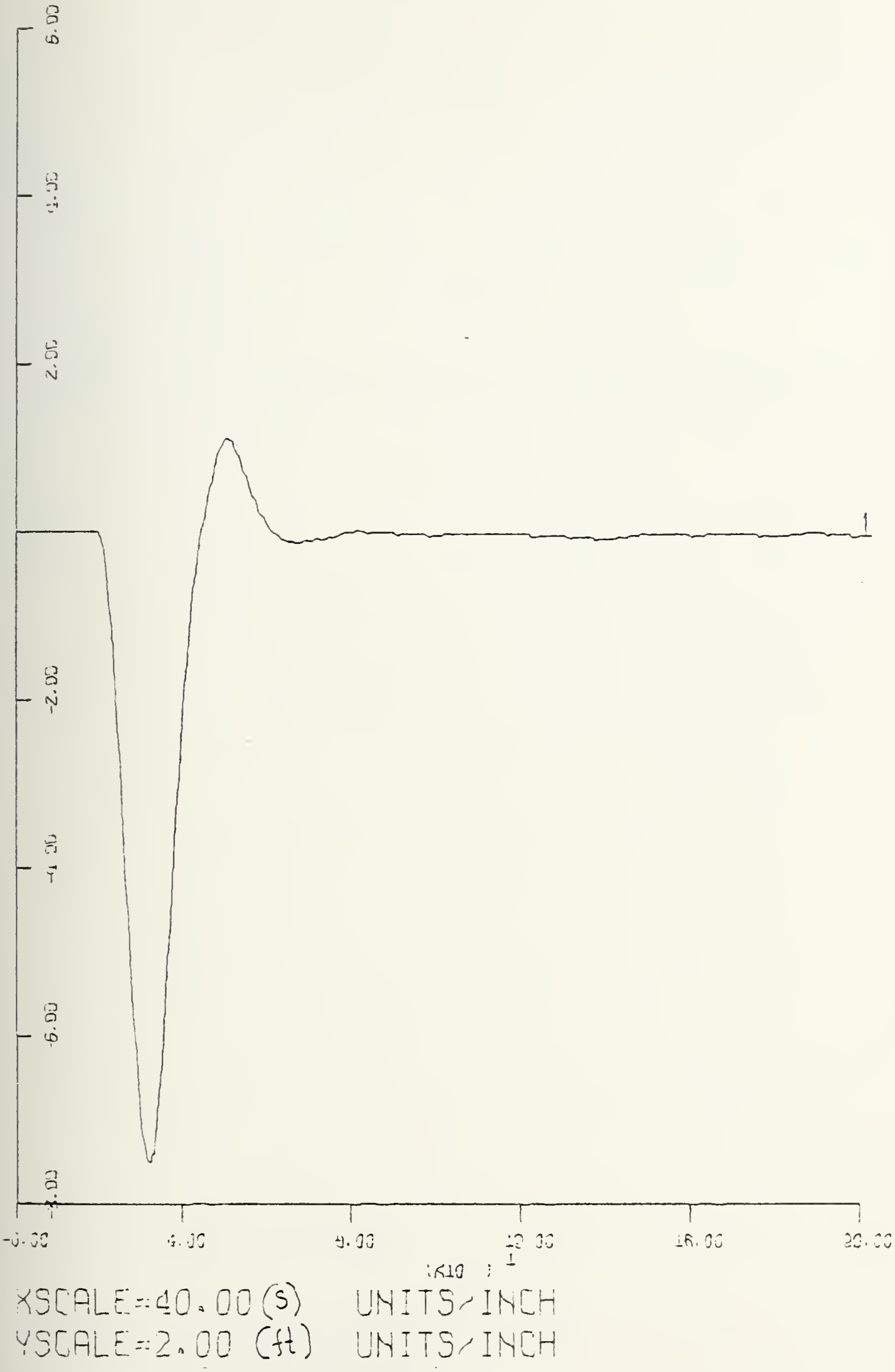
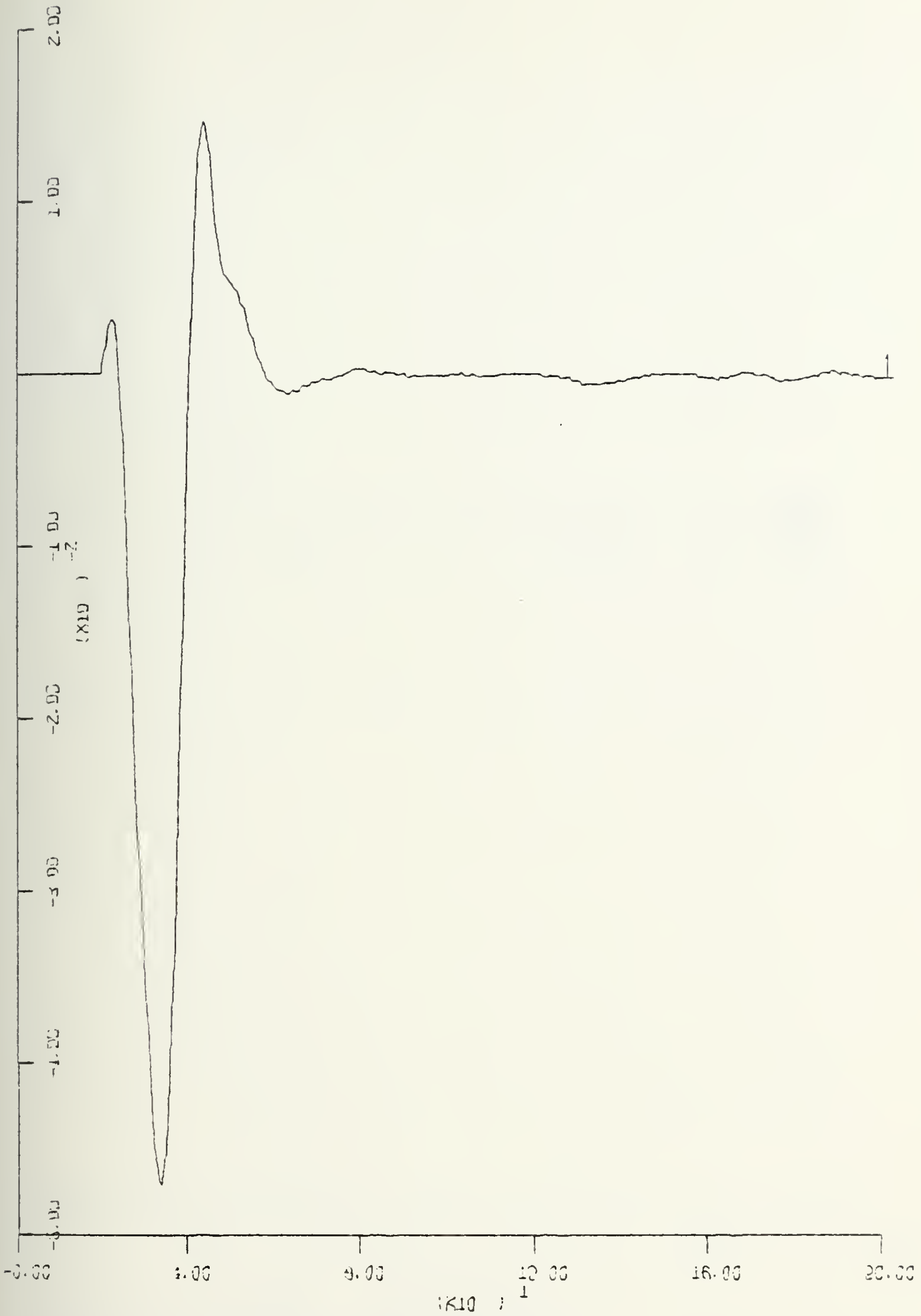


Fig. IV-50a. Depth vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with C=10, D=3000, E=1. Parameter X= 0.6



XSCALE=40.00 (s) UNITS/INCH
YSCALE=0.01 (rad) UNITS/INCH

Fig. IV-50b. Pitch vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with C=10, D=3000, E=1. Parameter X=0.6

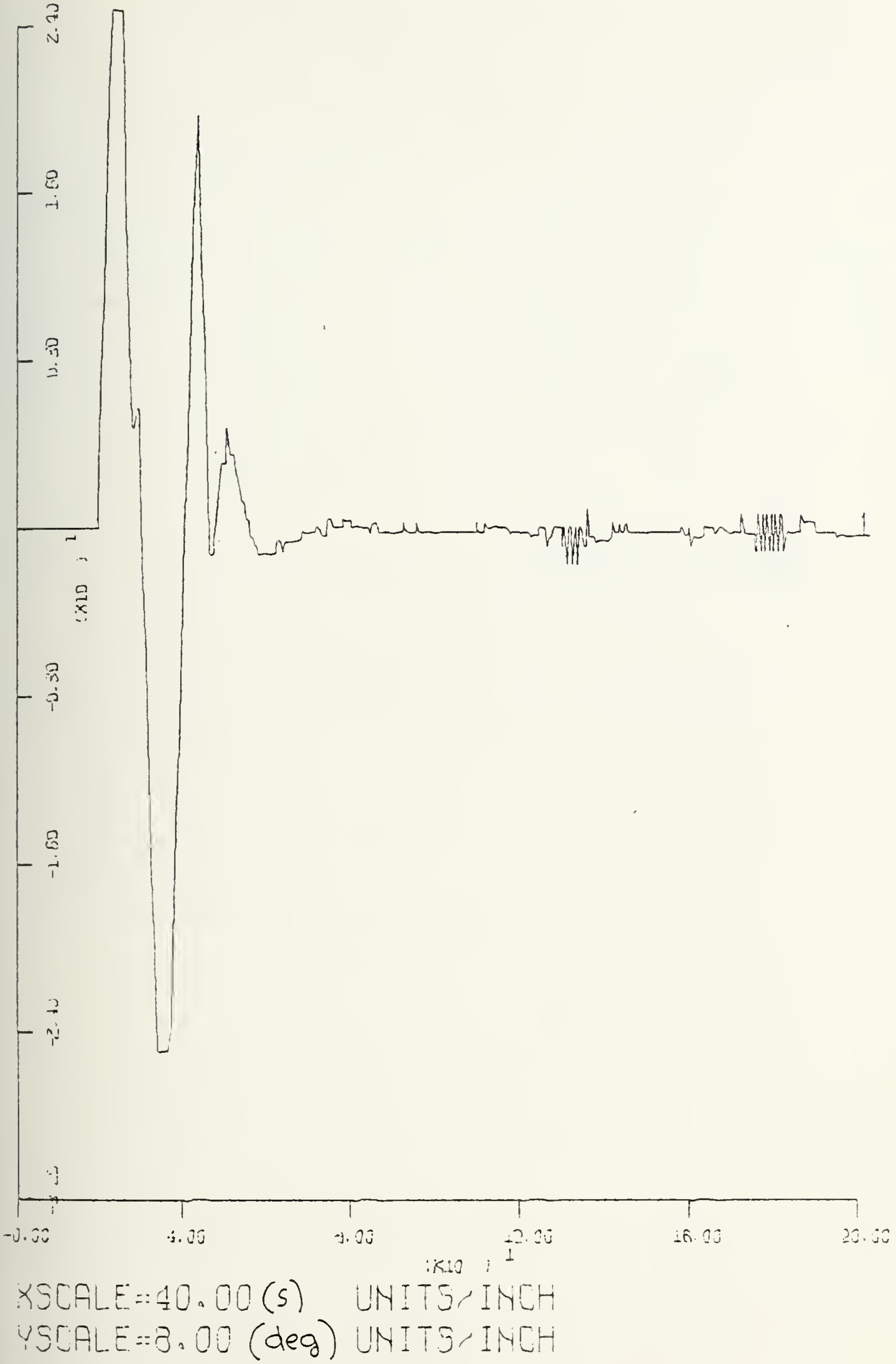
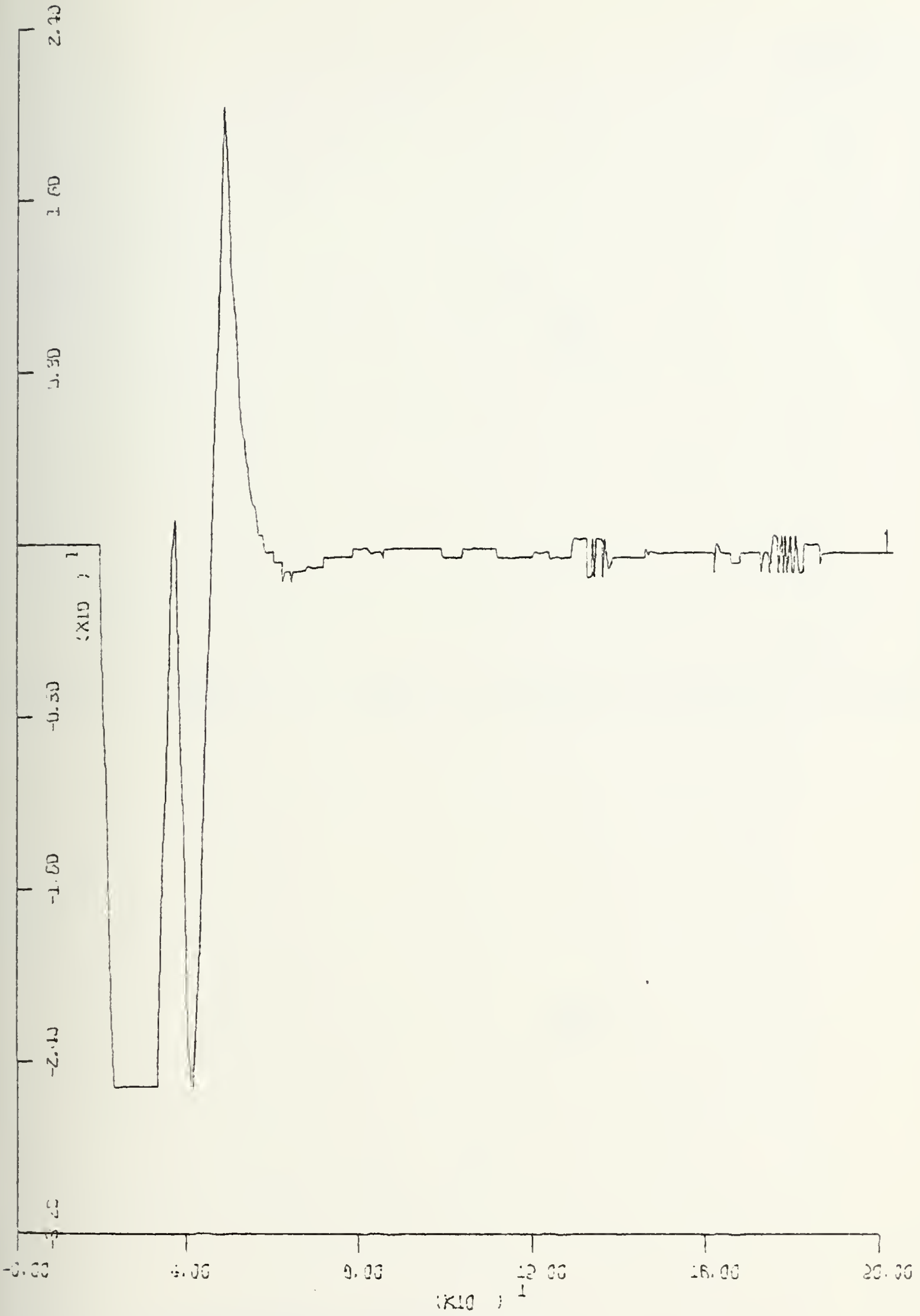


Fig. IV-50c. Stern Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with C=10, D=3000, E=1. Parameter X=0.6



XSCALE=40.00 (s) UNITS/INCH
YSCALE=8.00 (deg) UNITS/INCH

Fig. IV-50d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. CMC uses optimal design in combined controllers, with C=10, D=3000, E=1. Parameter X=0.6

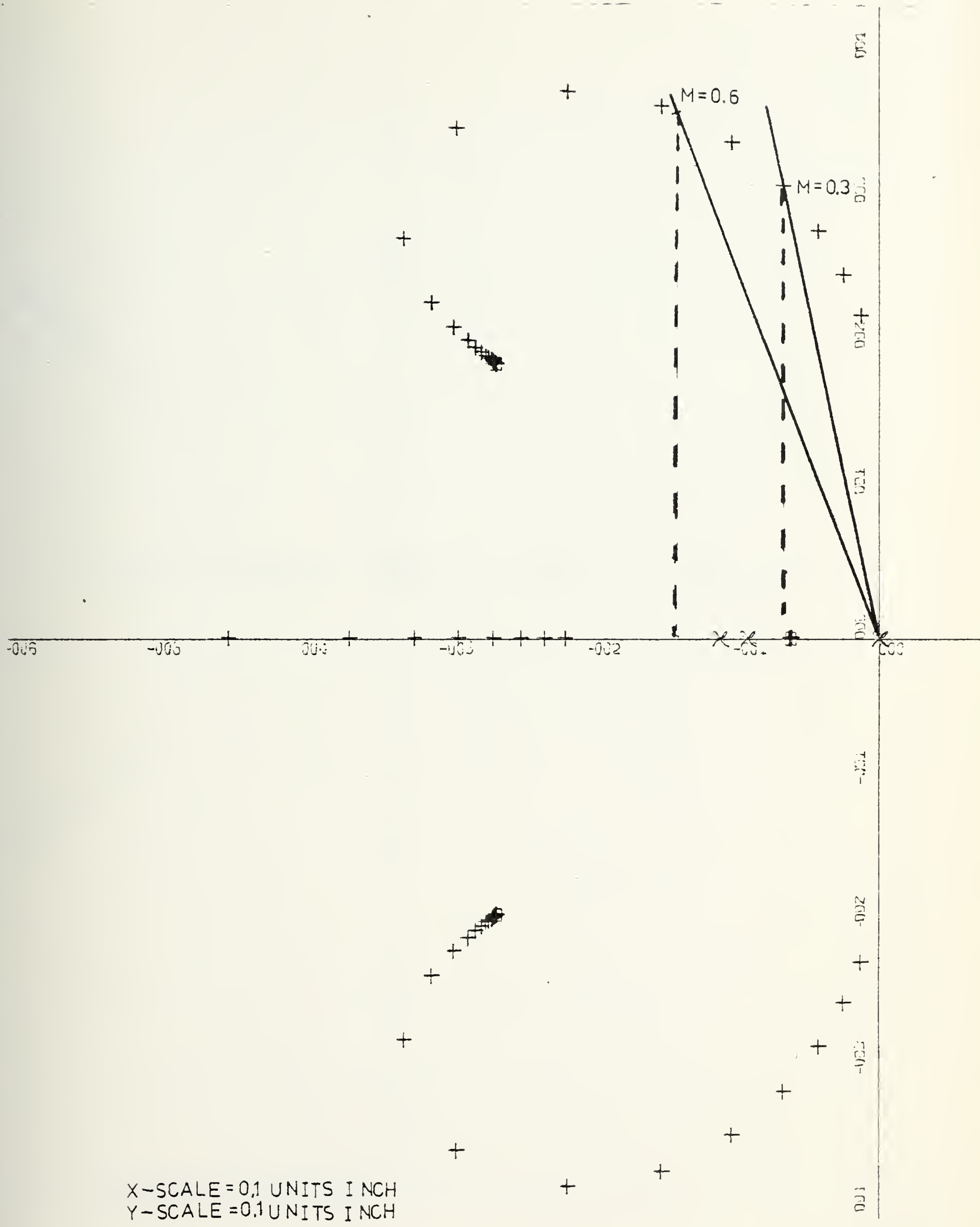


Fig. V-1a. Root locus for SOPOC with B=800, C=10, E=1.
M > 0

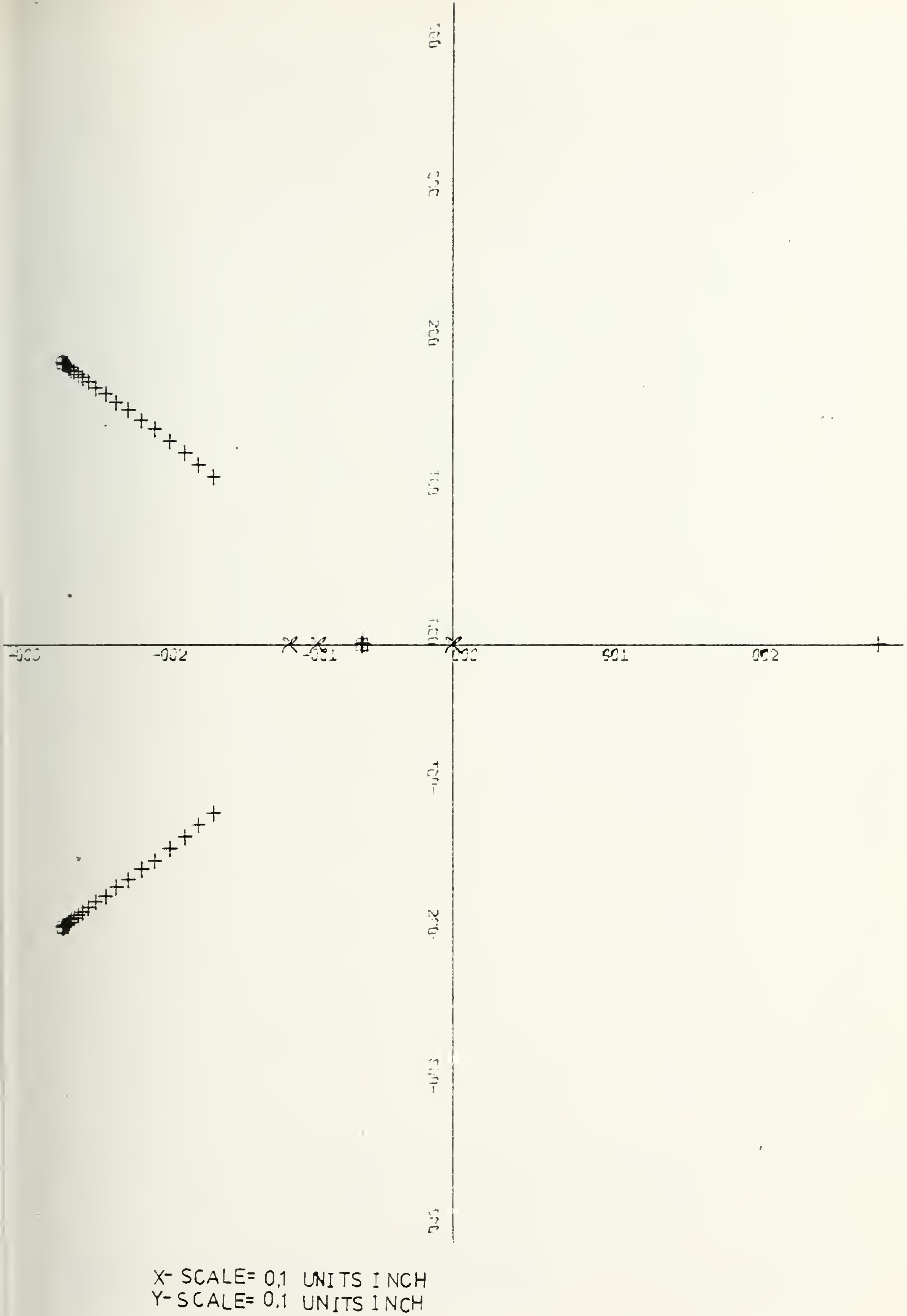


Fig. V-1b. Root locus for SOPOC with B=800, C=10, E=1.
$M < 0$

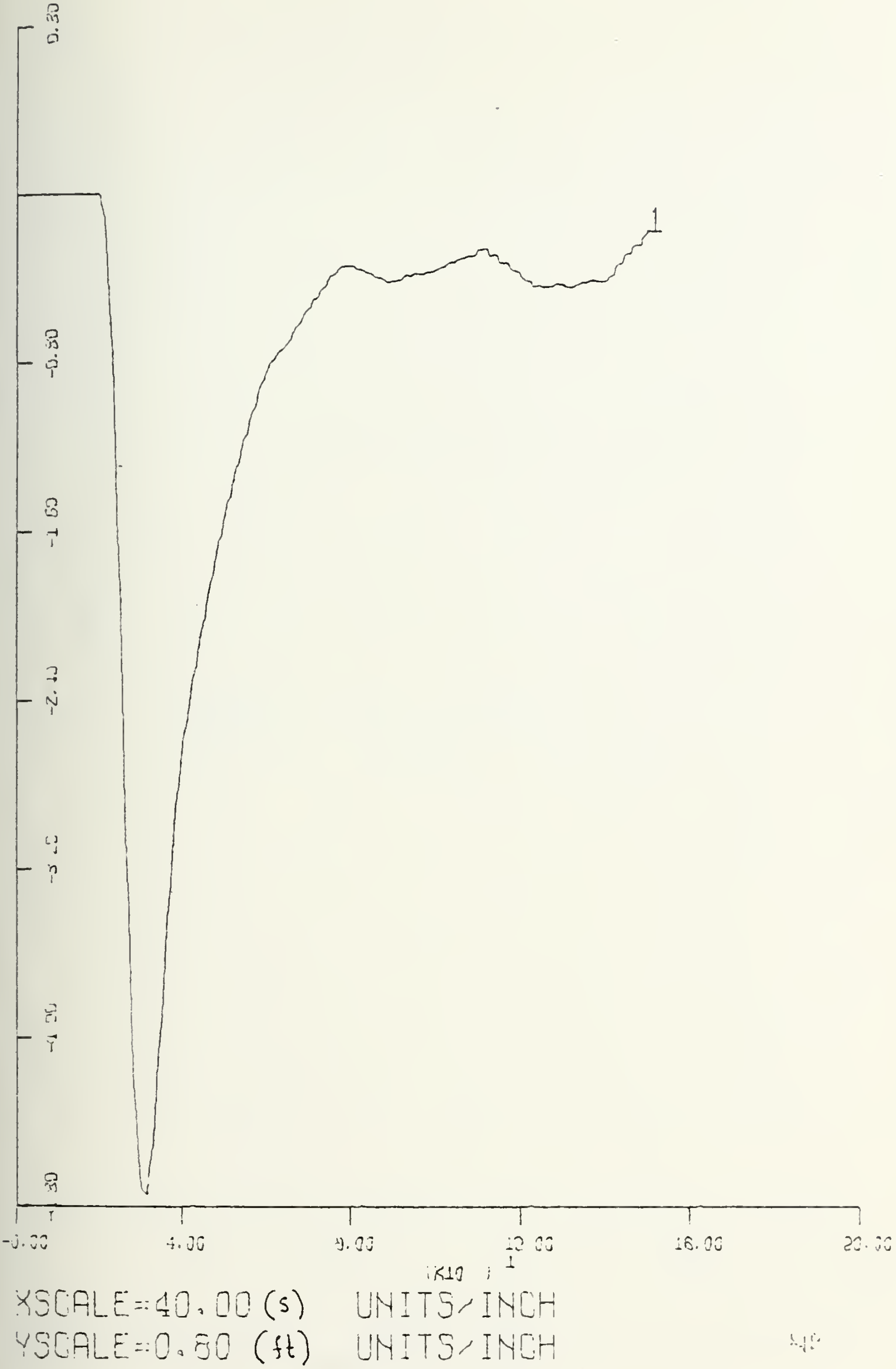


Fig. V-2a. Depth vs. Time. Response to a pulse force at FT. SOPC equal to 0.3·SOPOC (B=800, C=10, E=1)

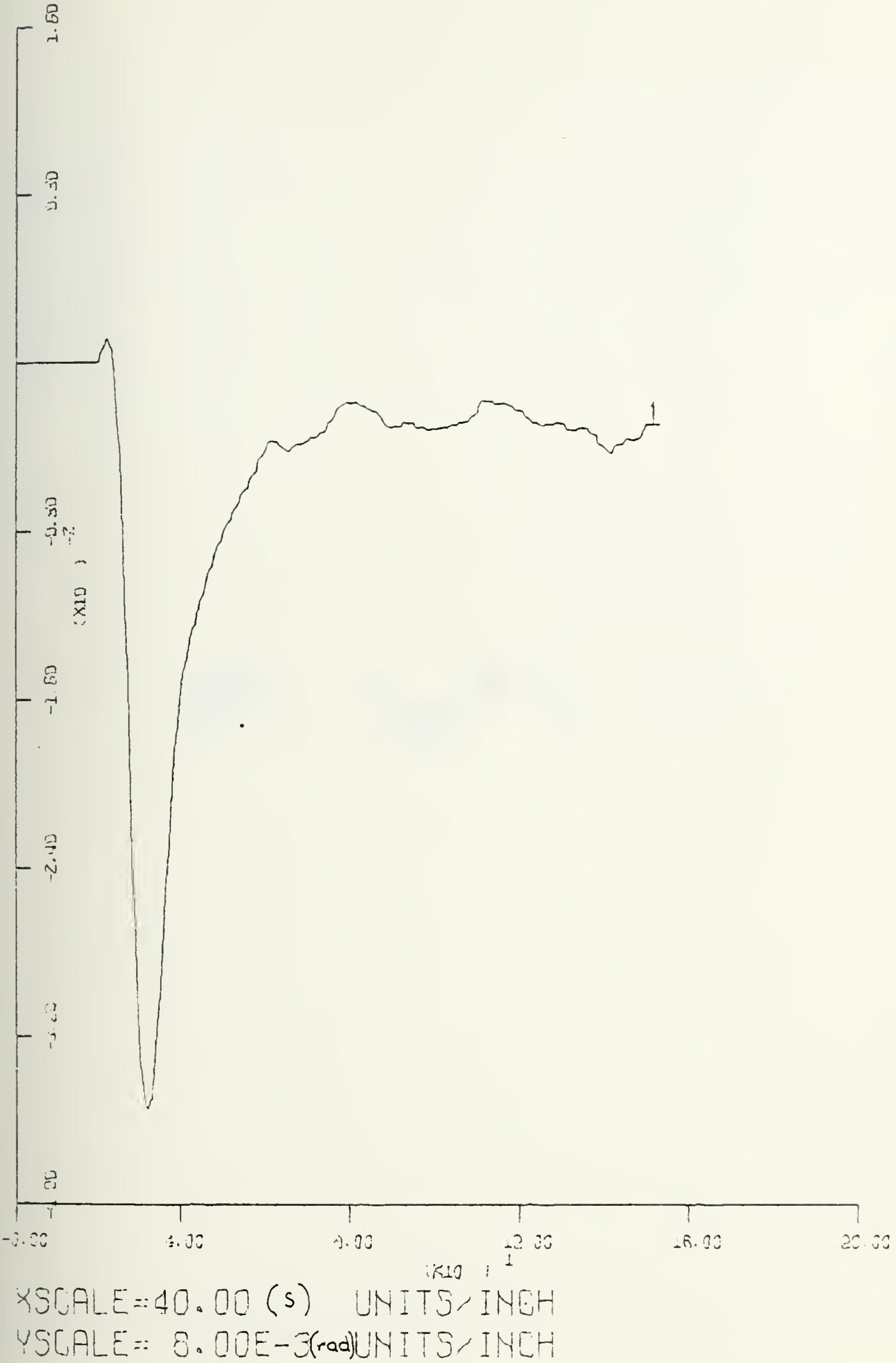


Fig. V-2b. Pitch vs. Time. Response to a pulse force at FT. SOPC equal to 0.3·SOPOC (B=800, C=10, E=1)

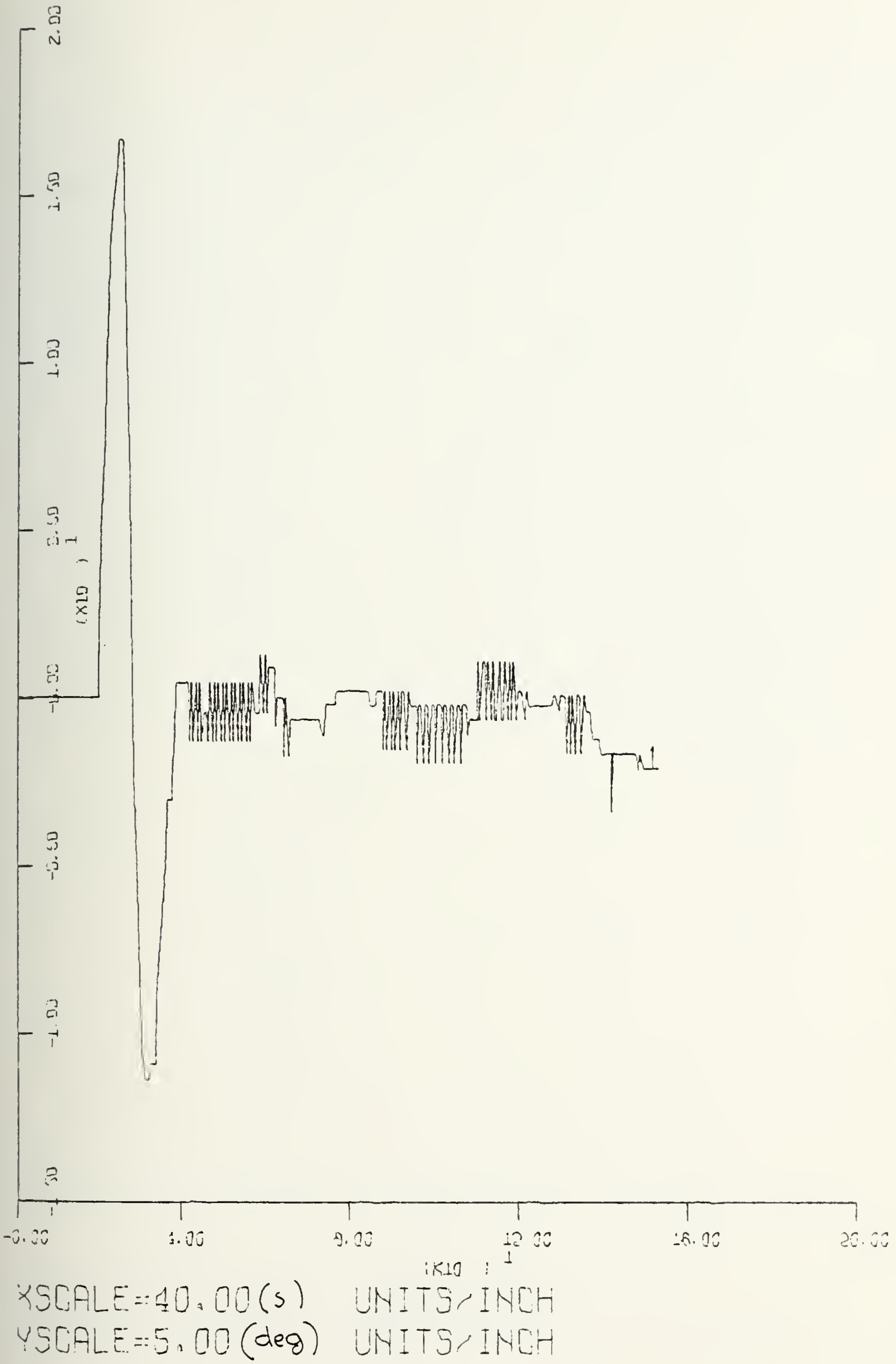


Fig. V-2c. Stern Plane Angle vs. Time. Response to a pulse force at FT. SOPC equal to 0.3·SOPOC (B=800, C=10, E=1)

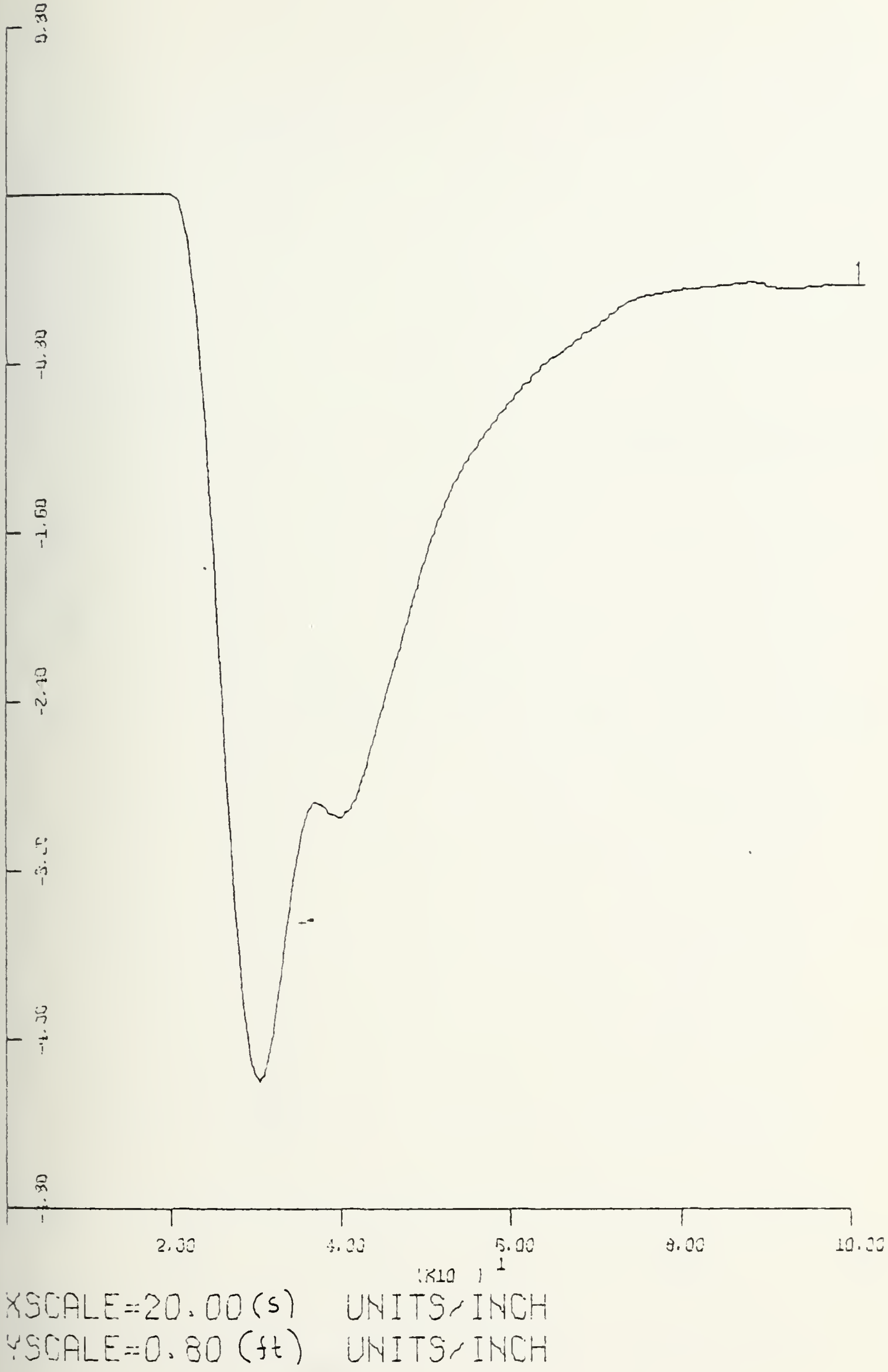


Fig. V-3a. Depth vs. Time. Response to a pulse force at FT. SOPC equal to 0.6·SOPOC (B=800, C=10, E=1)

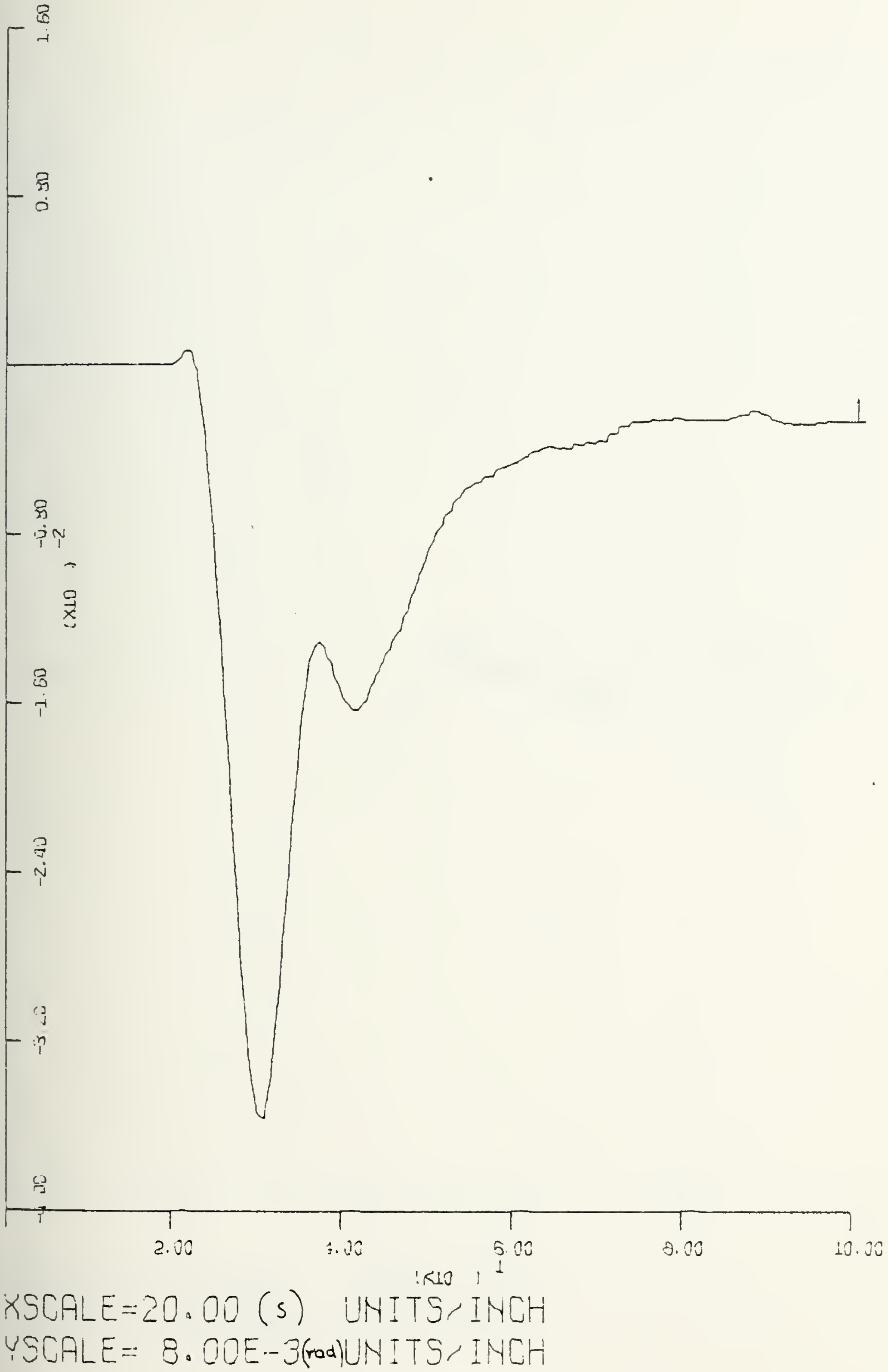


Fig. V-3b. Pitch vs. Time. Response to a pulse force at FT. SOPC equal to 0.6·SOPOC (B=800, C=10, E=1)

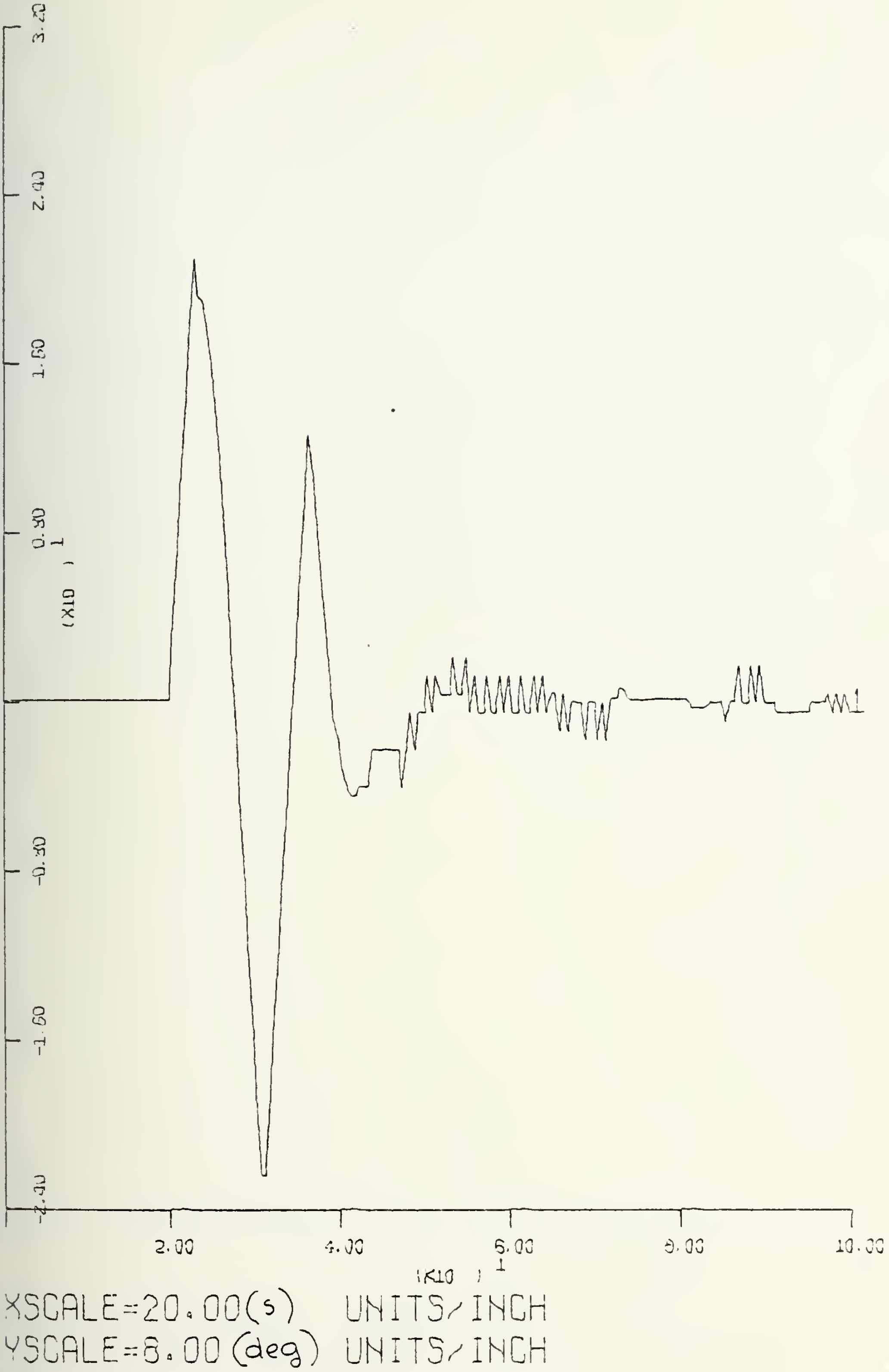


Fig. V-3c. Stern Plane Angle vs. Time. Response to a pulse force at FT. SOPC equal to 0.6·SOPOC (B=800, C=10, E=1)

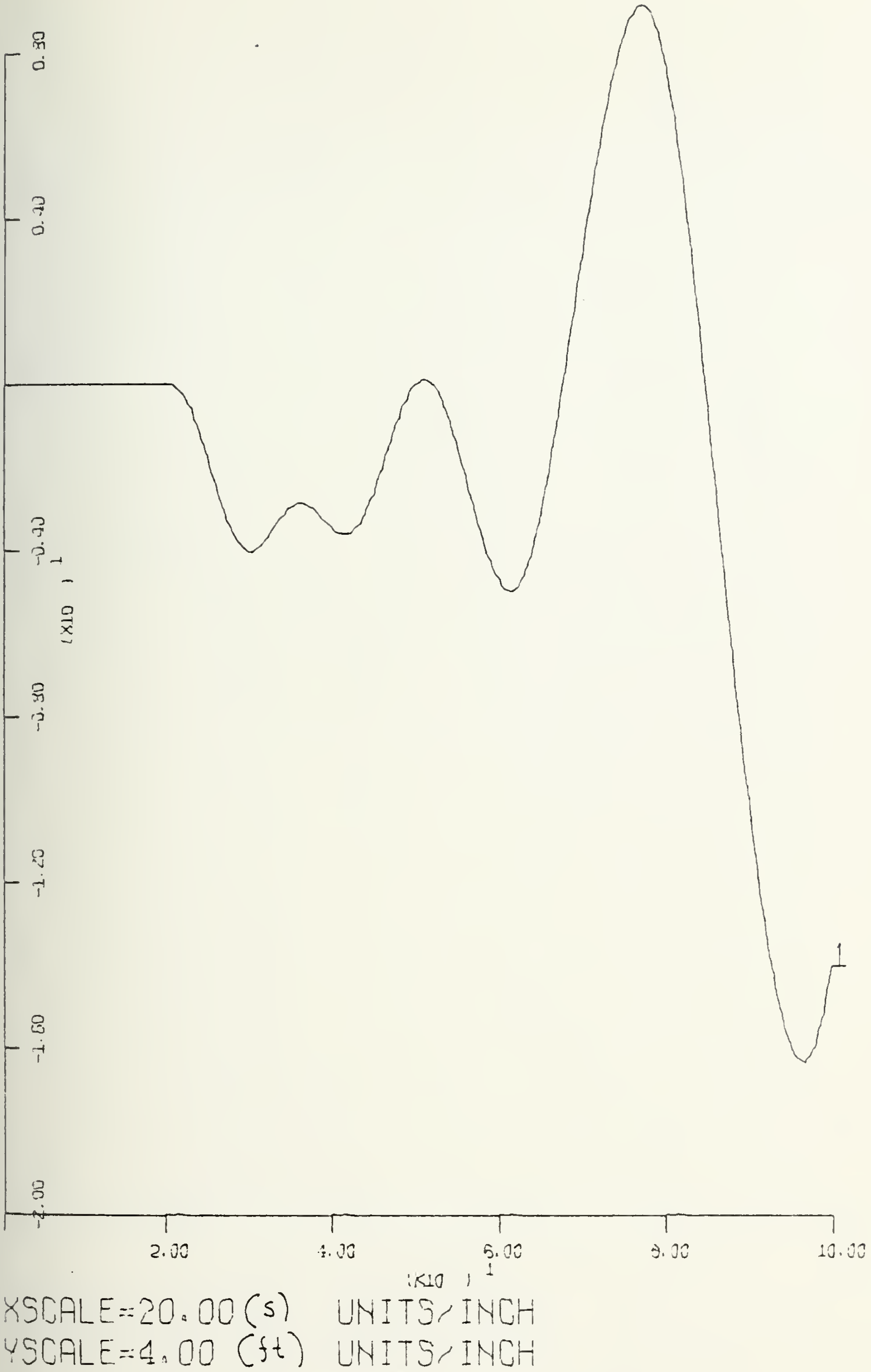
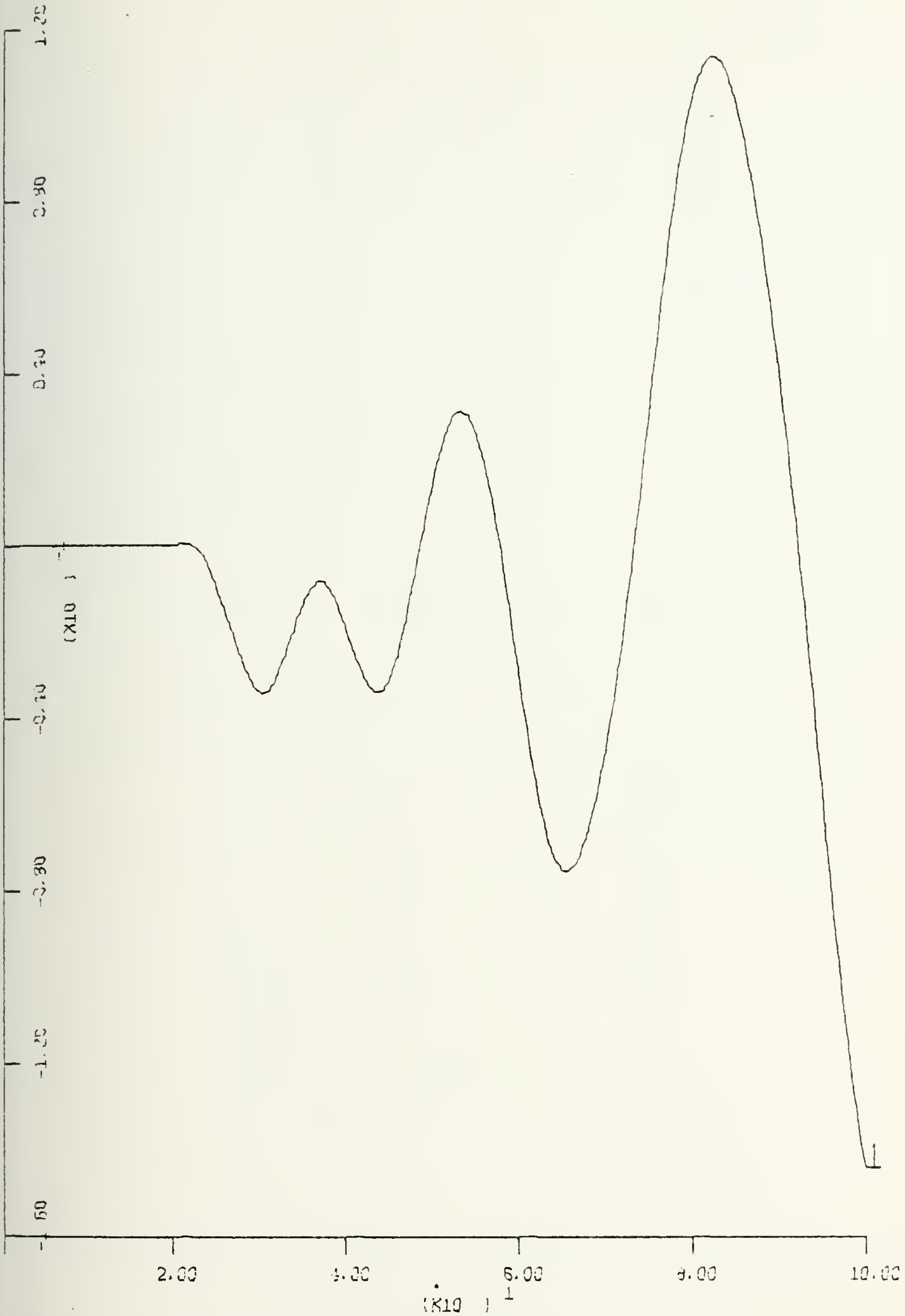


Fig. V-4a. Depth vs. Time. Response to a pulse force at FT. SOPC equal to 0.8 · SOPOC (B=800,C=10, E=1)



XSCALE=20.00(s) UNITS/INCH
YSCALE=0.04 (rad) UNITS/INCH

Fig. V-4b. Pitch vs. Time. Response to a pulse force at FT. SOPC equal to 0.8·SOPOC (B=800, C=10, E=1)

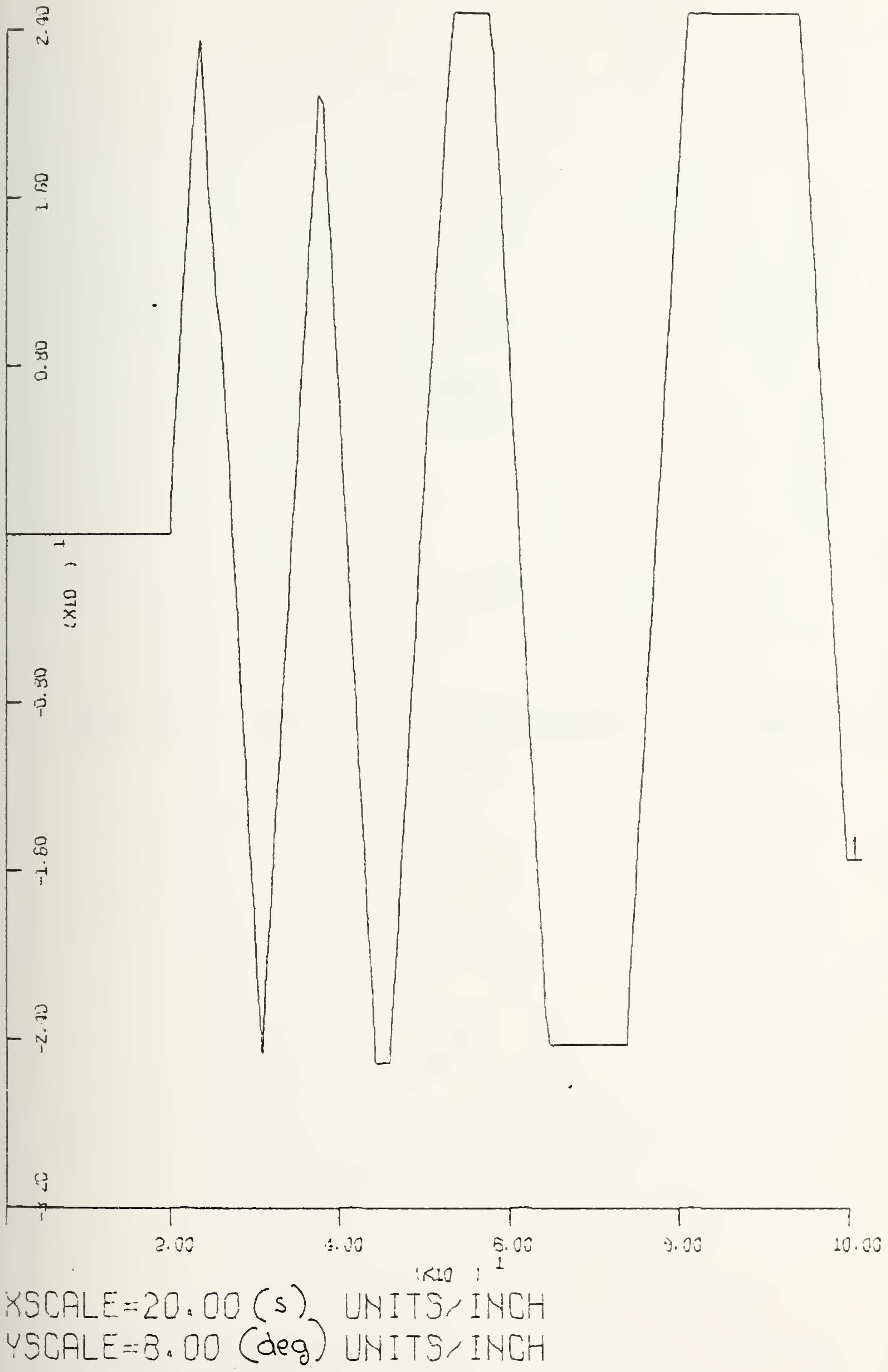


Fig. V-4c. Stern Plane Angle vs. Time. Response to a pulse force at FT. SOPC equal to 0.8·SOPOC (B=800, C=10, E=1)

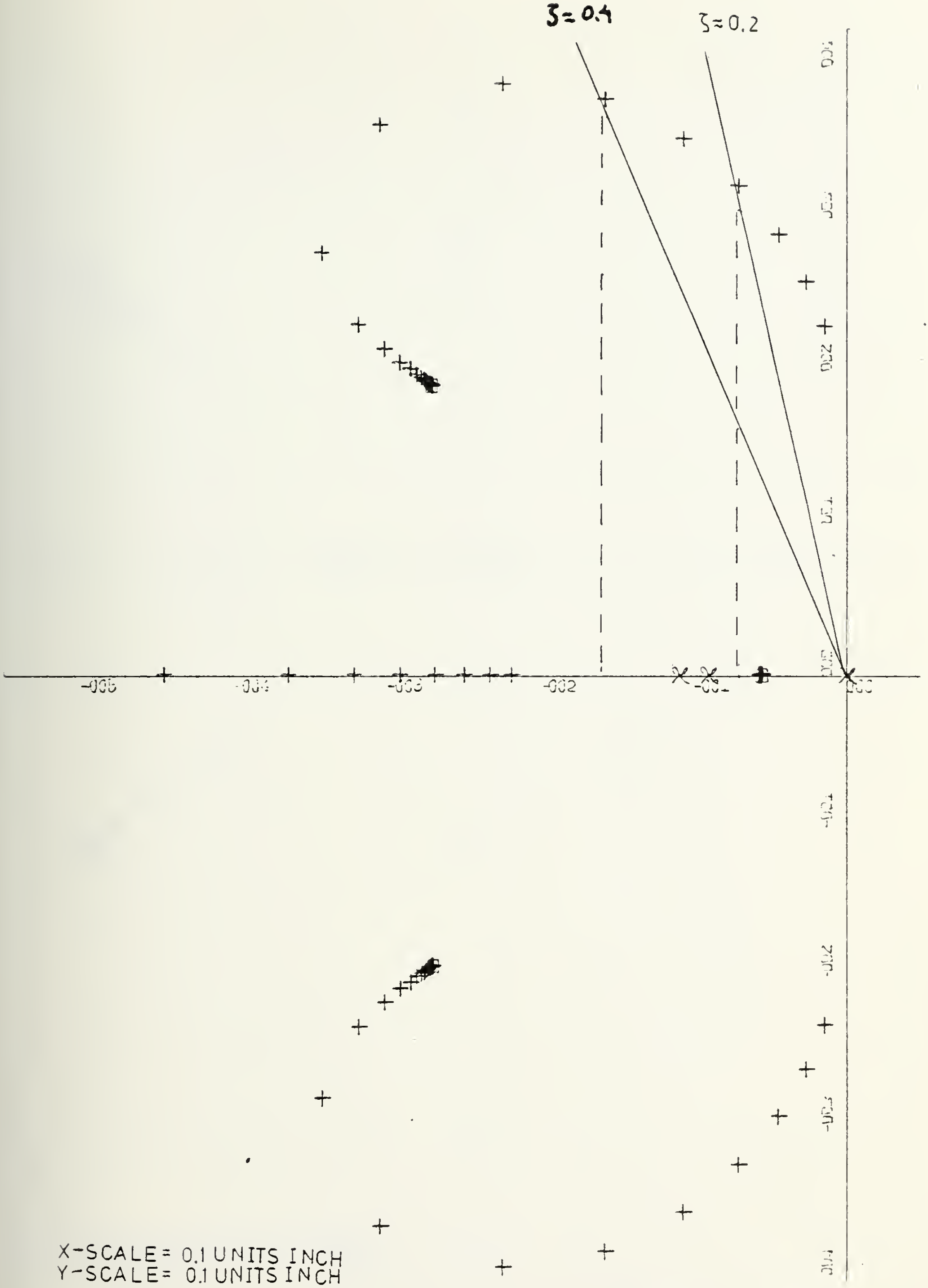
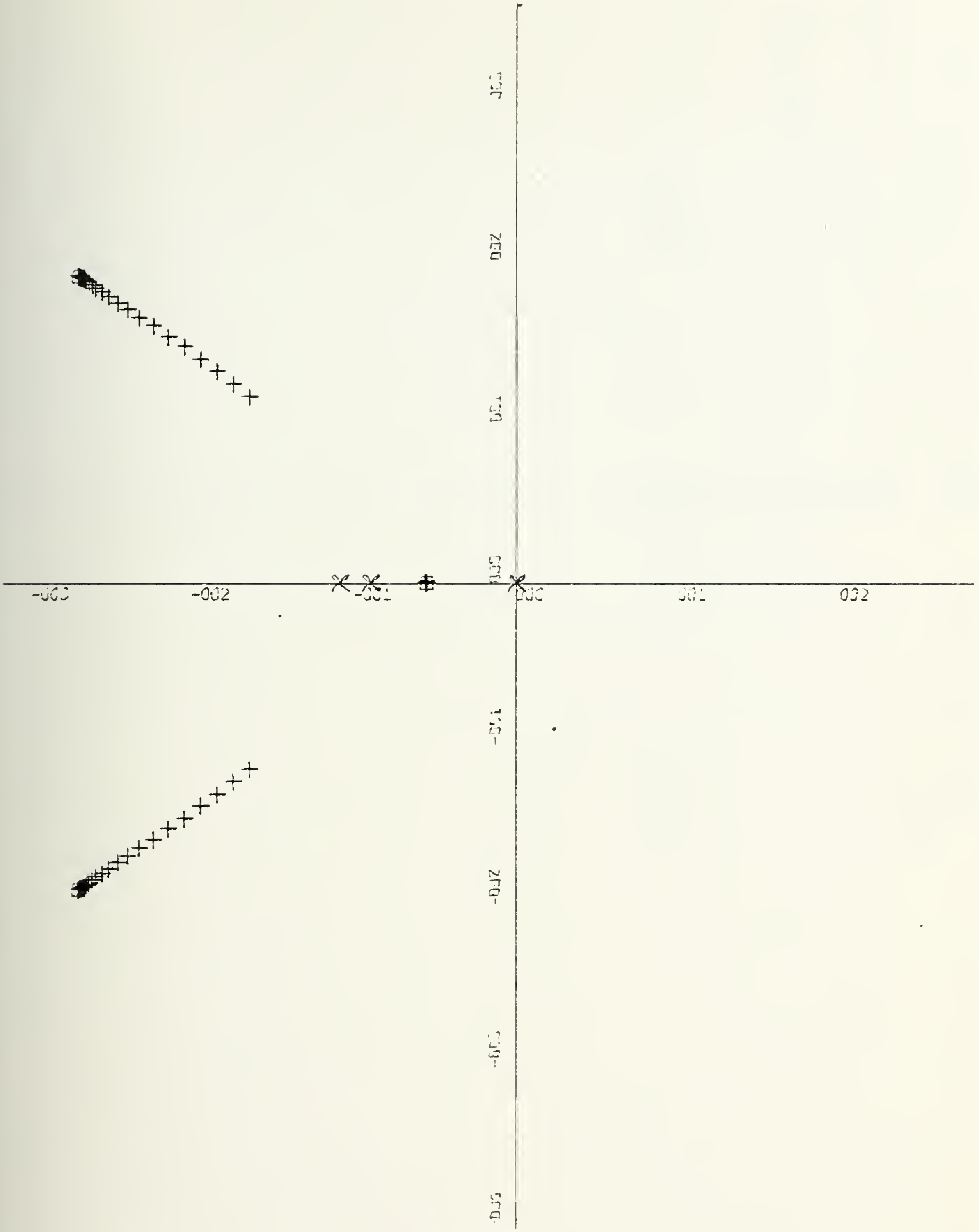
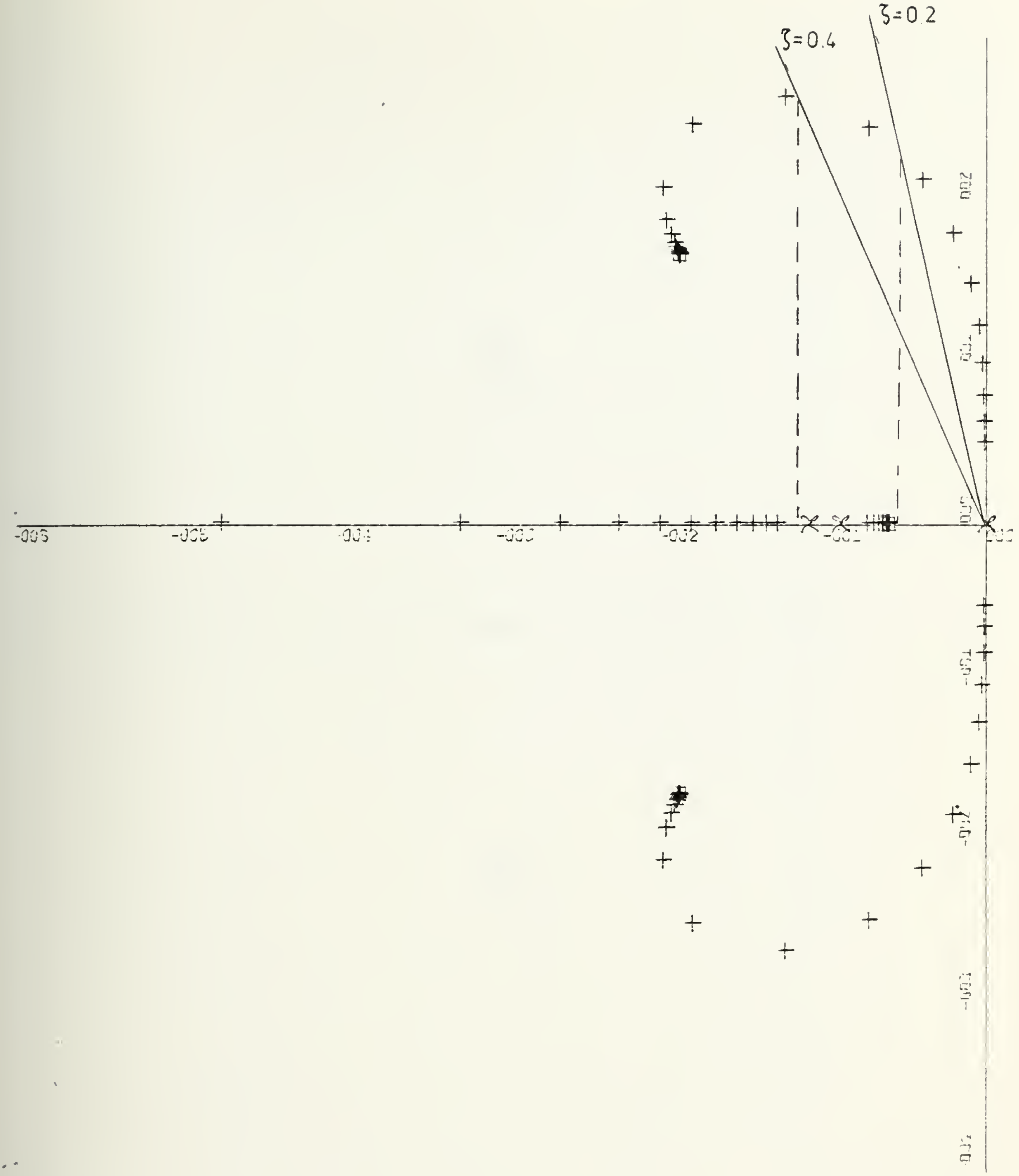


Fig. V-5a. Root locus for SOPC with CE same as for BPOC (B=800, C=10, E=1) M > 0



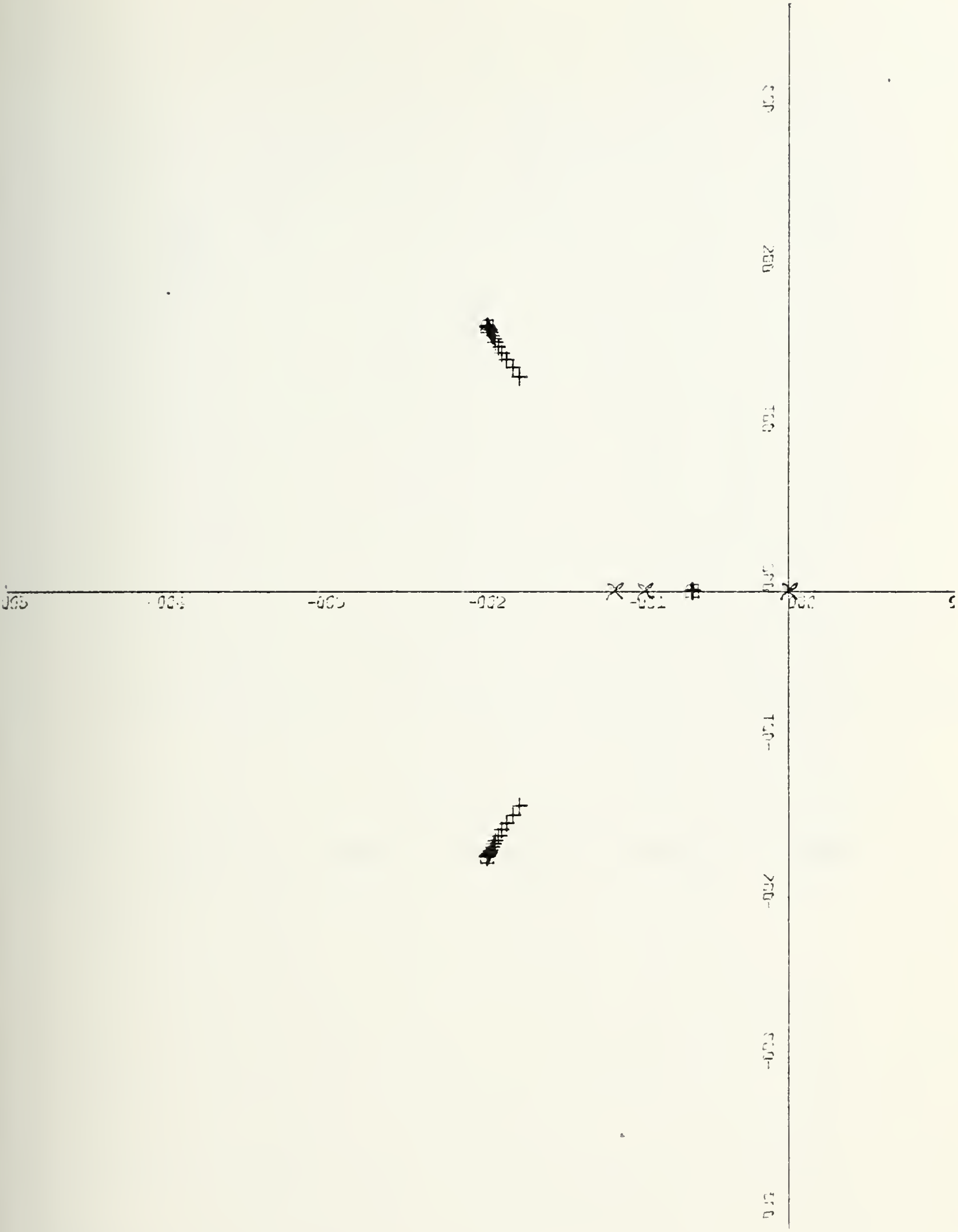
X-SCALE=0.1 UNITS INCH
Y-SCALE=0.1 UNITS INCH

Fig. V-5b. Root locus for SOPC with CE same as for BPOC (B=800, C=10, E=1) M <0



X-SCALE=1.00E-01 UNITS INCH.
Y-SCALE=1.00E-01 UNITS INCH.

Fig. V-6a. Root locus for SOPOC with B=164, C=0.001, E=0.001. $M > 0$



X-SCALE=0.1 UNITS INCH
Y-SCALE=0.1 UNITS INCH

Fig. V-6b. Root locus for SOPOC with B=164, C=0.001, E=0.001. $M < 0$

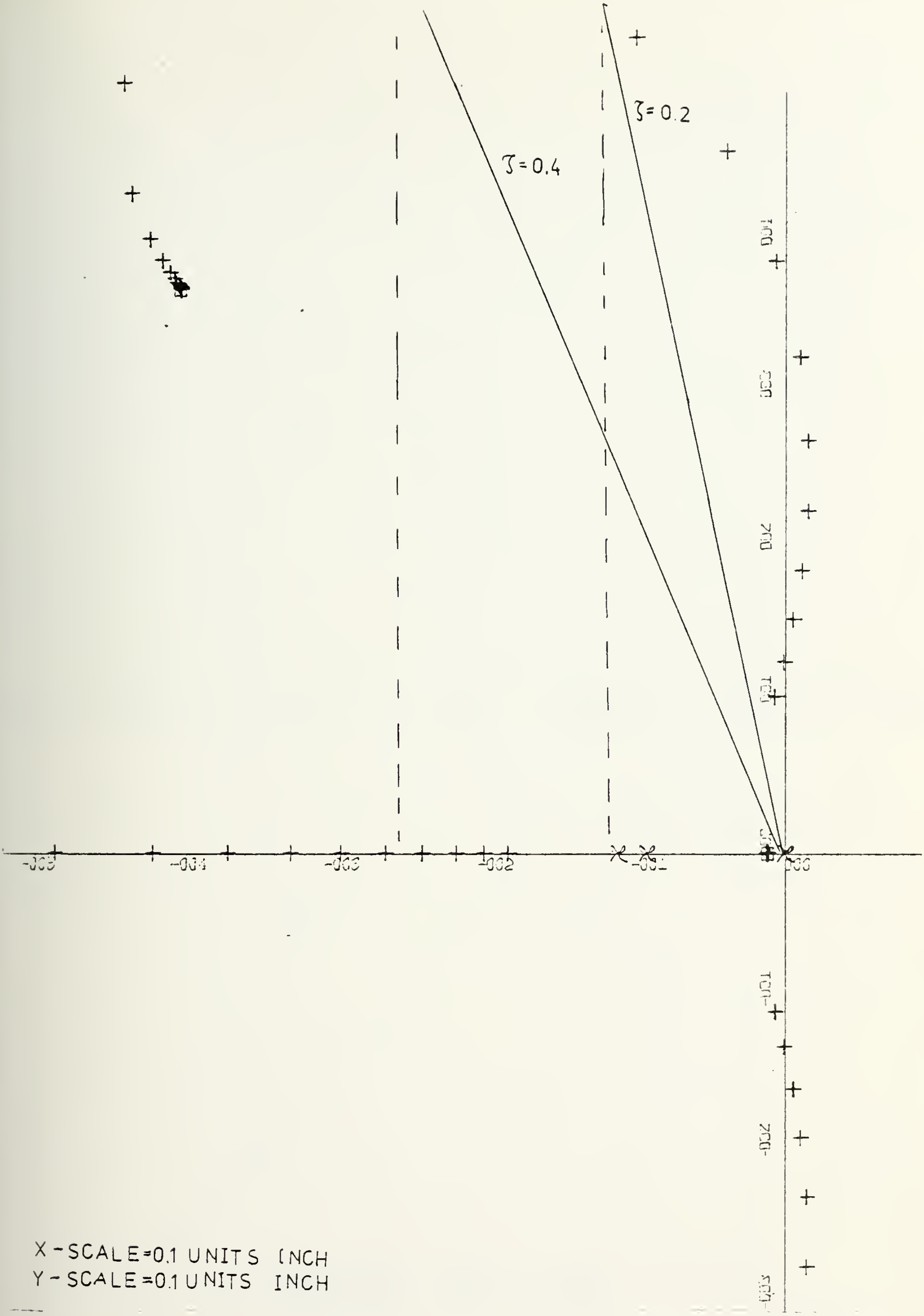


Fig. V-7a. Root locus for SOPC with CE same as for BPOC (B=164, C=0.001, E=0.001). M > 0

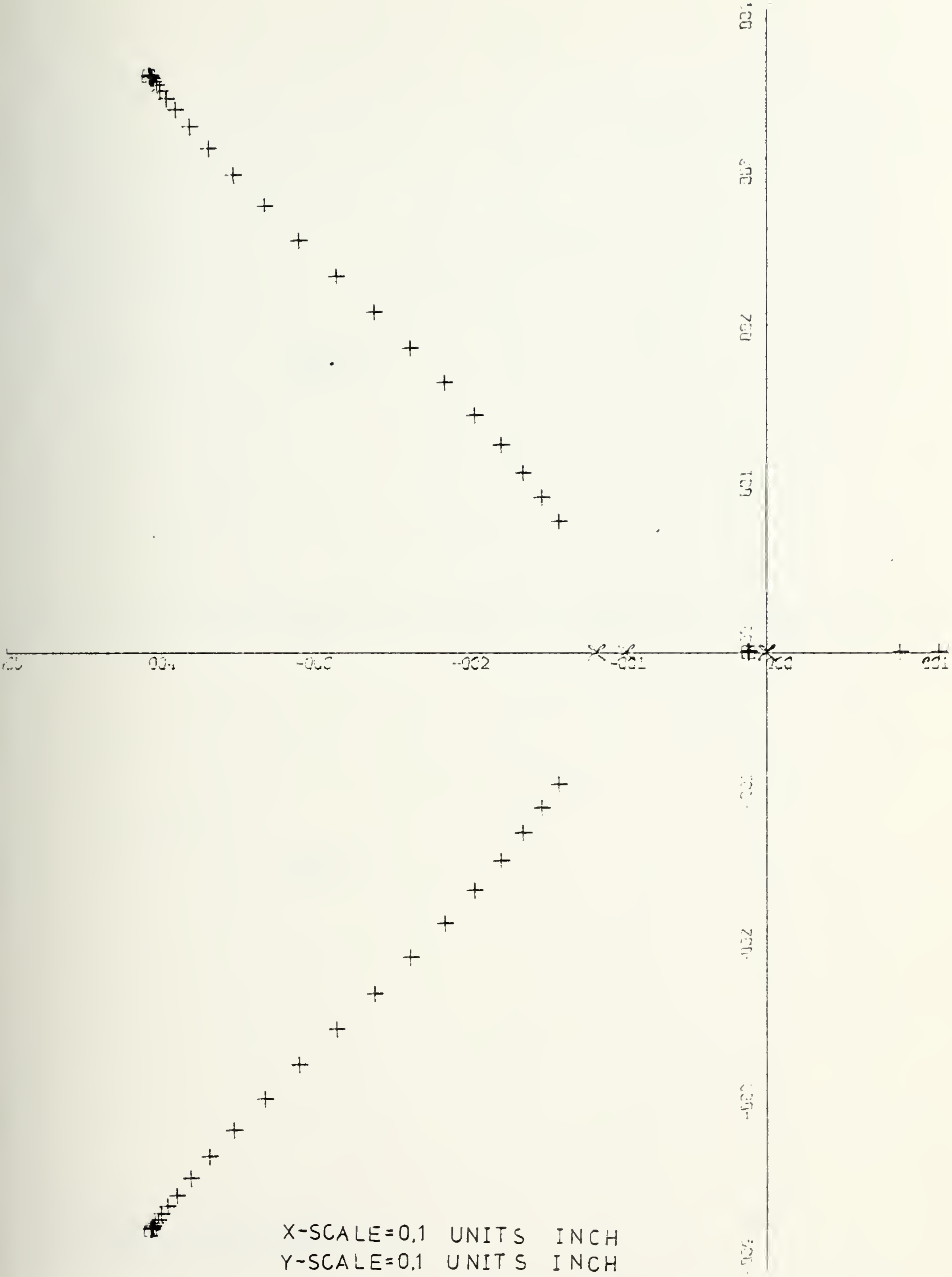


Fig. V-7b. Root locus for SOPC with CE same as for BPOC (B=164, C=0.001, E=0.001). M $<$ 0

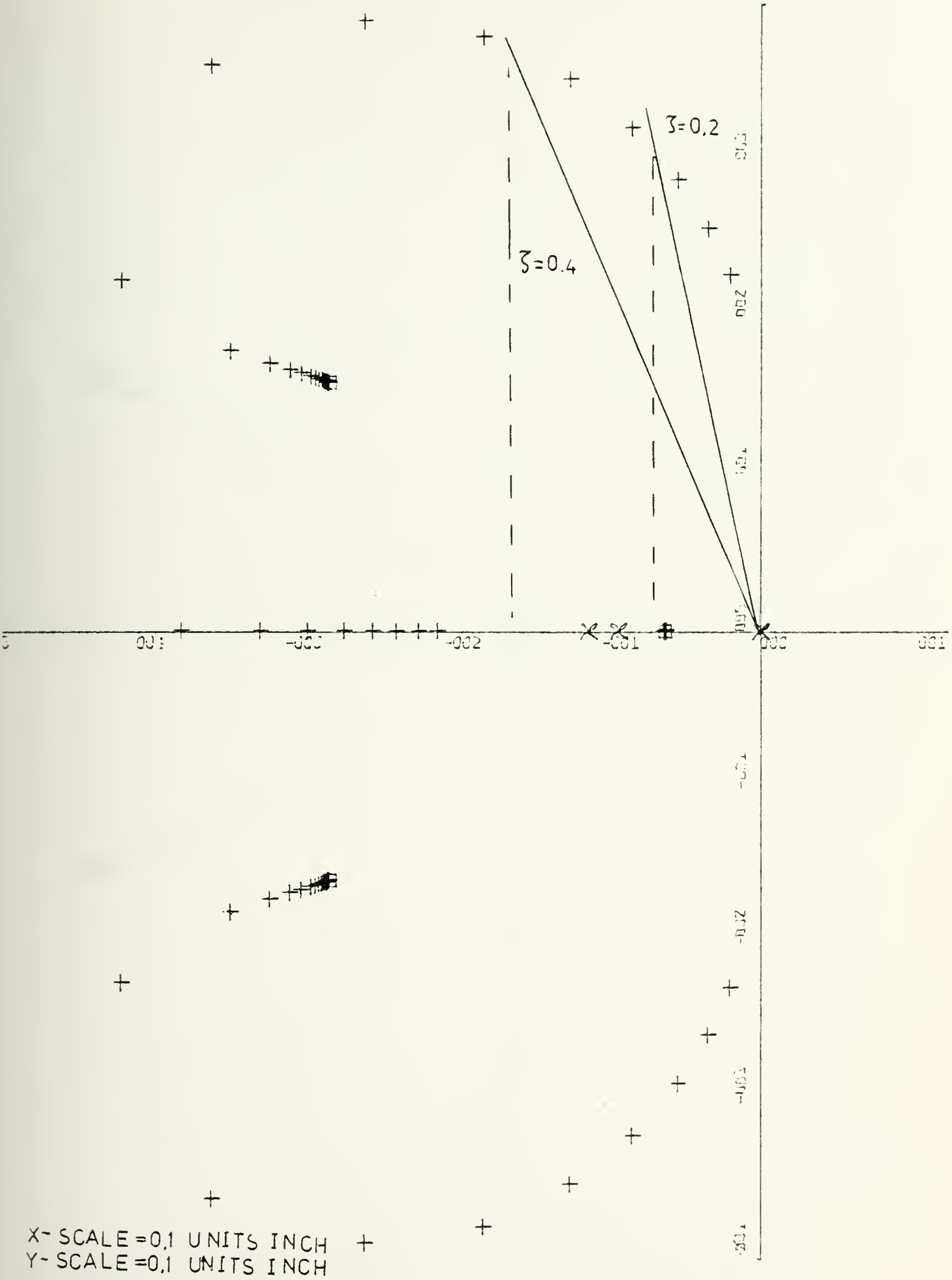


Fig. V-8a. Root locus for SOPOC with D=3000, C=10, E=1. M ⟩ 0

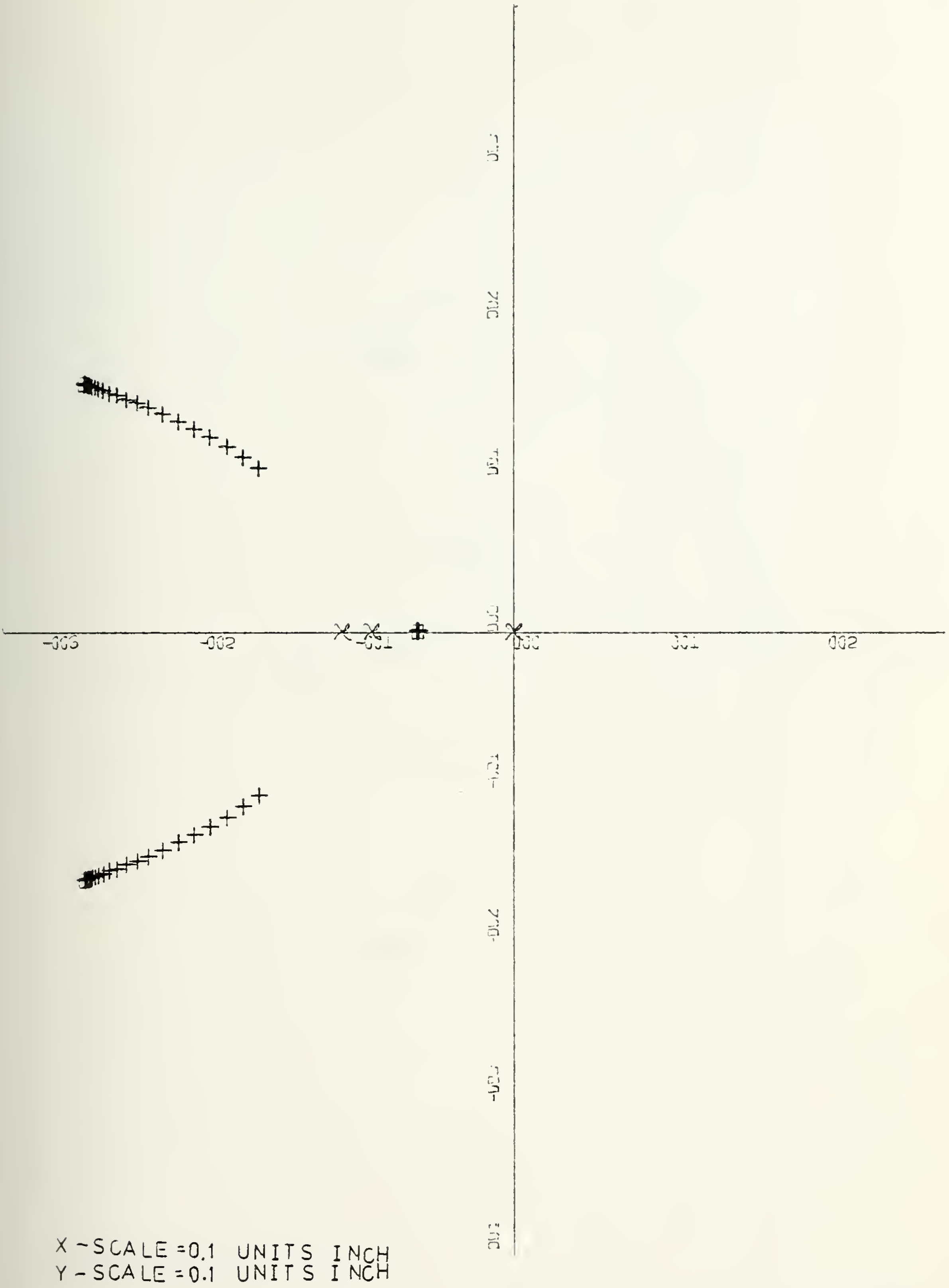


Fig. V-8b. Root locus for SOPOC with D=3000, C=10, E=1. M < 0

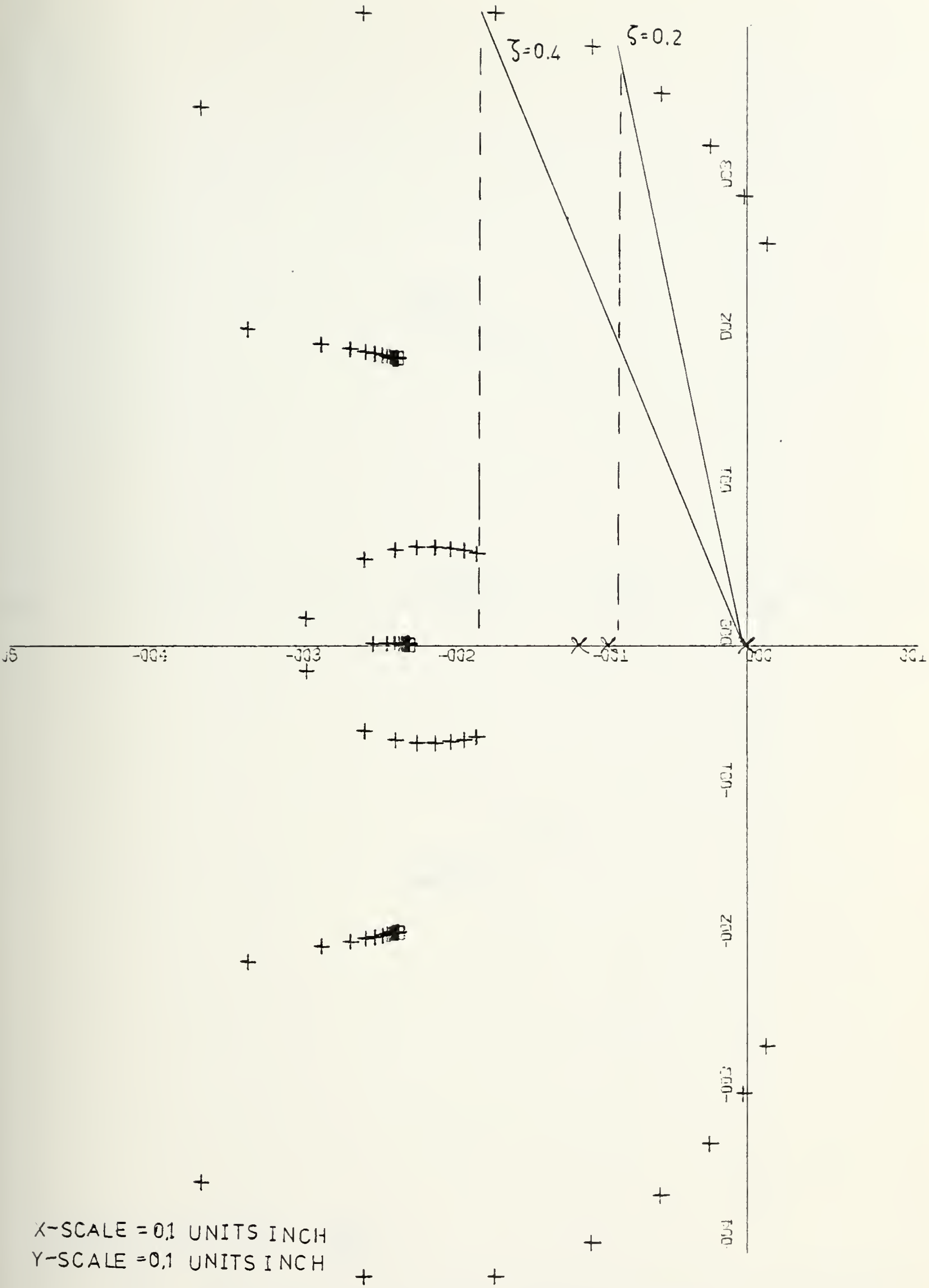


Fig. V-9a. Root locus for SOPC with CE same as for BPOC (D=3000, C=10, E=1). M > 0

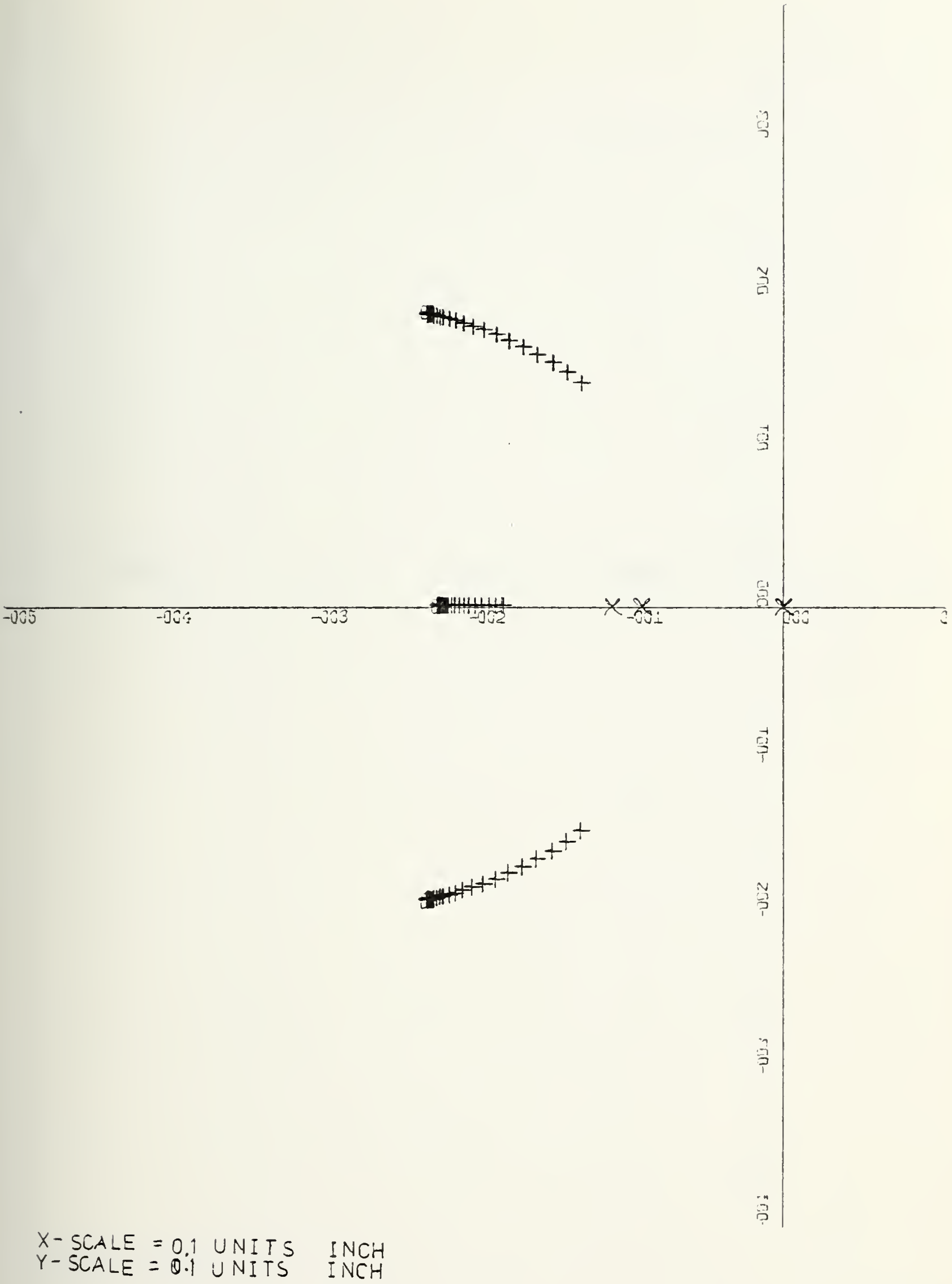
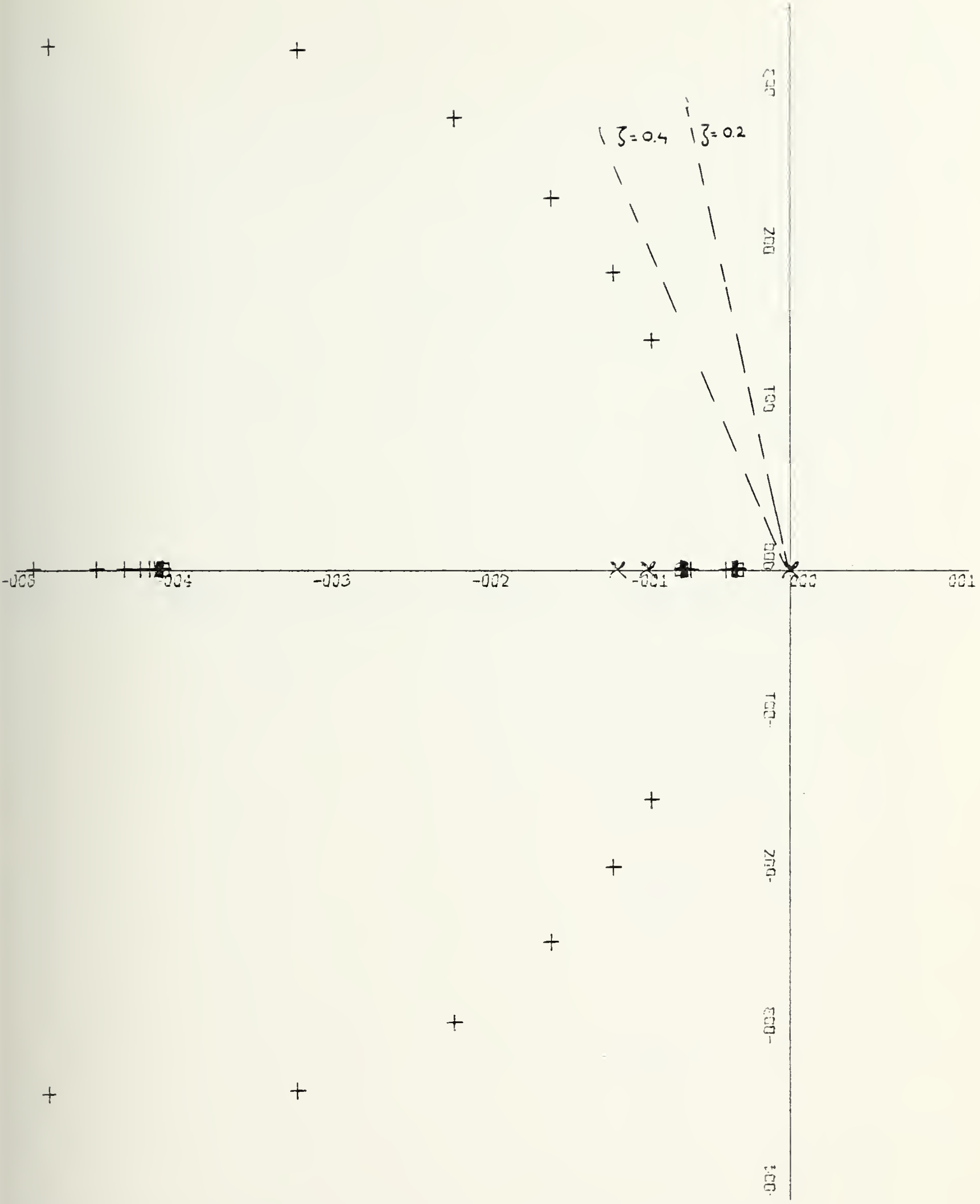
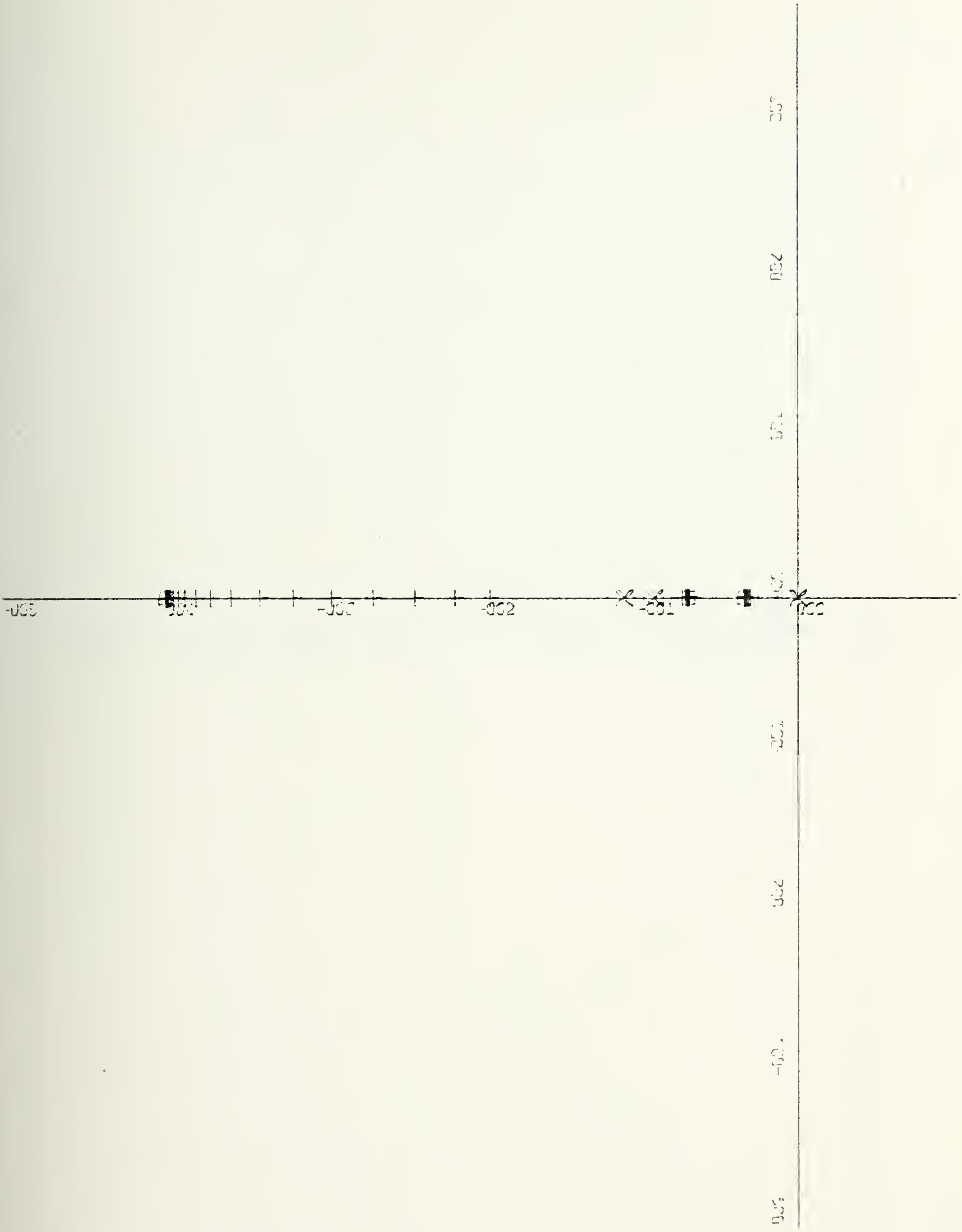


Fig. V-9b. Root locus for SOPC with CE same as for BPOC (D=3000, C=10, E=1). M < 0



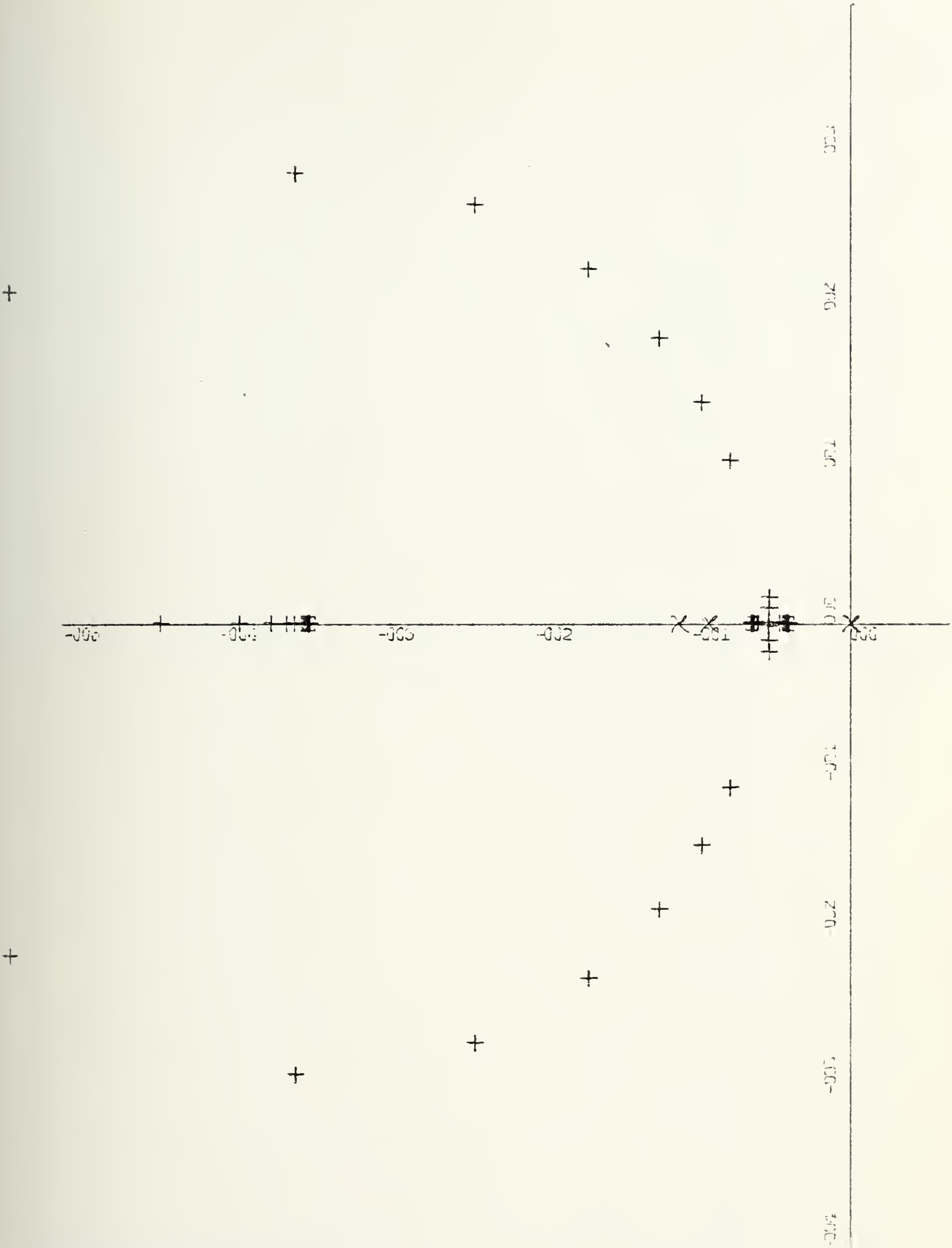
X-SCALE = 0.1 UNITS INCH
Y-SCALE = 0.1 UNITS INCH

Fig. V-10a. Root locus for SOPC using δ_{S1} of BPOC (D=3000, C=10, E=1). $M_1 > 0$



X-SCALE =0.1 UNITS INCH
Y-SCALE =0.1 UNITS INCH

Fig. V-10b. Root locus for SOPC using δ_{s1} of BPOC (D=3000, C=10, E=1). $M_1 < 0$



X-SCALE = 0.1 UNITS INCH
Y-SCALE = 0.1 UNITS INCH

Fig. V=10c. Root locus for SOPC using δ_{s1} of BPOC (B=800, C=10, E=1). $M_1 > 0$

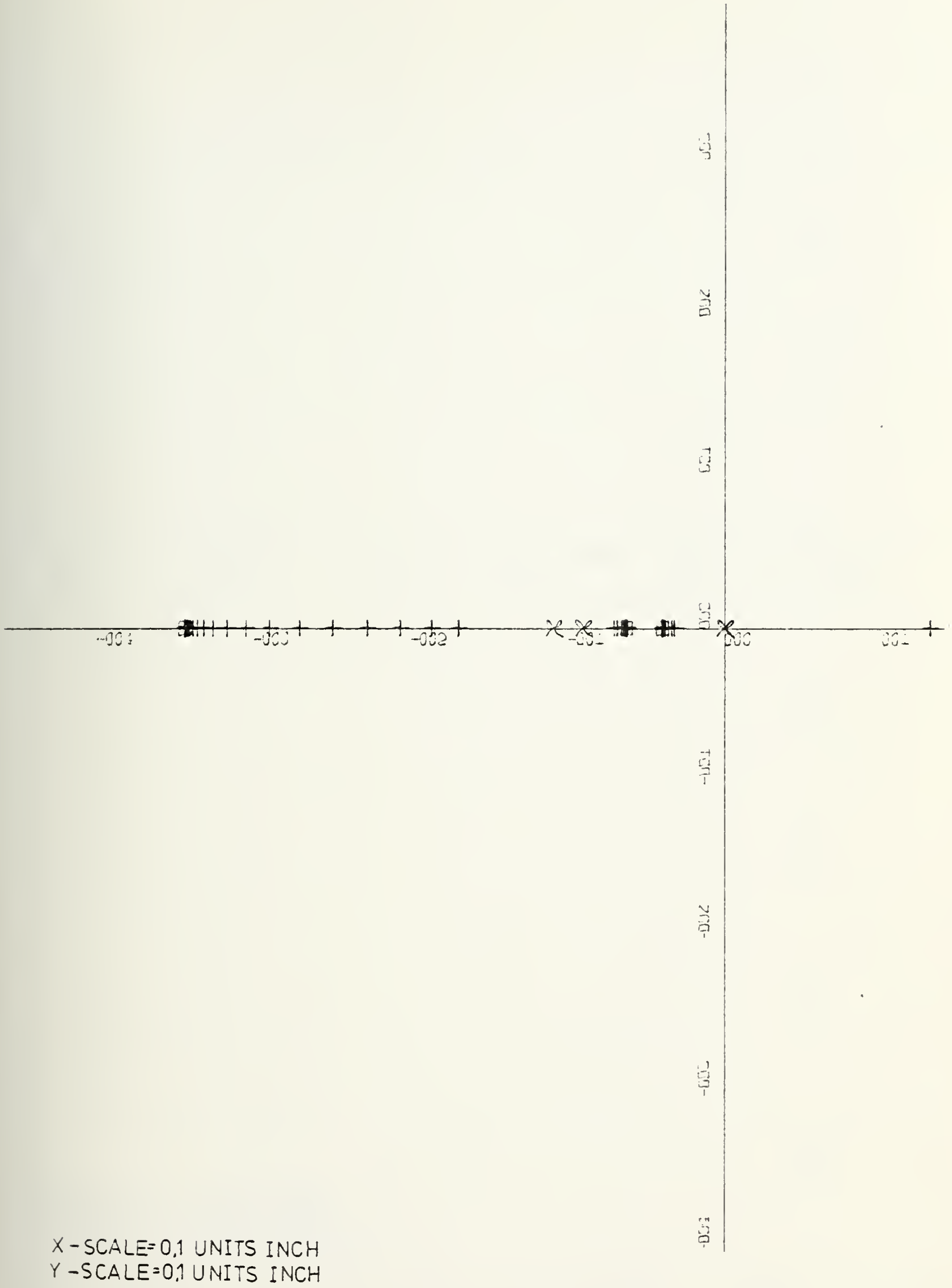
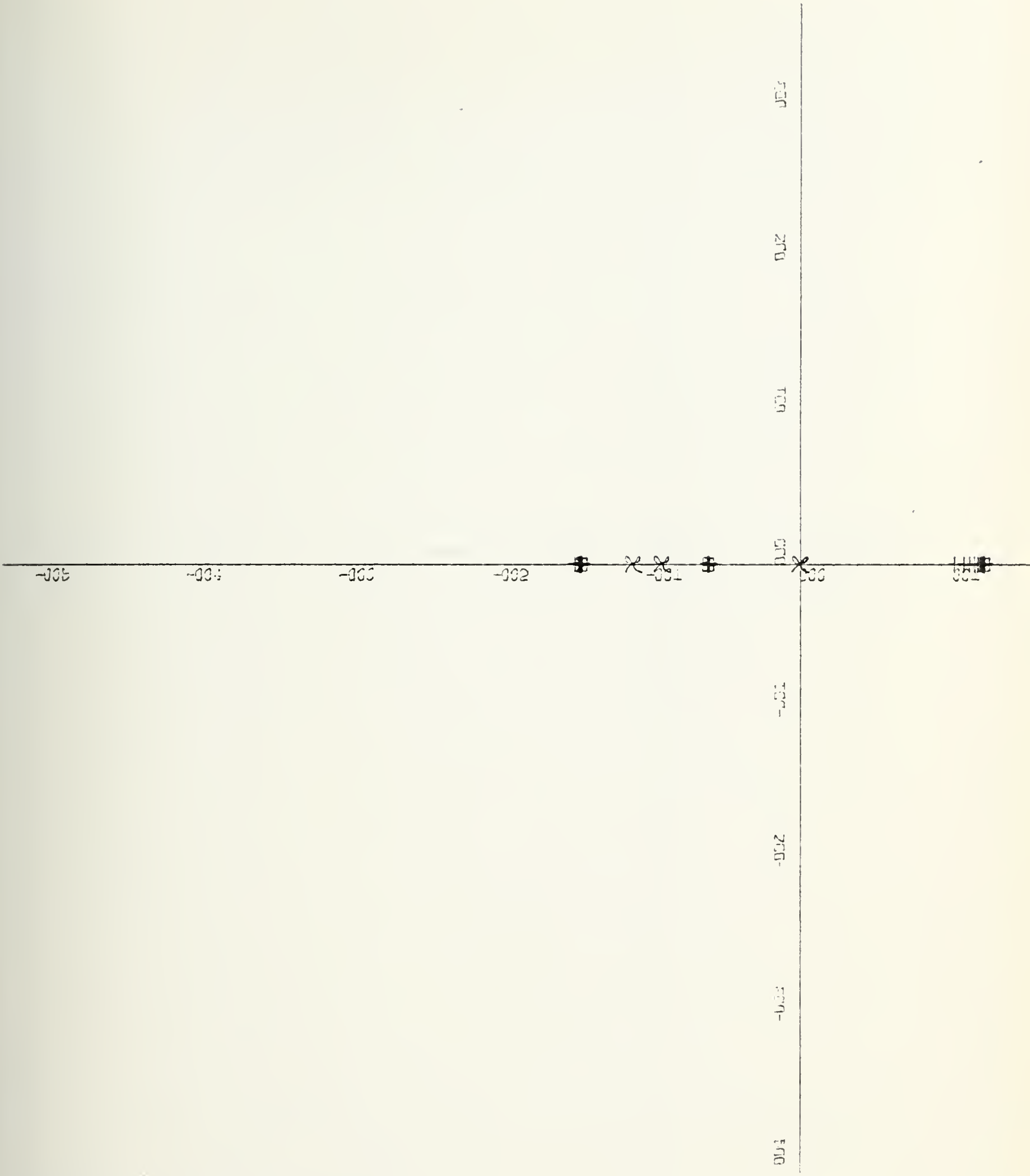
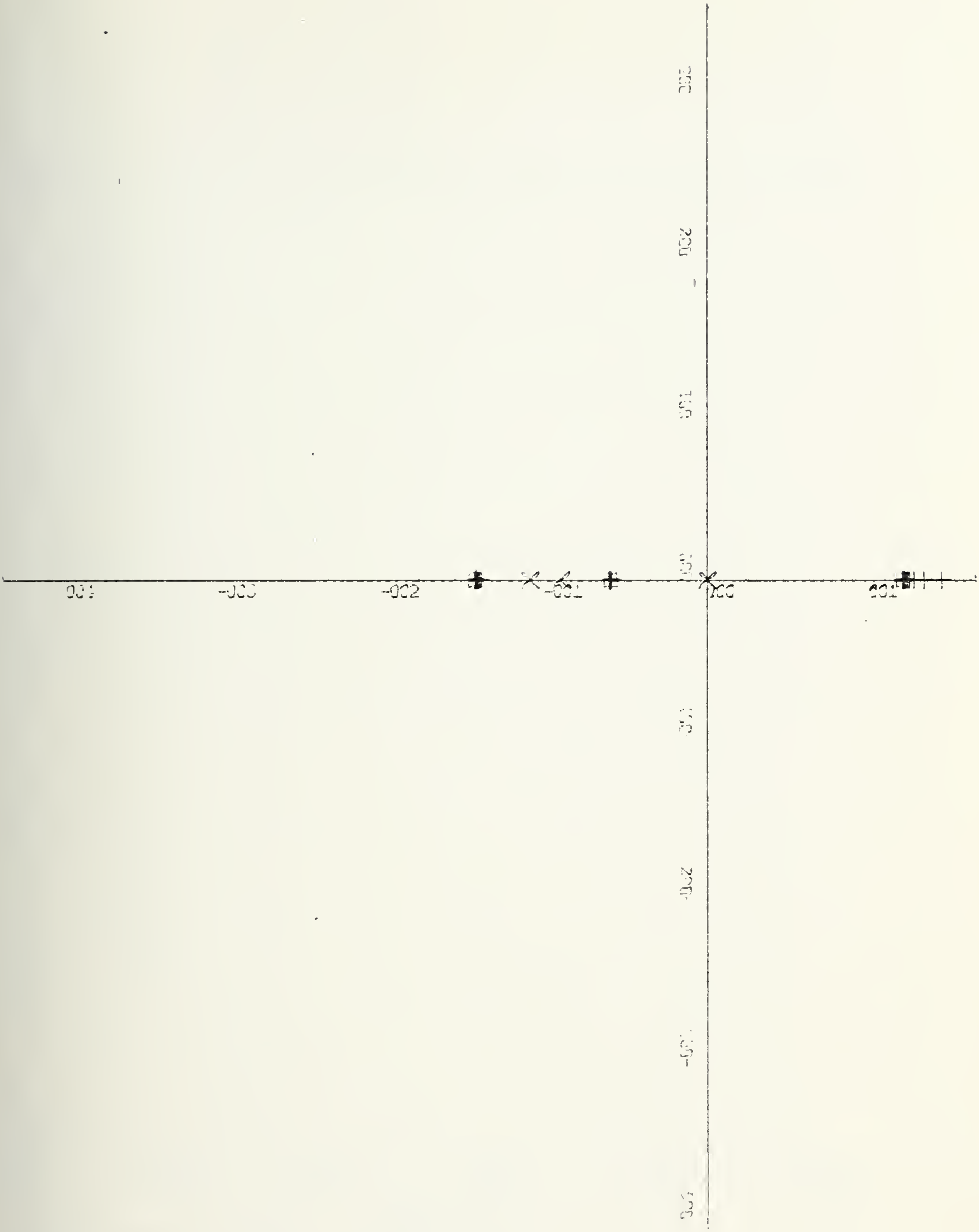


Fig. V-10d. Root locus for SOPC using δ_{S1} of BPOC (B=800, C=10, E=1). $M_1 < 0$



X-SCALE = 0.1 UNITS INCH
Y-SCALE 0.1 UNITS INCH

Fig. V-10e. Root locus for SOPC using δ_{s1} of BPOC (B=164, C=0.001, E=0.001). $M_1 > 0$



X-SCALE = 0.1 UNITS INCH
Y-SCALE = 0.1 UNITS INCH

Fig. V-10f. Root locus for SOPC using δ_{S1} of BPOC (B=164, C=0.001, E=0.001). $M_1 < 0$

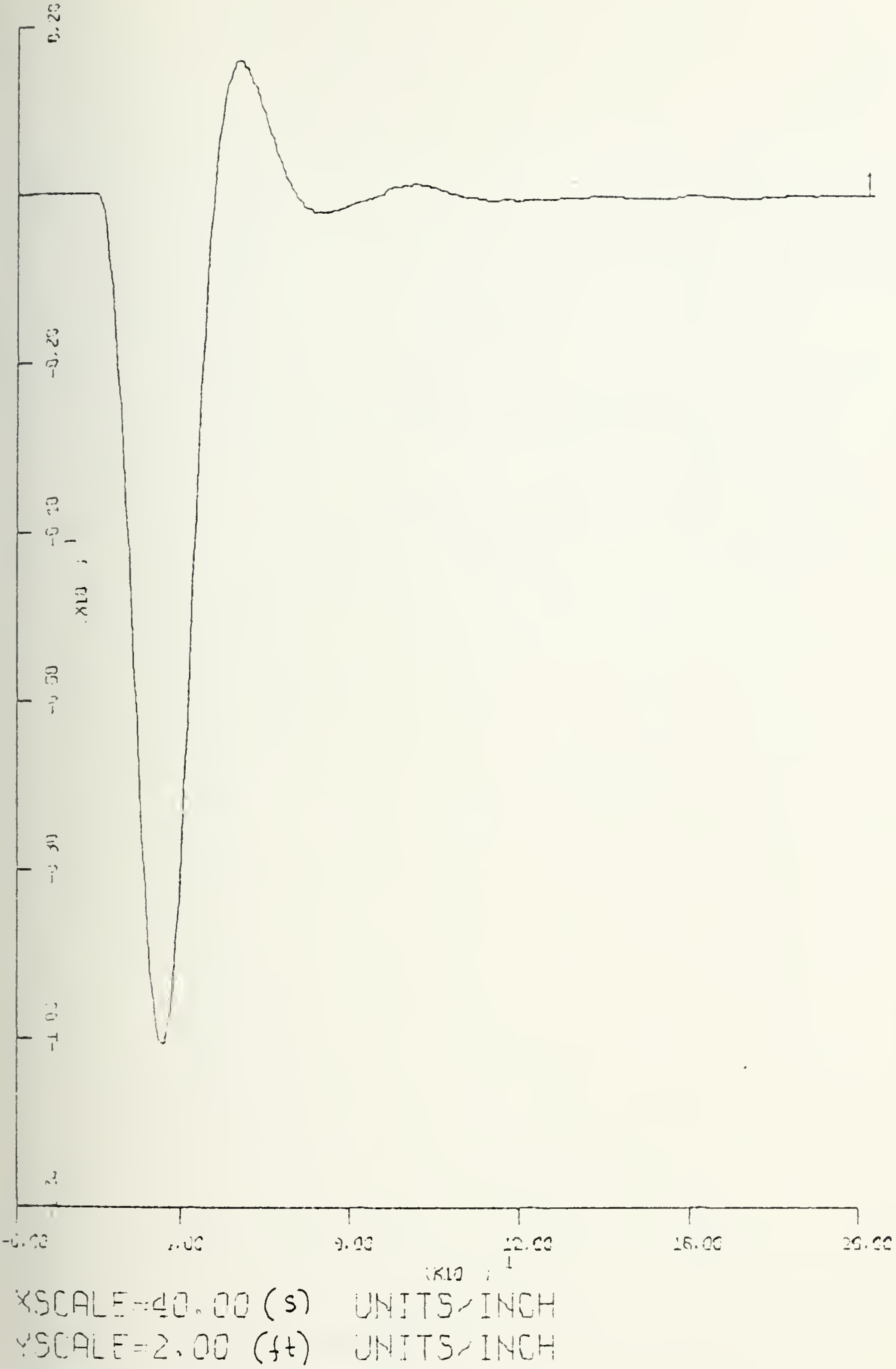


Fig. V-11a. Depth vs. Time. Response to a pulse force at FT. Both planes submarine with $\delta_{scom} = \delta_{s1} \cdot f(x)$. X=0.5

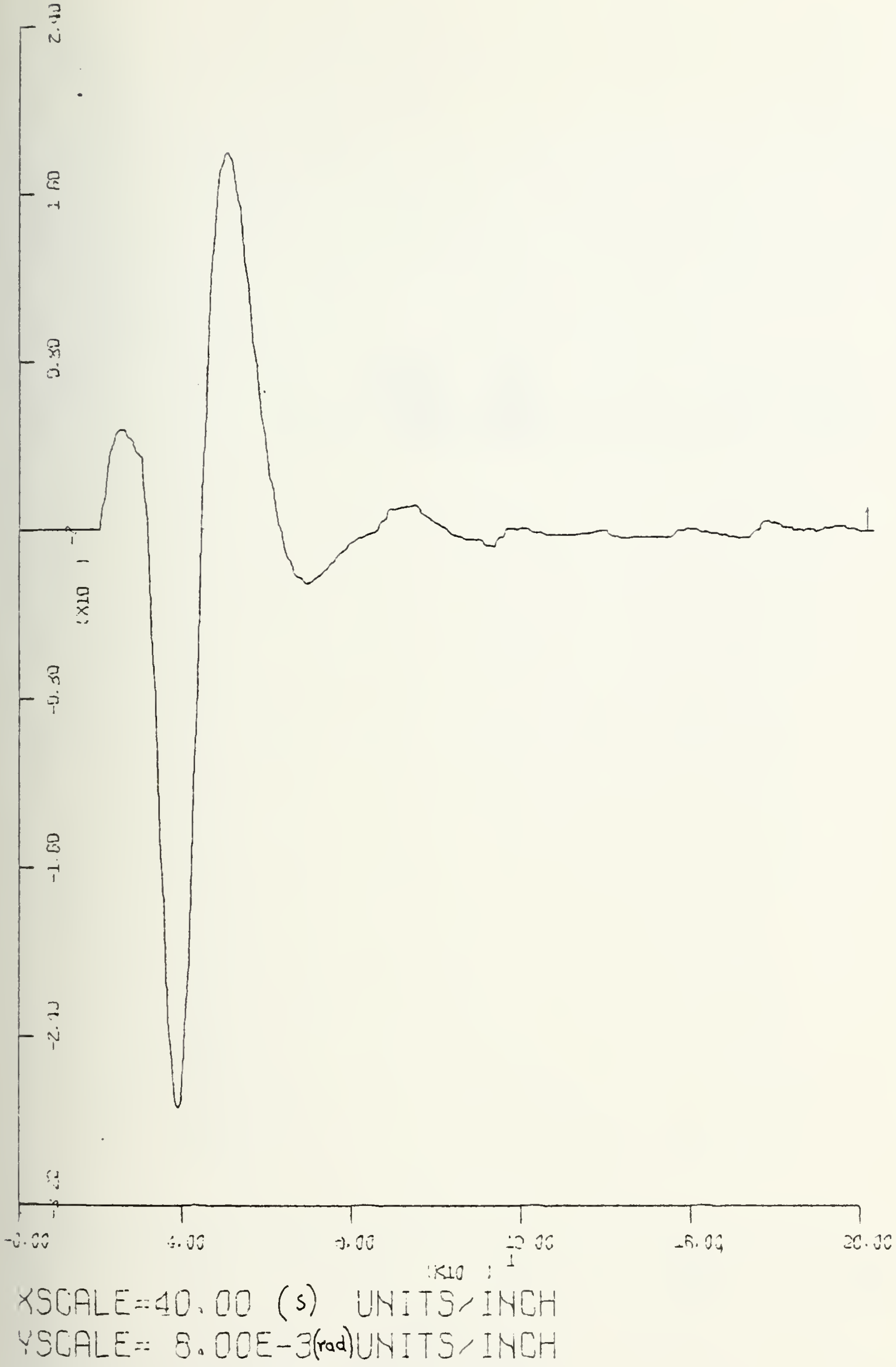
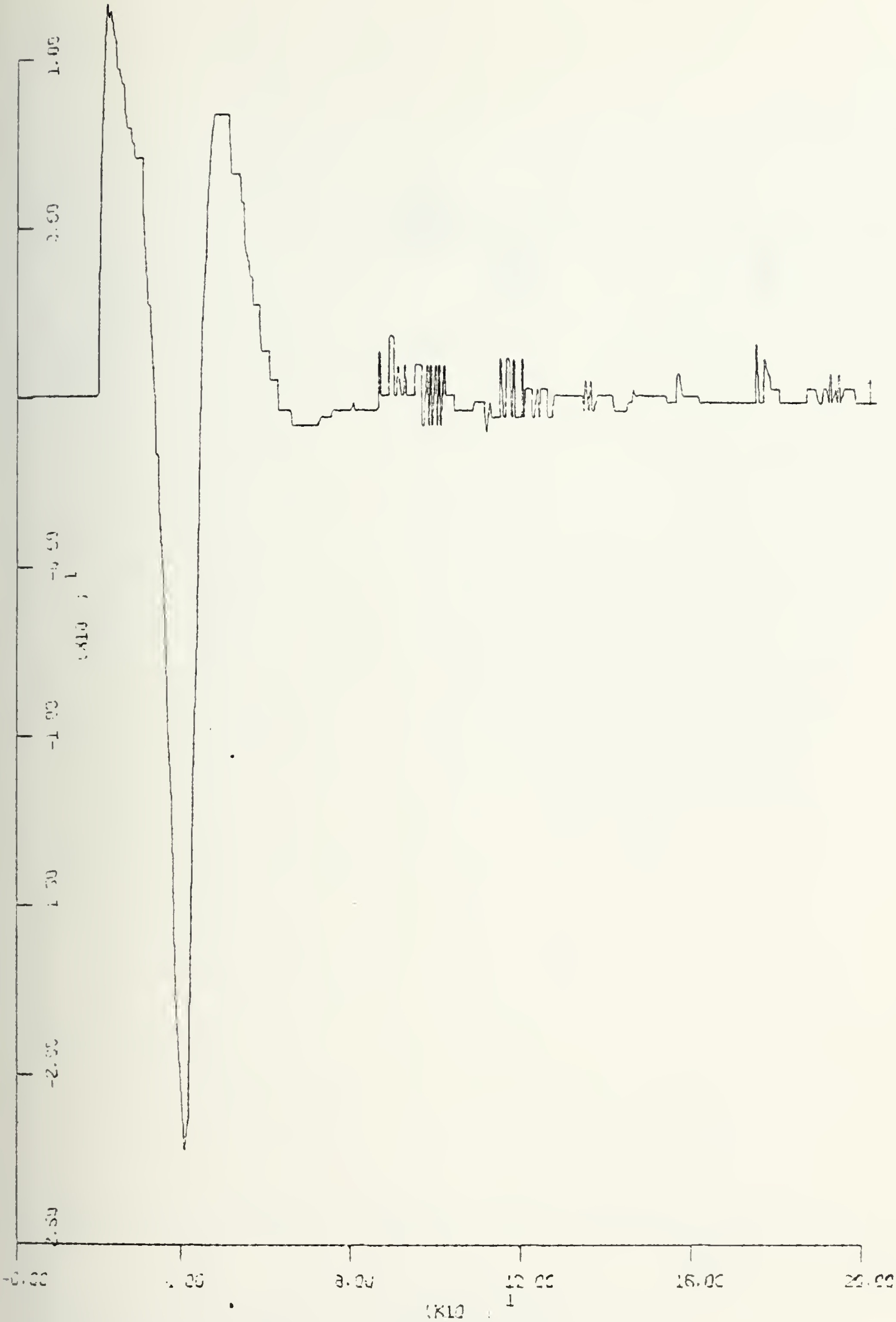


Fig. V-11b. - Pitch vs. Time. Response to a pulse force at FT. Both planes submarine with $\delta s_{com} = \delta s_1 \cdot f(K)$. X=0.5



XSCALE=40.00 (s) UNITS/INCH
YSCALE=5.00 (deg) UNITS/INCH

Fig. V-11c. Stern Plane Angle vs. Time. Response to a pulse force at FT. Both planes submarine with $\delta_{scom} = \delta_{s1} \cdot f(K)$. X=0.5

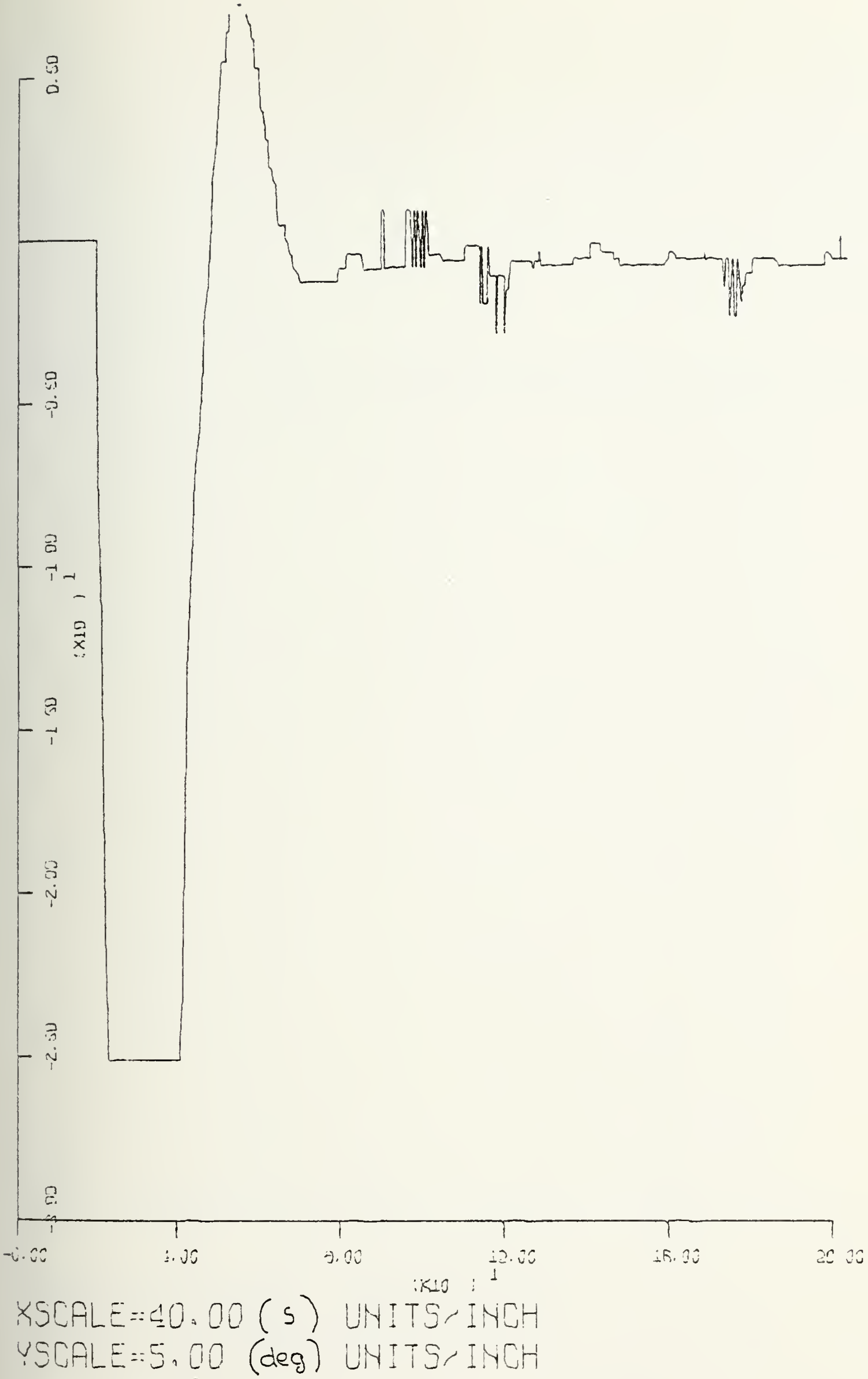
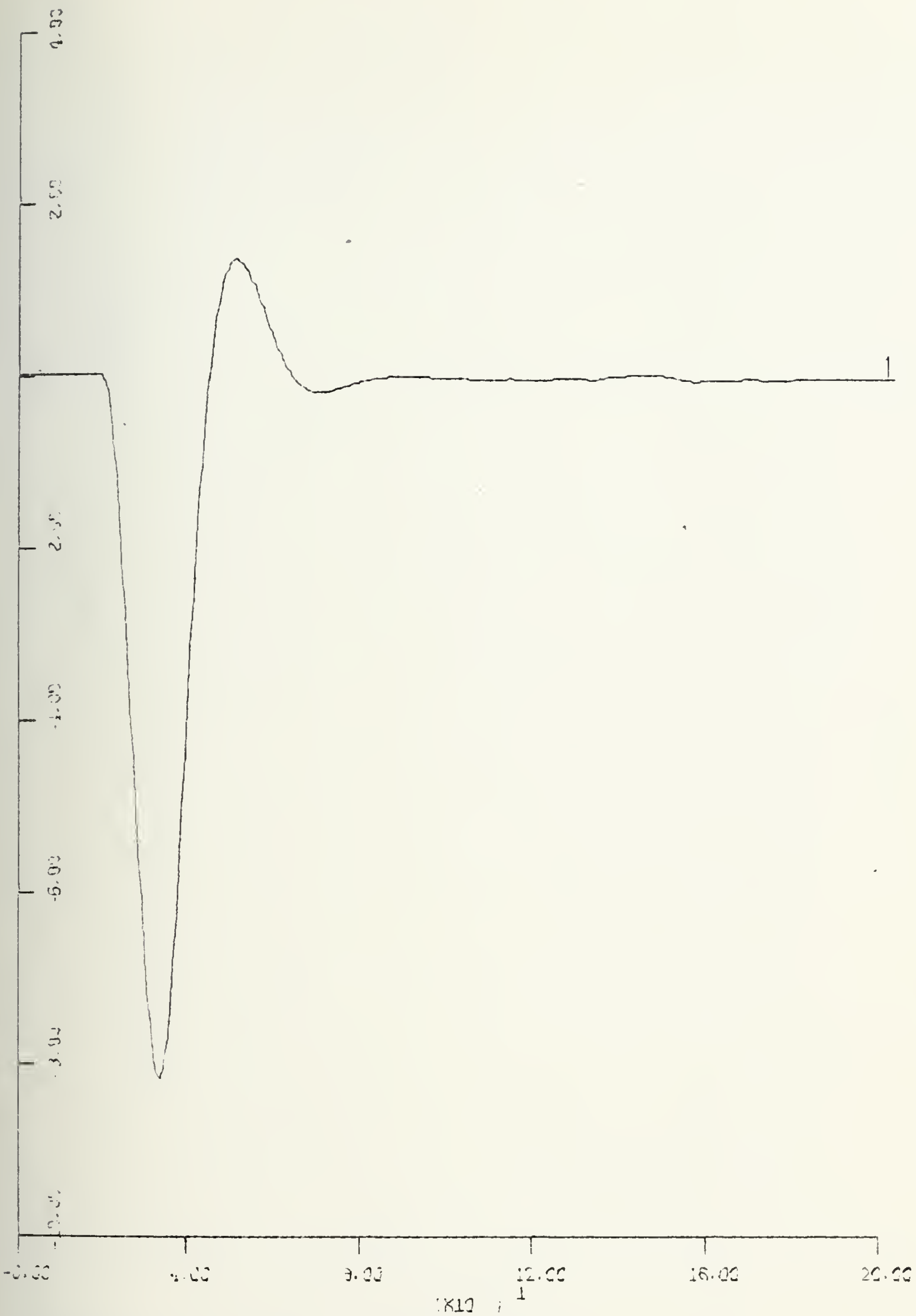


Fig. V-11d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. Both planes submarine with $\delta s_{com} = \delta s_1 \cdot f(K)$. X=0.5



XSCALE=40.00 (s) UNITS/INCH
YSCALE=2.00 (ft) UNITS/INCH

Fig. V-12a. Depth vs. Time. Response to a pulse force at FT. Both planes submarine with $\delta scom = \delta s1 \cdot f(K)$. X=-0.5

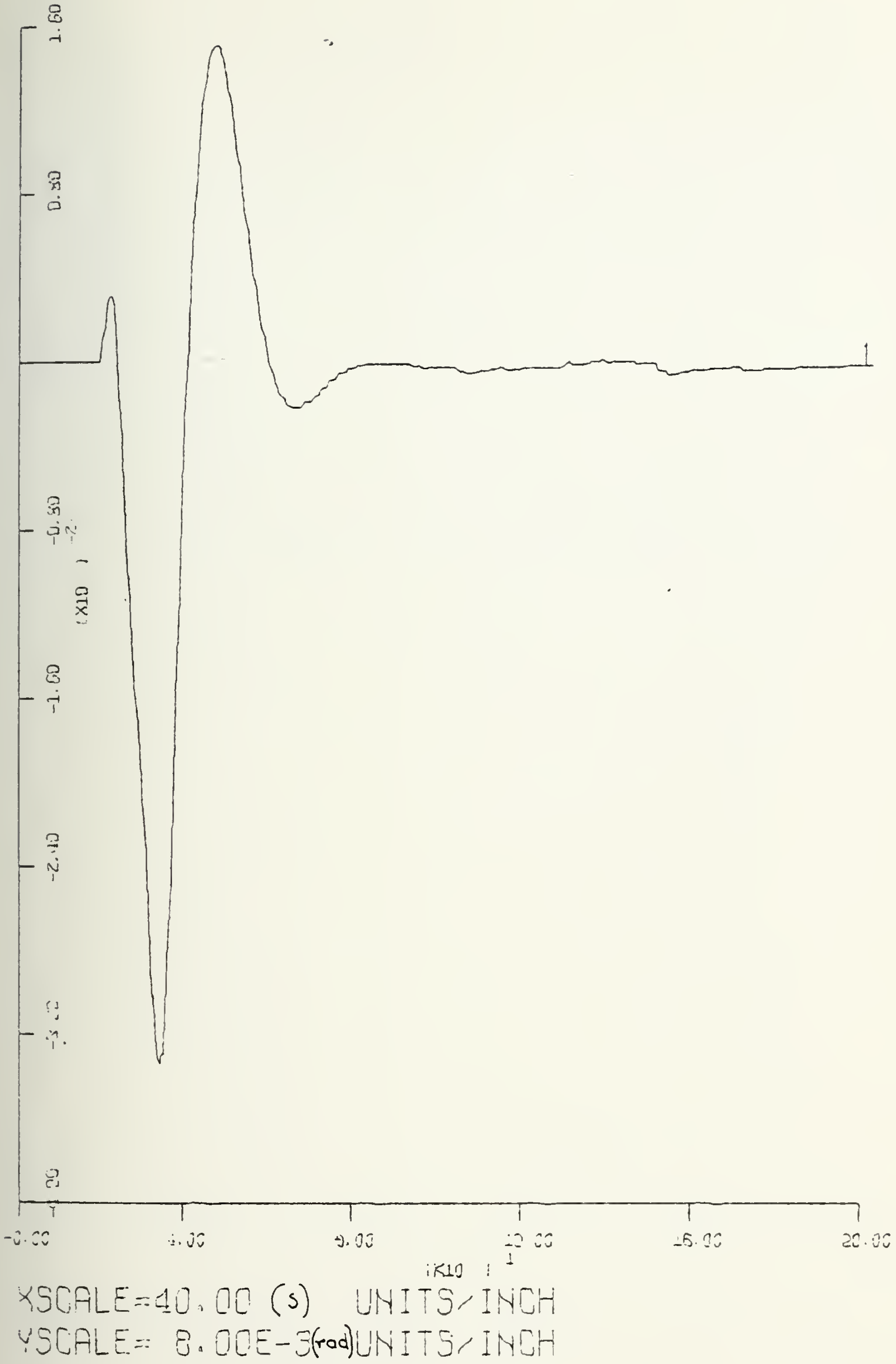
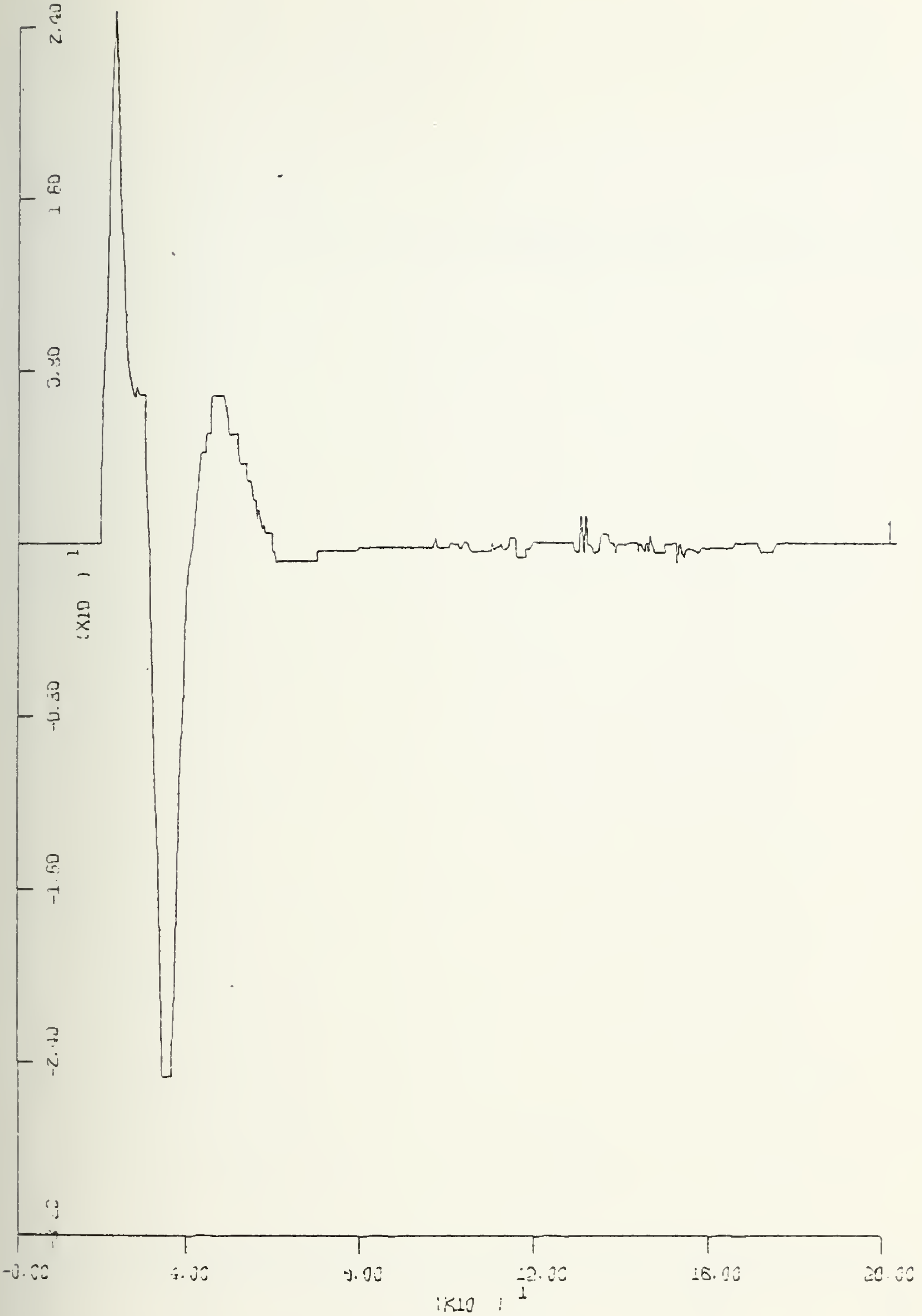


Fig. V-12b. Pitch vs. Time. Response to a pulse force at FT. Both planes submarine with $\delta_{scom} = \delta_{s1} \cdot f(k)$. X=-0.5



XSCALE=40.00 (s) UNITS/INCH
YSCALE=8.00 (deg) UNITS/INCH

Fig. V-12c. Stern Plane Angle vs. Time. Response to a pulse force at FT. Both planes submarine with $\delta_{scom} = \delta_{s1} \cdot f(k)$. X=-0.5

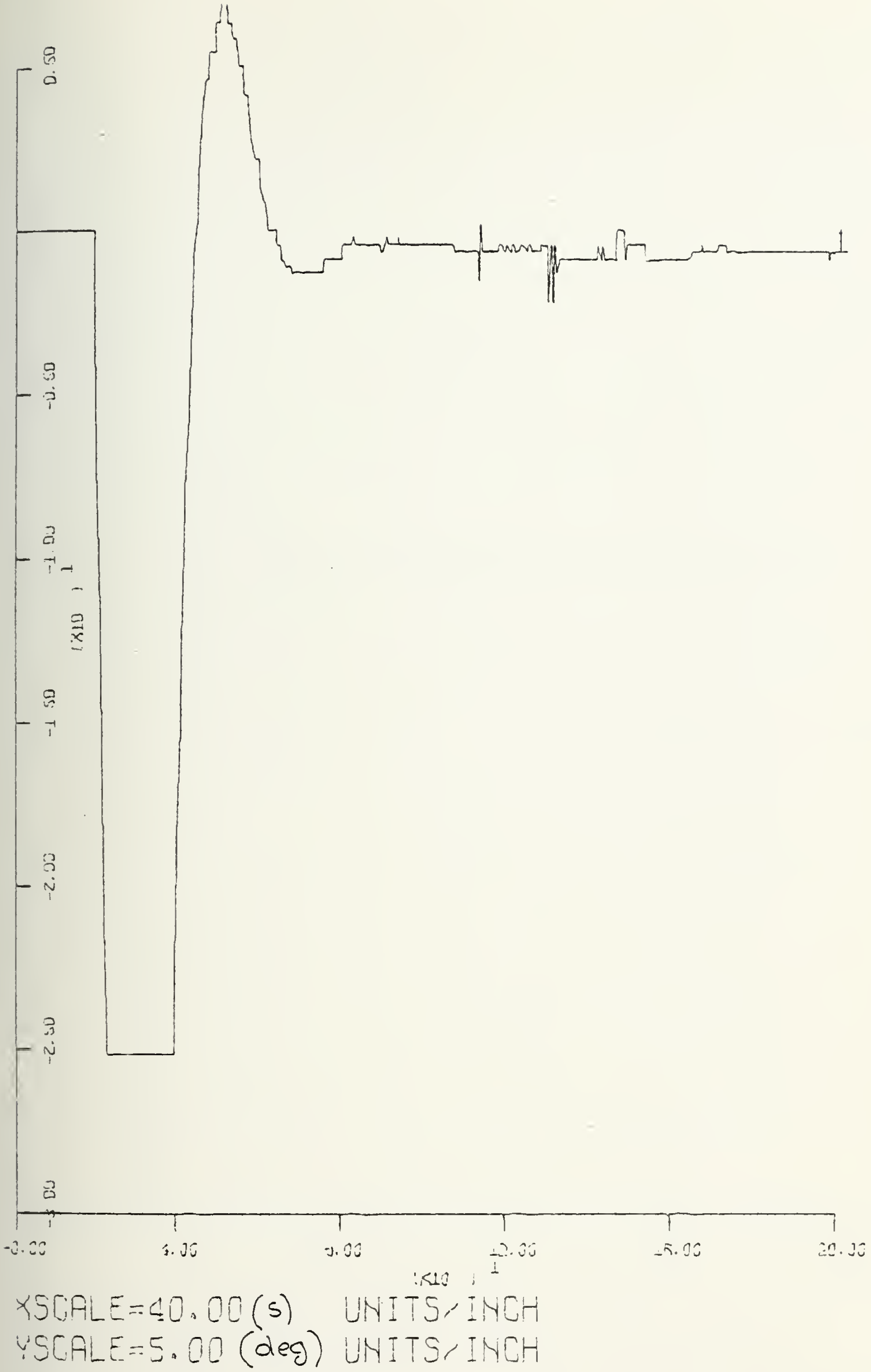


Fig. V-12d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. Both planes submarine with $\delta s_{com} = \delta s_1 \cdot f(\kappa)$. X=-0.5

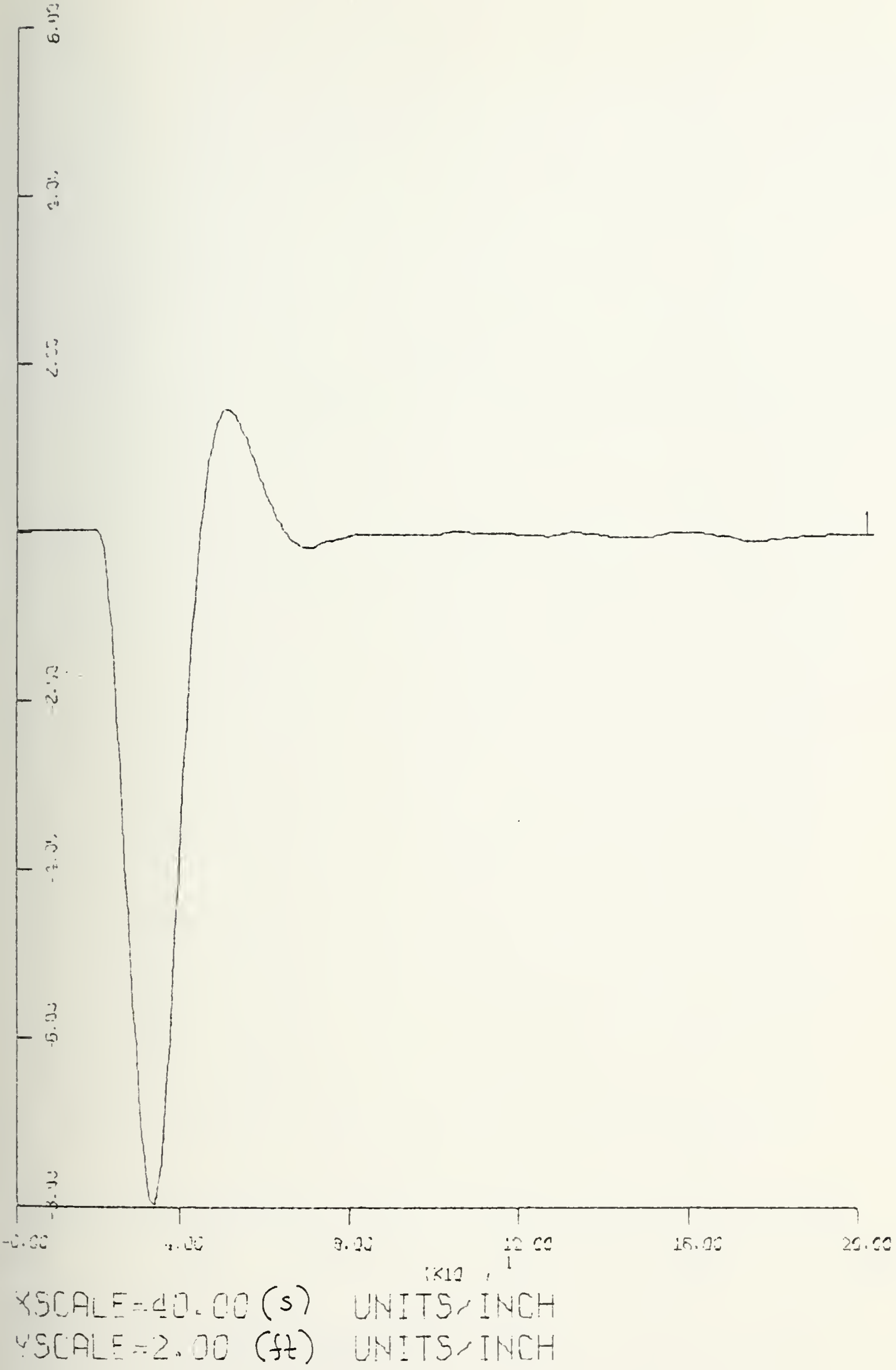
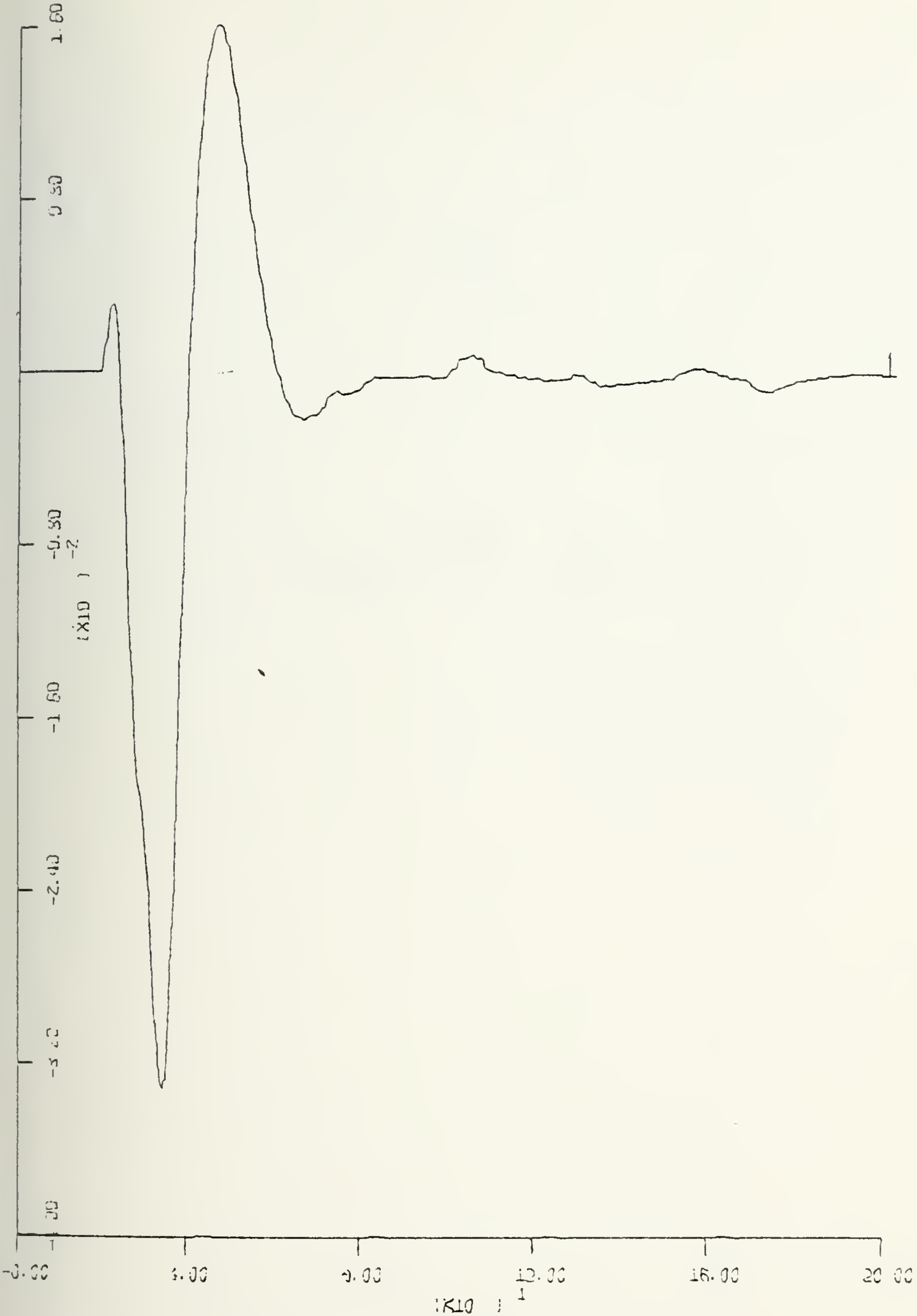


Fig. V-13a. Depth vs. Time. Response to a pulse force at FT. Both planes submarine with $\delta s_{com} = \delta s_1 \cdot f(\kappa)$. X=-0.75



XSCALE=40.00 (s) UNITS/INCH
YSCALE= 8.00E-3(rad)UNITS/INCH

Fig. V-13b. Pitch vs. Time. Response to a pulse force at FT. Both planes submarine with $\delta_{scom} = \delta_{sL} \cdot f(k)$. X=-0.75

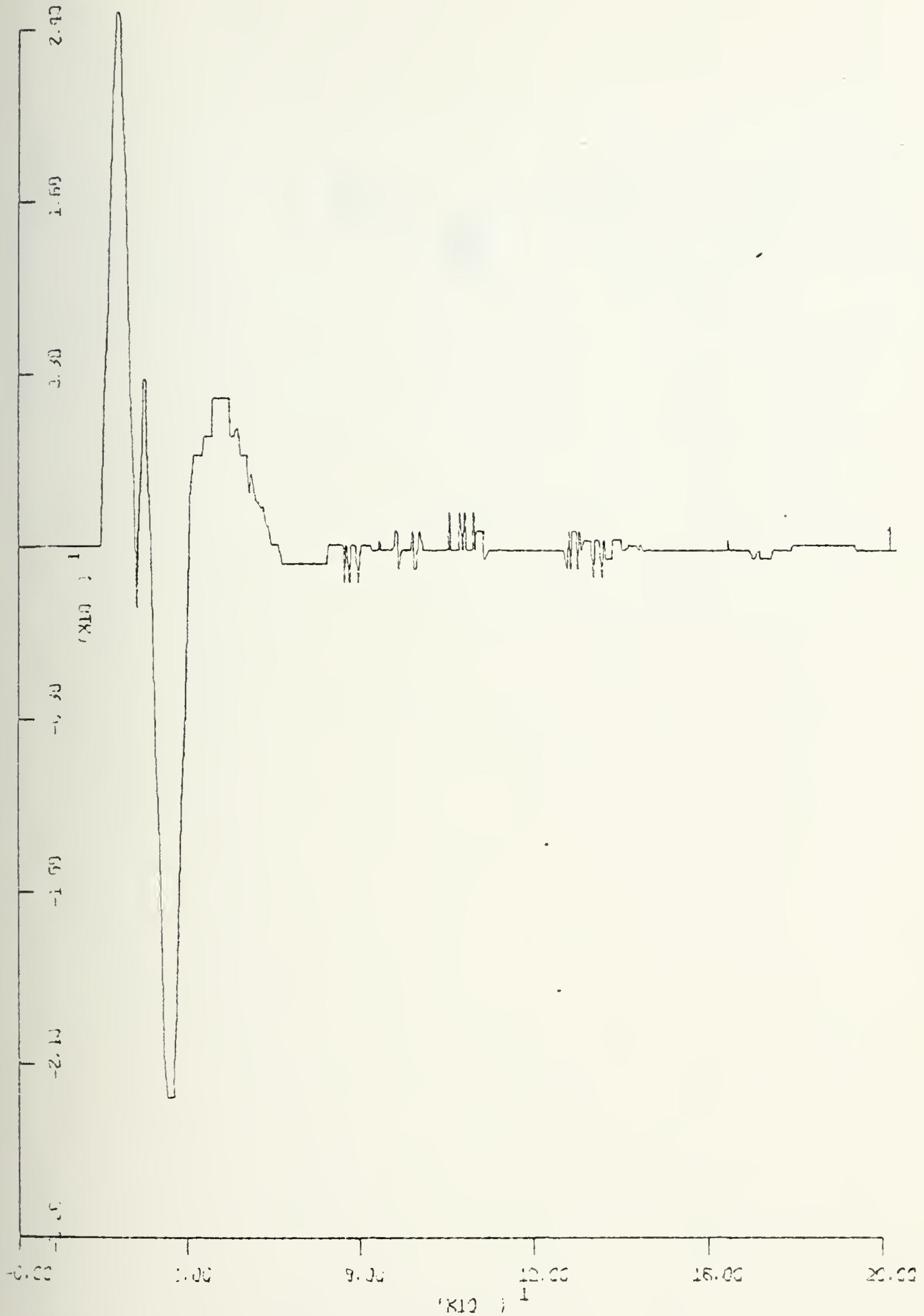
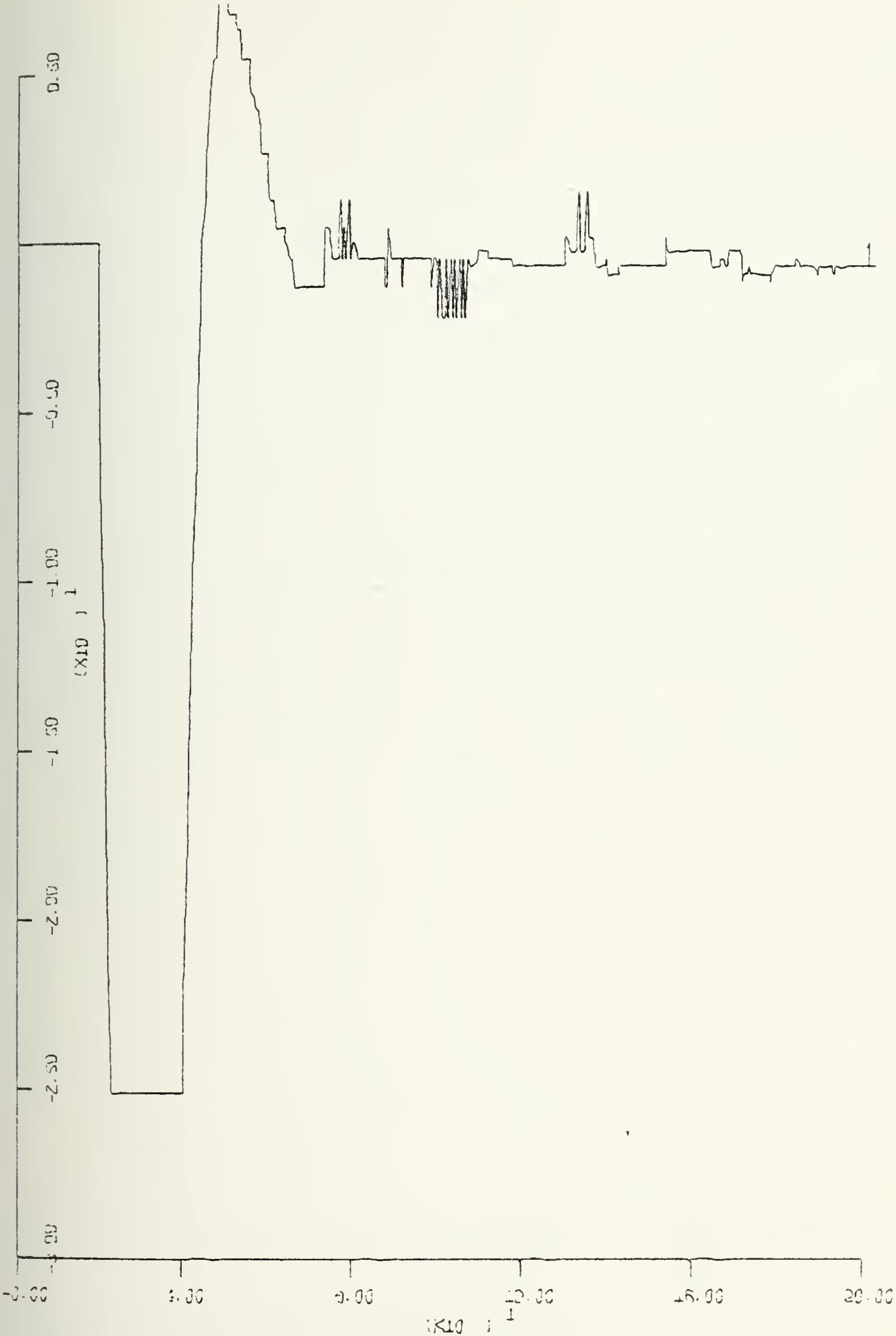


Fig. V-13c. Stern Plane Angle vs. Time. Response to a pulse force at FT. Both planes submarine with $\delta scom = \delta s_1 \cdot f(k)$. X=-0.75



XSCALE=40.00 (s) UNITS/INCH
YSCALE=5.00 (deg) UNITS/INCH

Fig. V-13d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. Both planes submarine with $\delta s_{com} = \delta s_1 \cdot f(K)$. X=-0.75

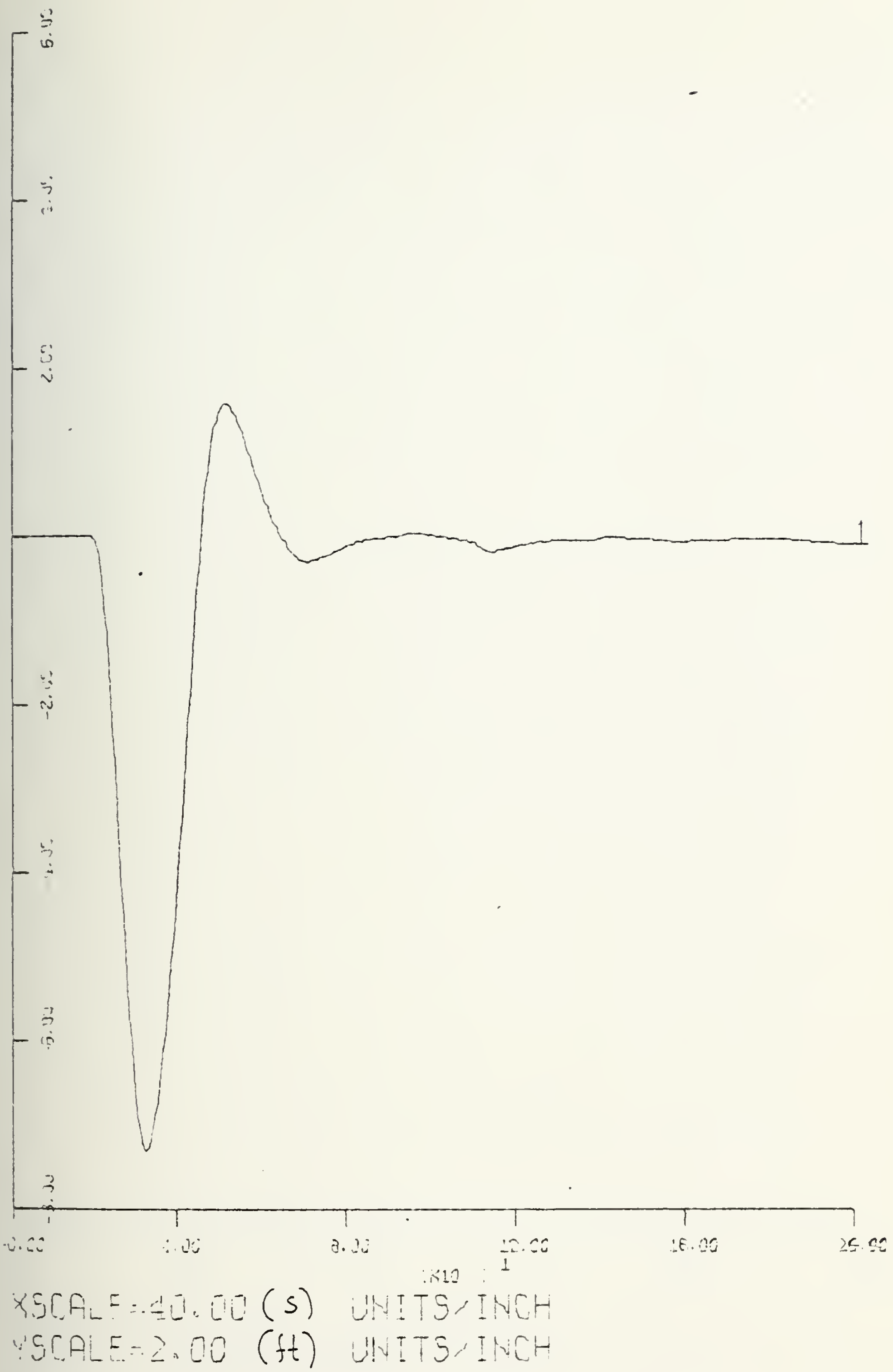
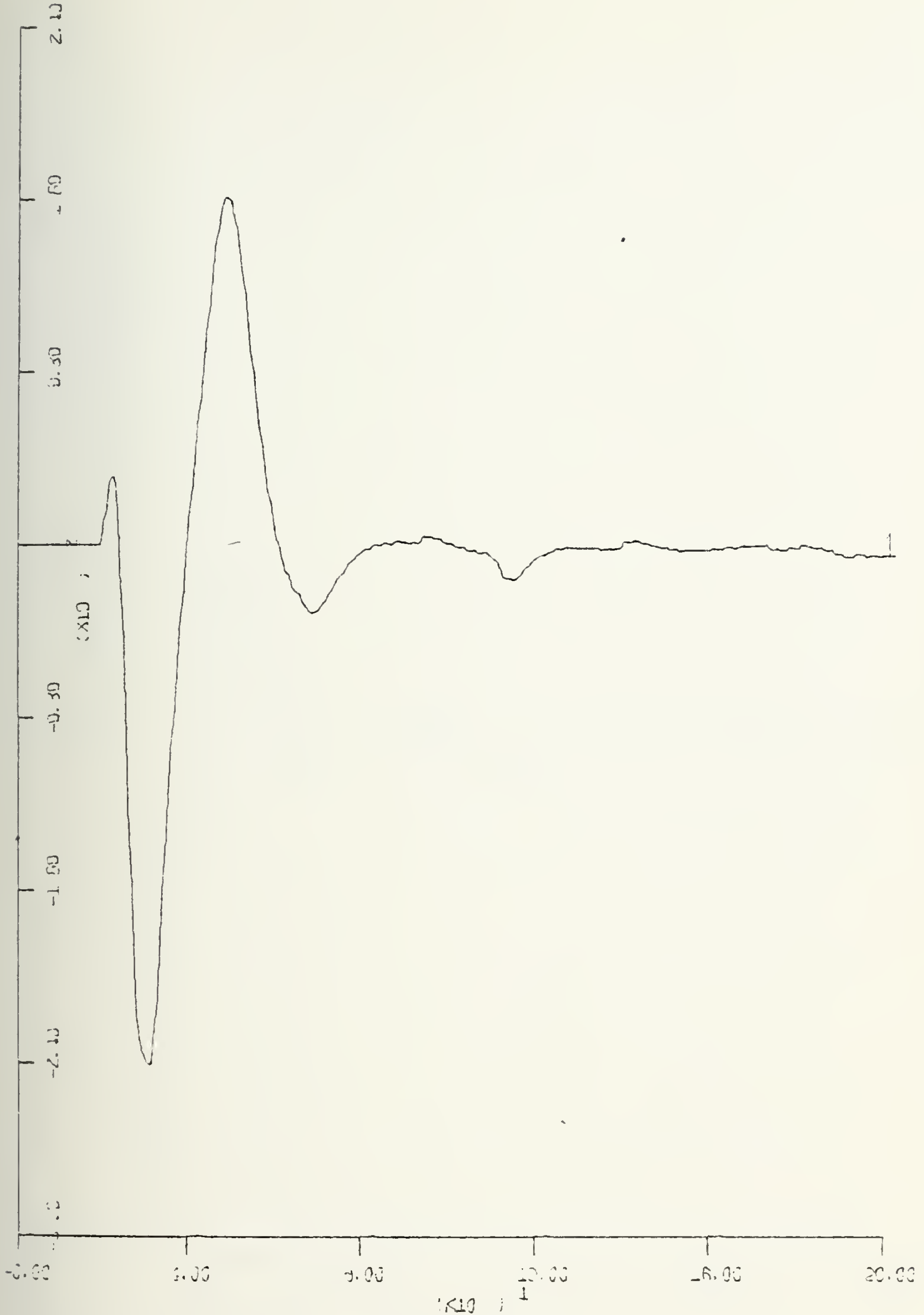
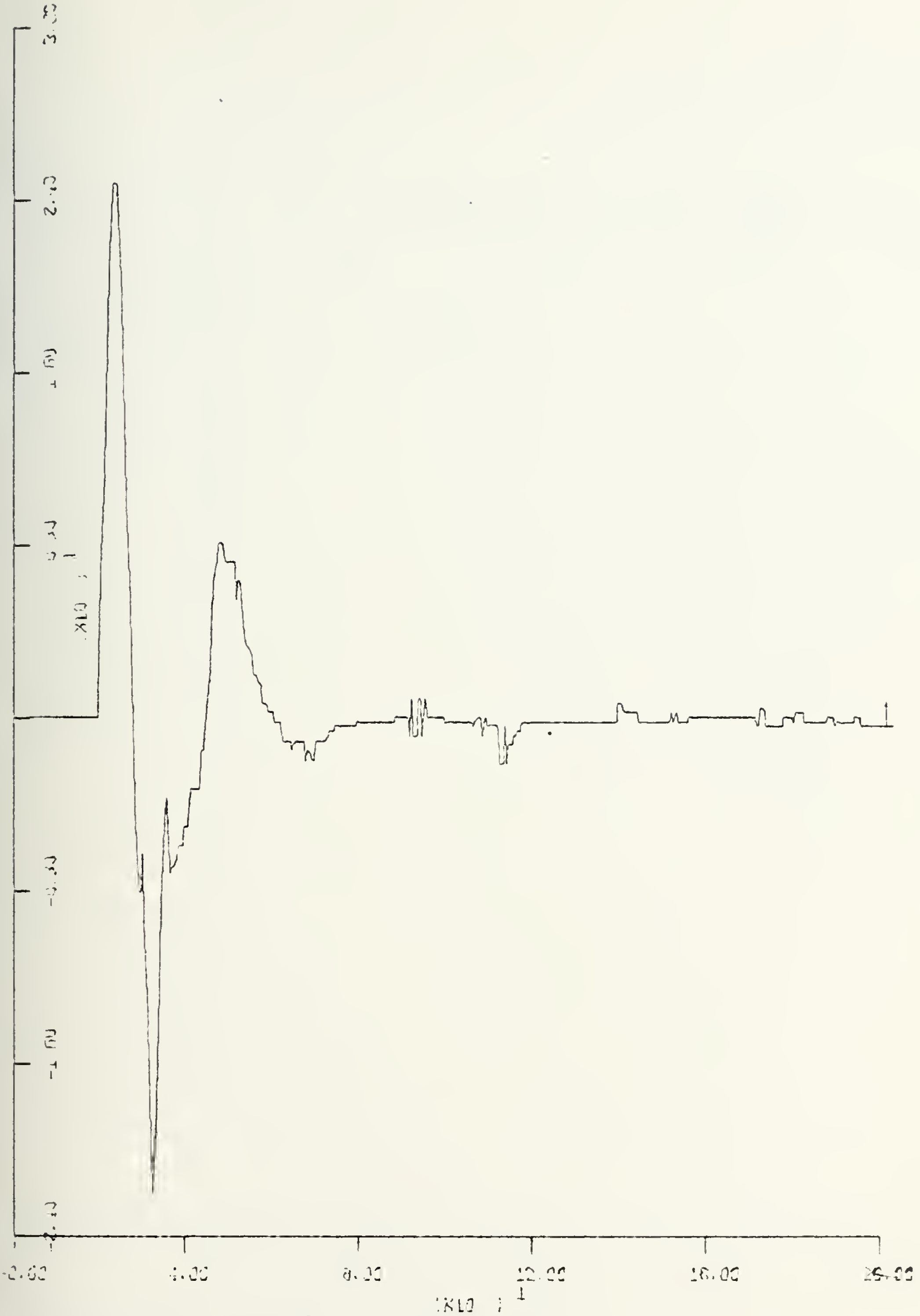


Fig. V-14a. Depth vs. Time. Response to a pulse force at FT. Both planes submarine with $\delta s_{com} = \delta s_1 \cdot f(k)$. X=-1.0



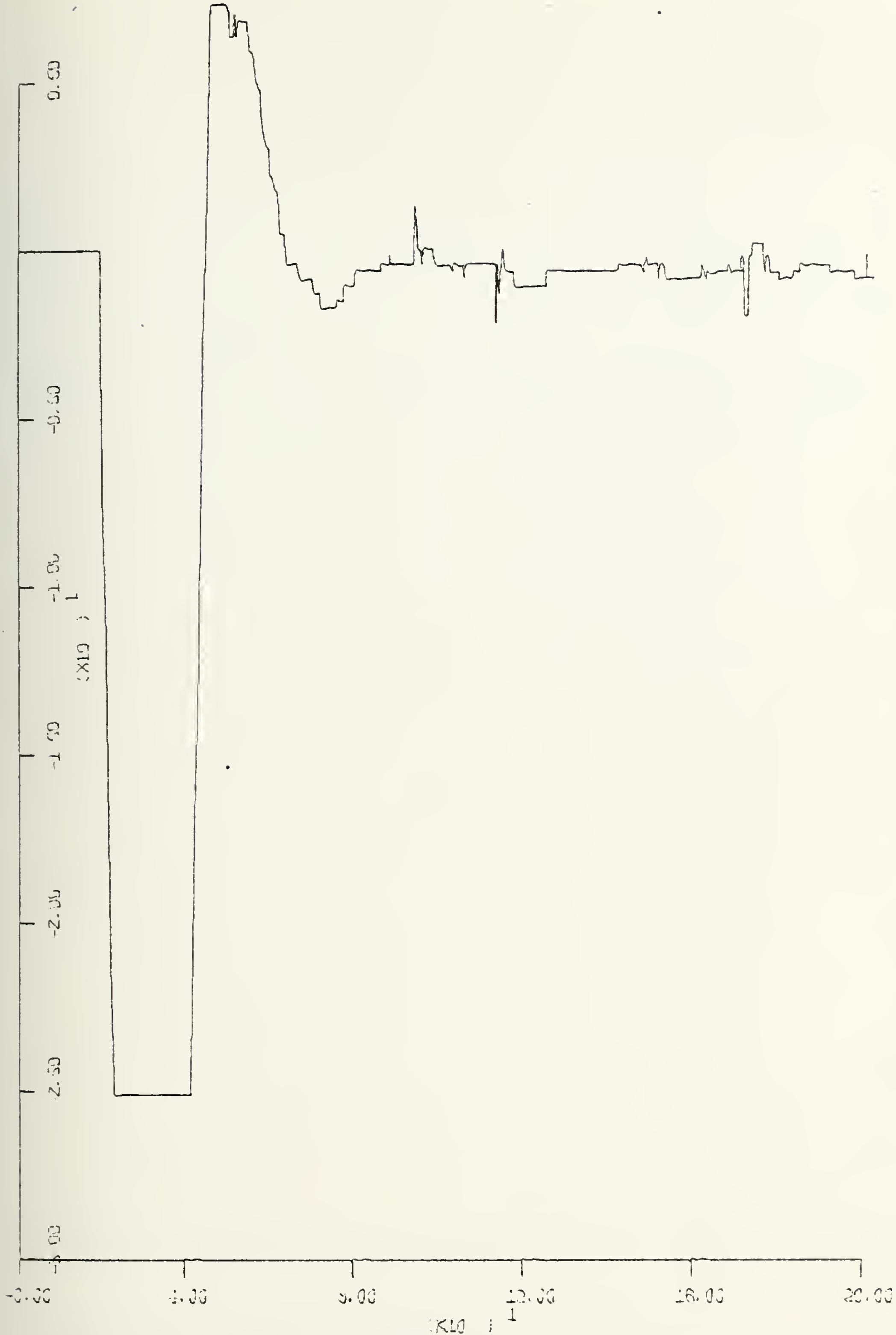
XSCALE=40.00 (s) UNITS/INCH
YSCALE= 8.00E-3(rad)UNITS/INCH

Fig. V-14b. Pitch vs. Time. Response to a pulse force at FT. Both planes submarine with $\delta s com = \delta s_1 \cdot f(x)$. X=-1.0



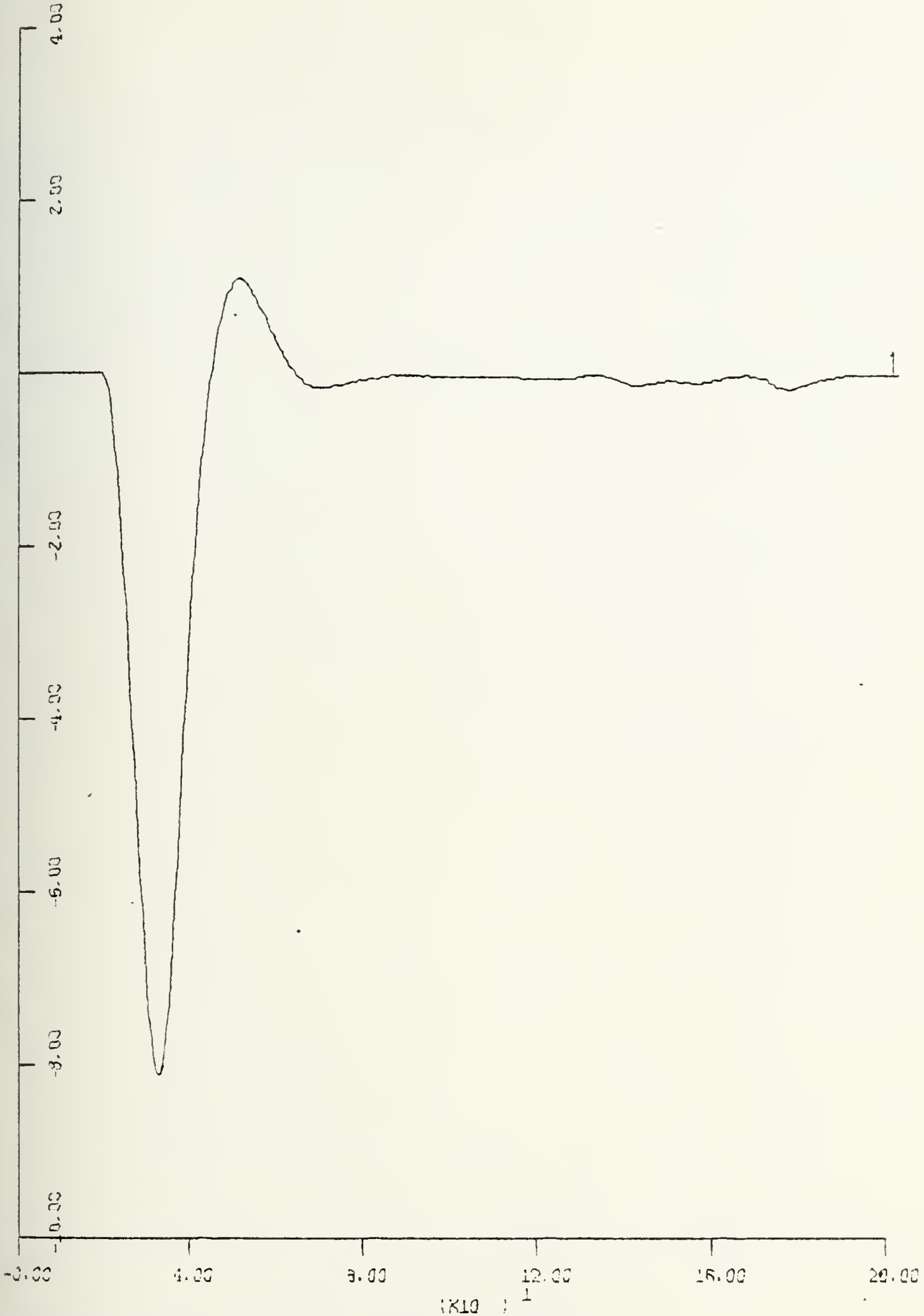
XSCALE=40.00 (s) UNITS/INCH
YSCALE=8.00 (deg) UNITS/INCH

Fig. V-14c. Stern Plane Angle vs. Time. Response to a pulse force at FT. Both planes submarine with $\delta s_{com} = \delta s_1 \cdot f(K)$. X=-1.0



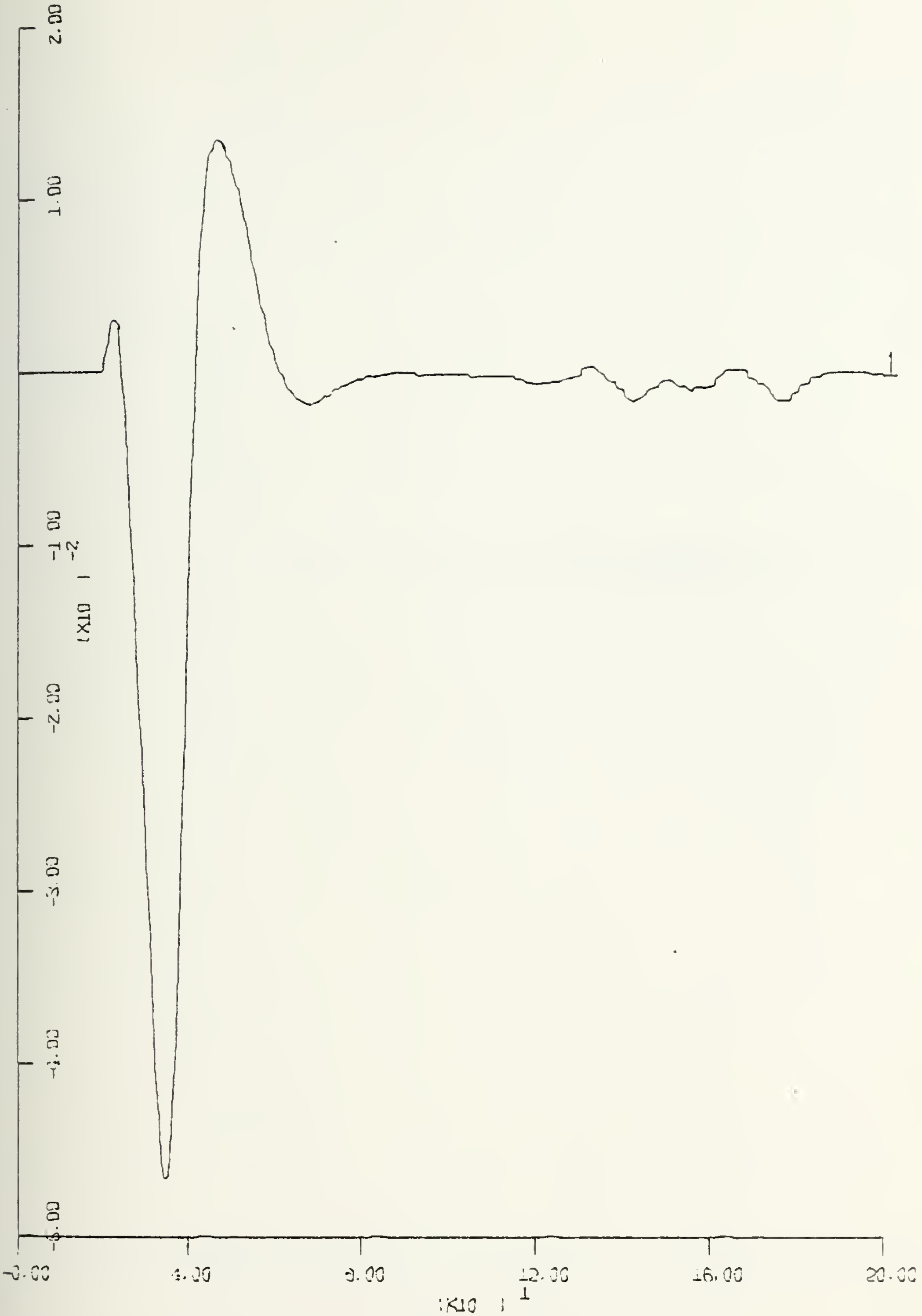
XSCALE=40.00 (s) UNITS/INCH
YSCALE=5.00 (deg) UNITS/INCH

Fig. V-14d. Fairwater Plane Angle vs. Time. Response to a pulse force at FT. Both planes submarine with $\delta_{scom} = \delta_{s1} \cdot f(k)$. X=-1.0



XSCALE=40.00 (s) UNITS/INCH
YSCALE=2.00 (ft) UNITS/INCH

Fig. V-15a. Depth vs. Time. CMC



XSCALE=40.00 (s) UNITS/INCH
YSCALE=0.01 (rad) UNITS/INCH

Fig. V-15b. Pitch vs. Time. CMC

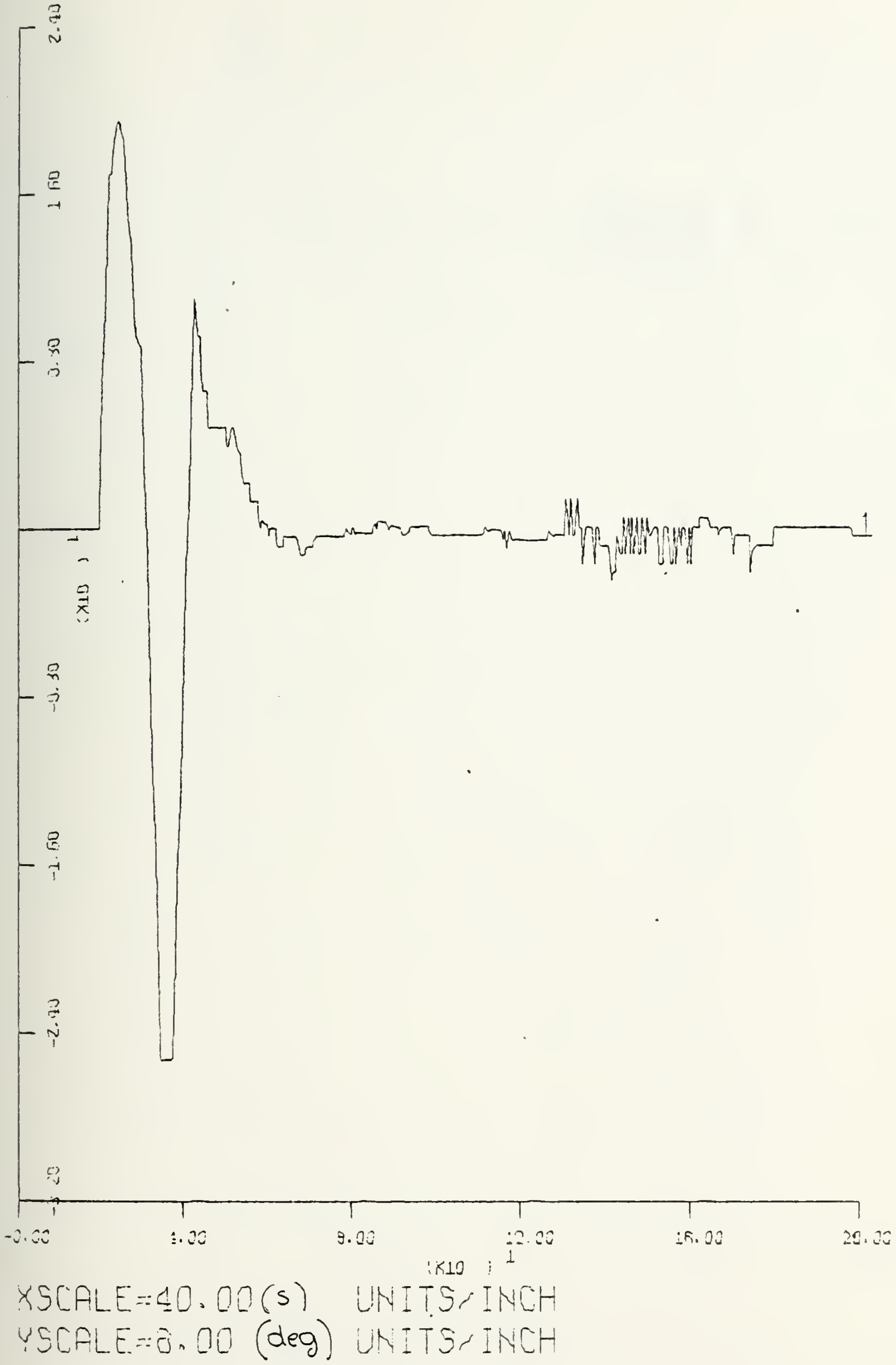


Fig. V-15c. Stern Plane Angle vs. Time. CMC

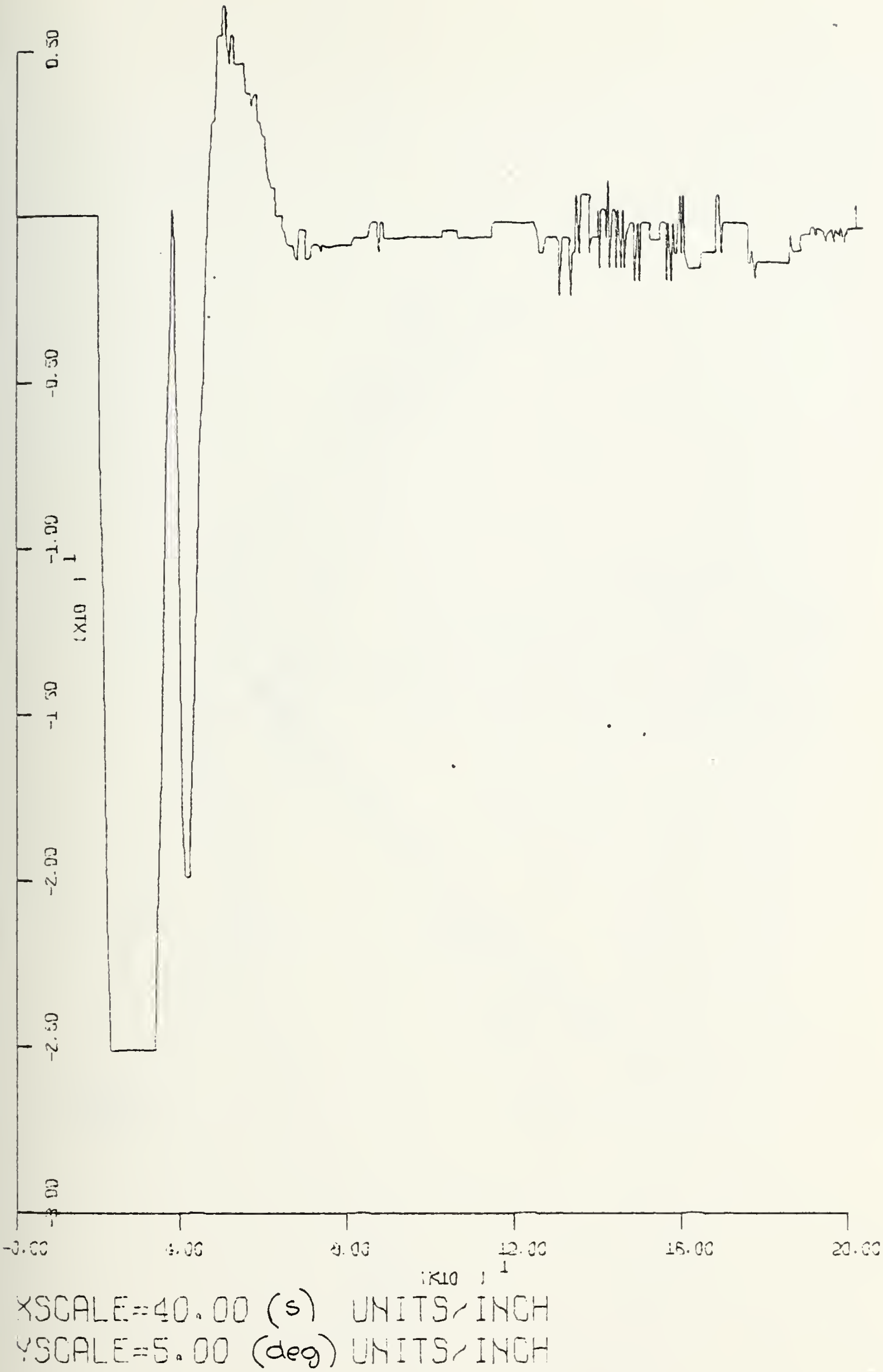


Fig. V-15d. Fairwater Plane Angle vs. Time. CMC

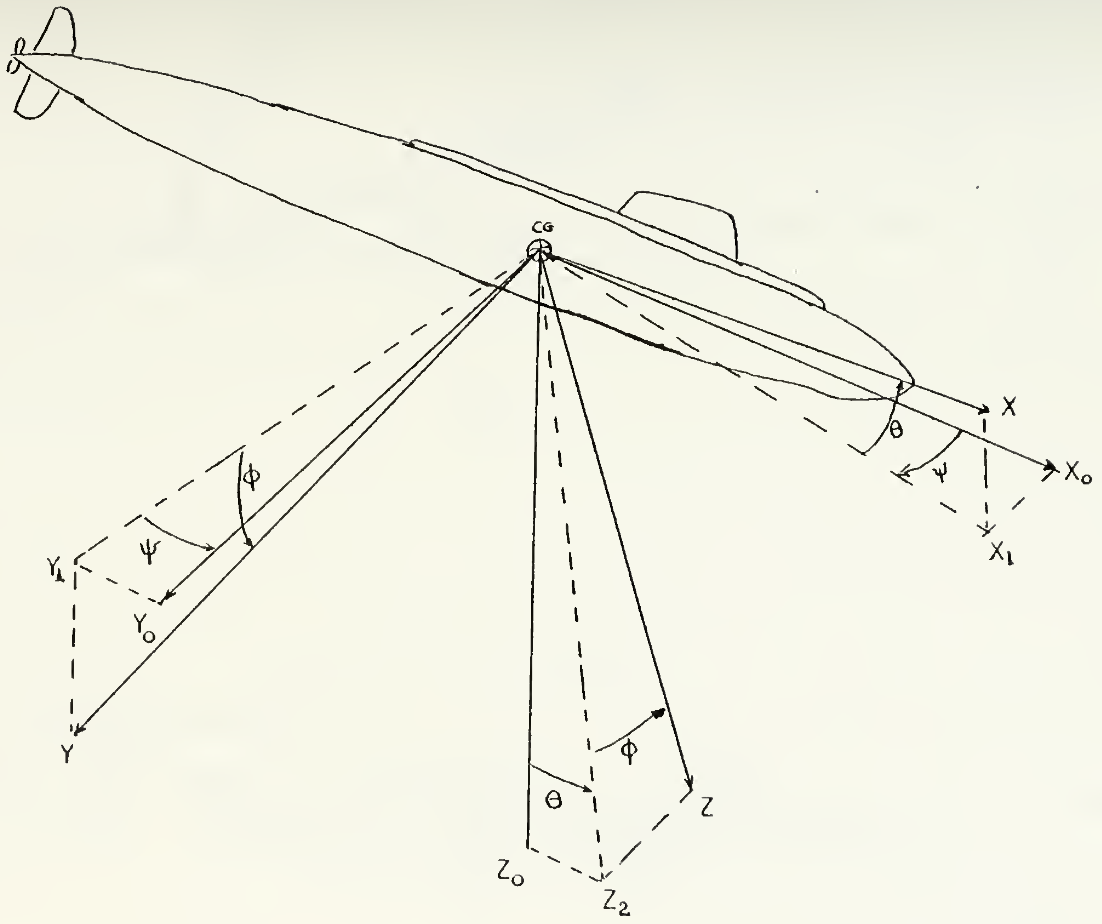


Fig. B-1a.Relation Between Body Axes and Fixed Axes

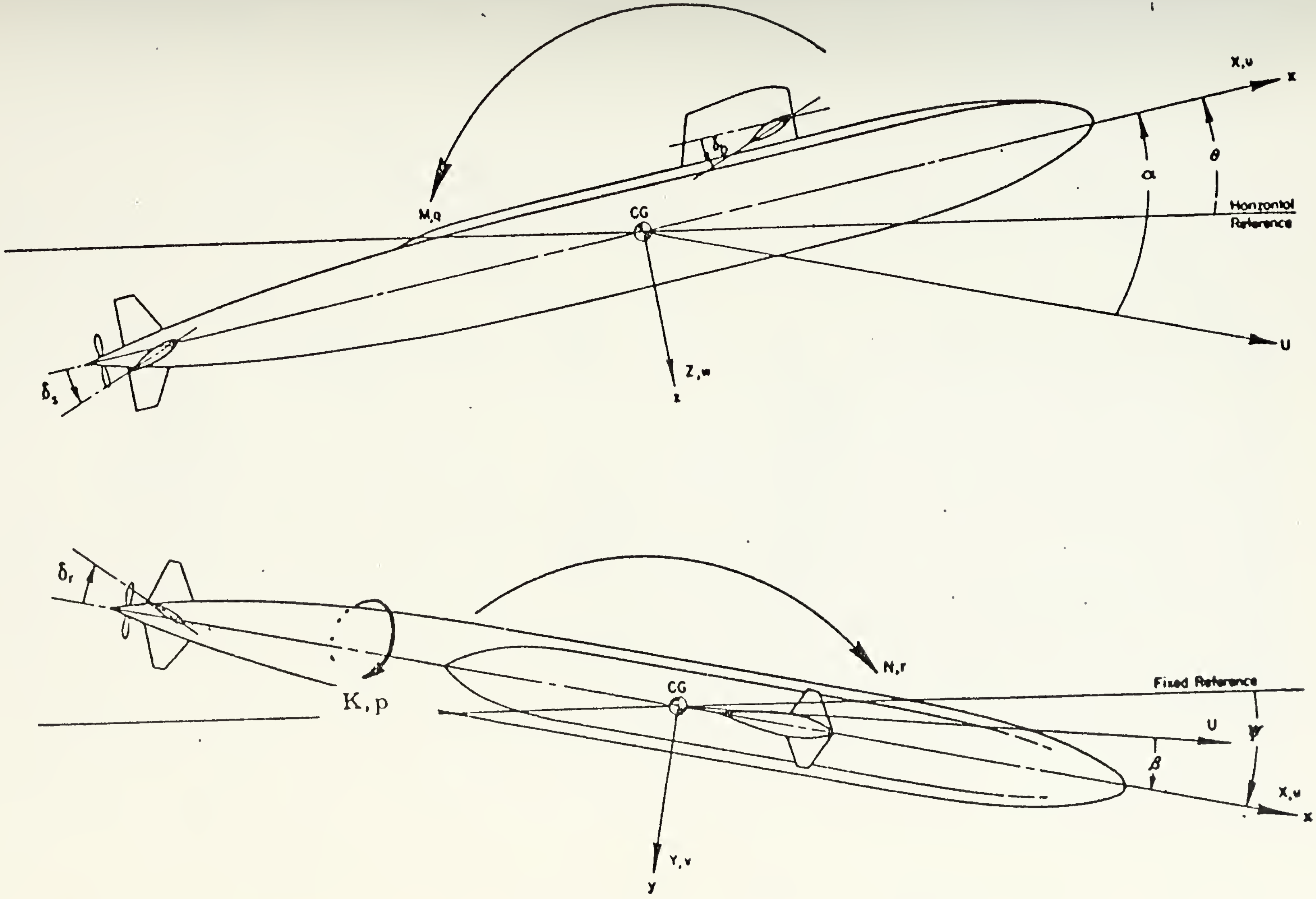


Figure B-1b. Submarine Axes

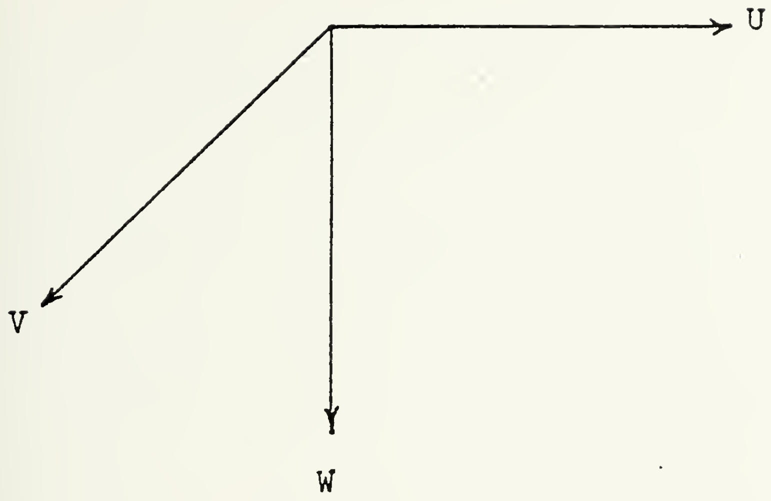


Figure B-1c.

Axes Definitions

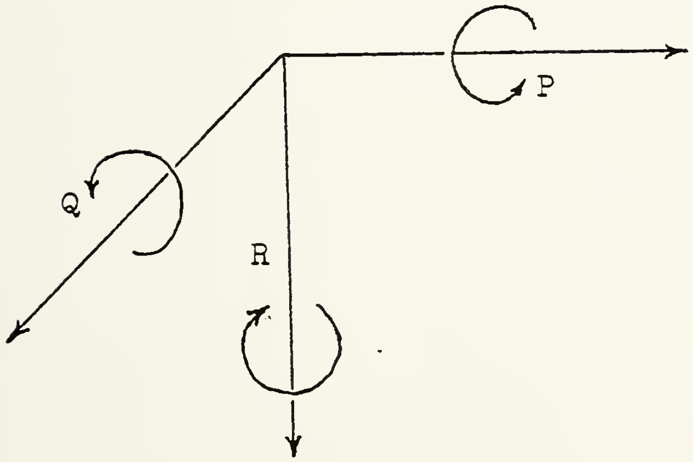
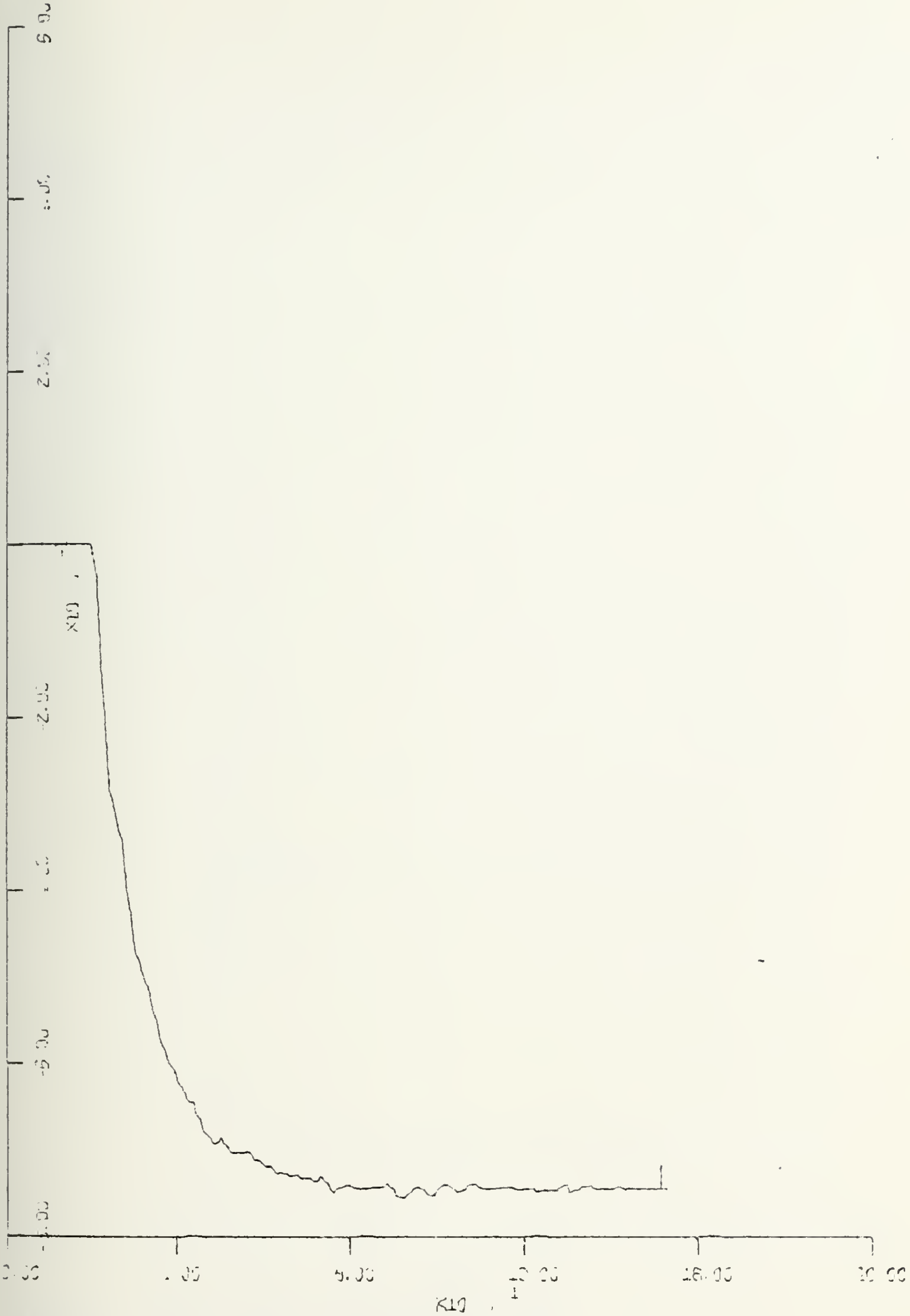


Figure B-1d.

Axes Definitions



XSCALE=40.00 (s) UNITS/INCH
YSCALE=0.20 (ft) UNITS/INCH

Fig. C-1a. Depth vs. Time. Stern planes only submarine, not "in trim" with feedback controller (SOA=-0.5, SOB=-7.03, SOC=32.5, SOD=360) and zero pitch and depth ordered.

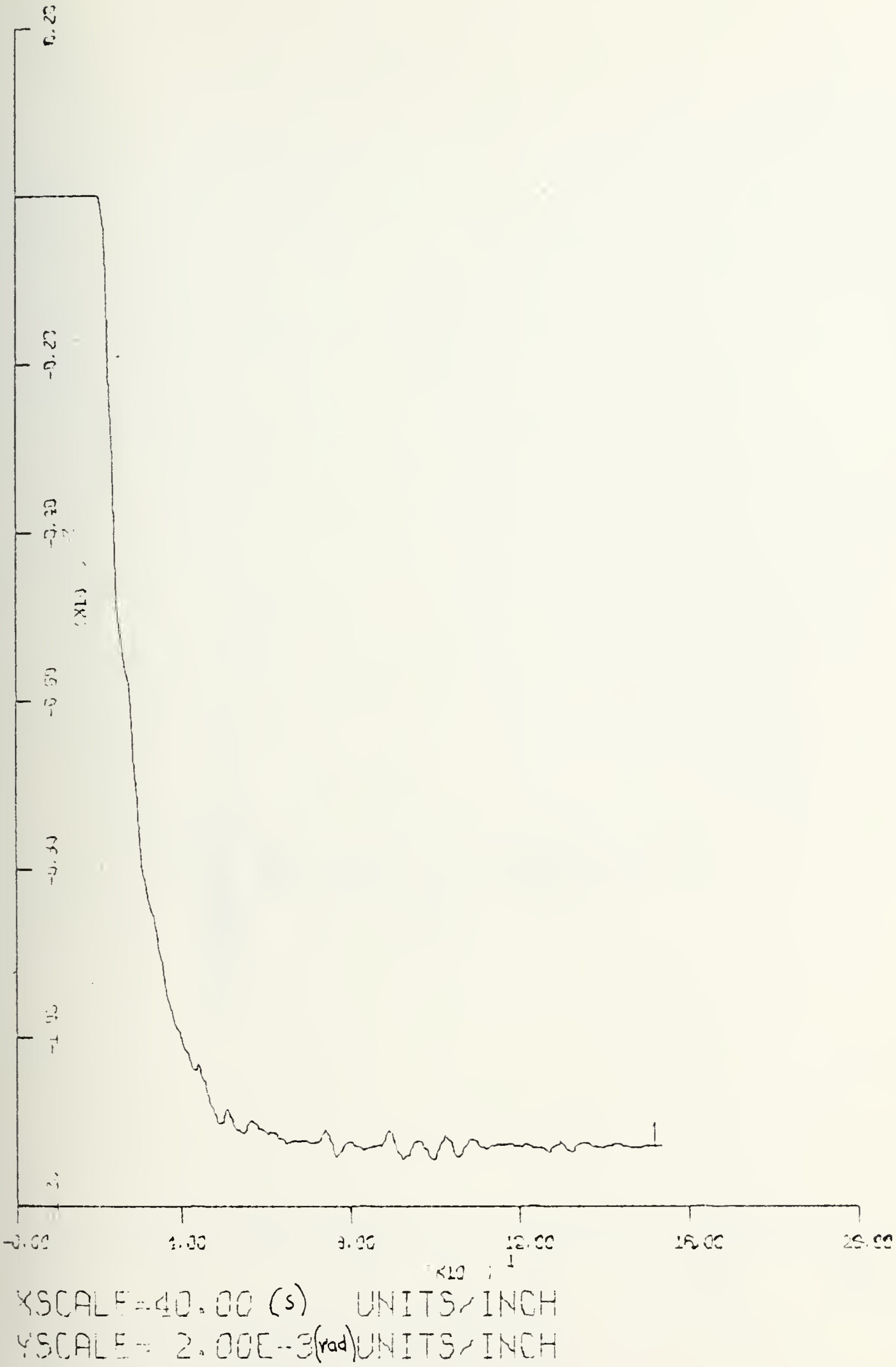


Fig. C-1b. Pitch vs. Time. Stern planes only submarine, not "in trim" with feedback controller (SOA=-0.5, SOB=-7.03, SOC=32.5, SOD=360) and zero pitch and depth ordered.

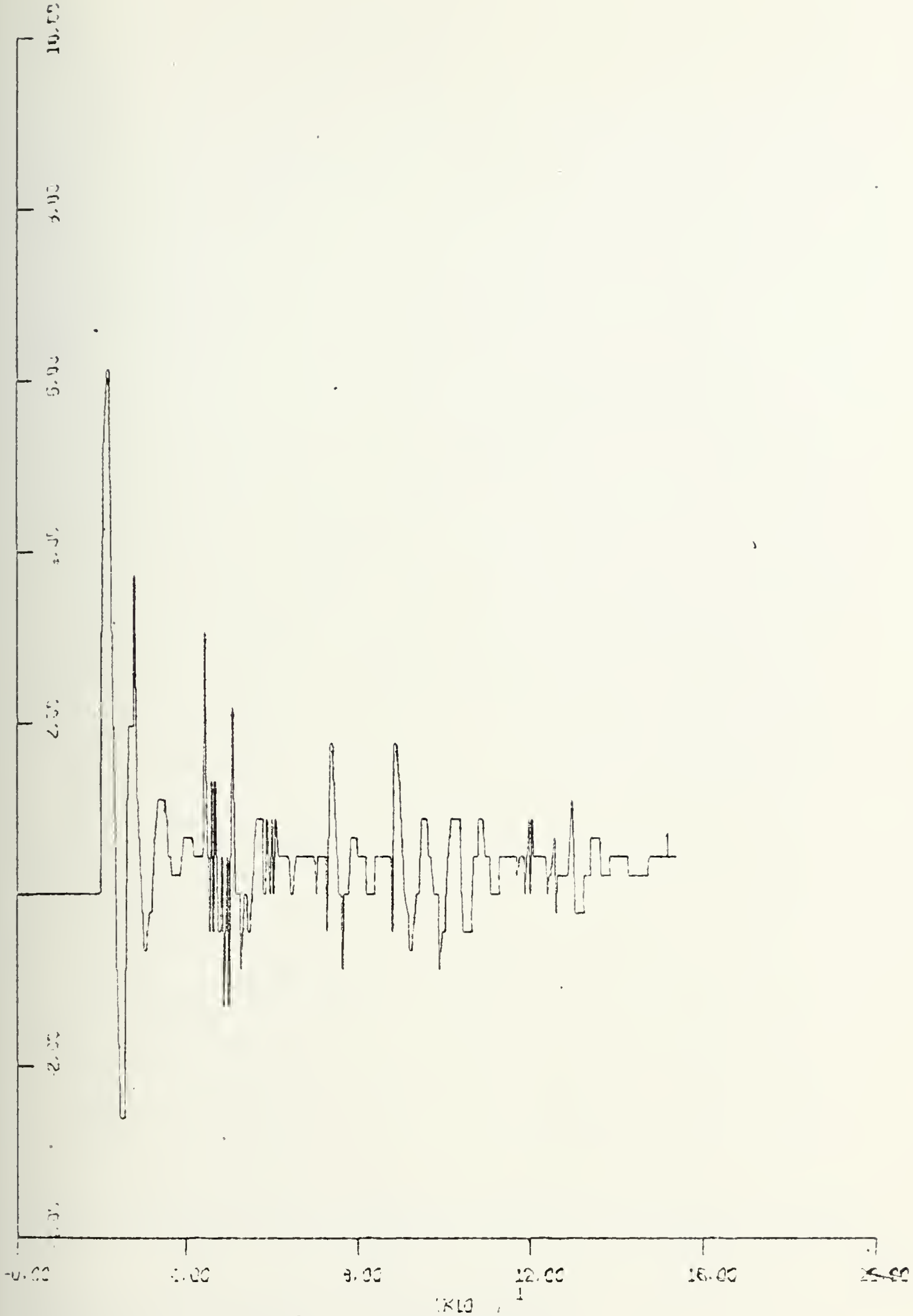


Fig. C-1c. Stern Plane Angle vs. Time. Stern planes only submarine, not "in trim" with feedback controller (SOA=-0.5, SOB=-7.03, SOC=32.5, SOD=360) and zero pitch and depth ordered.

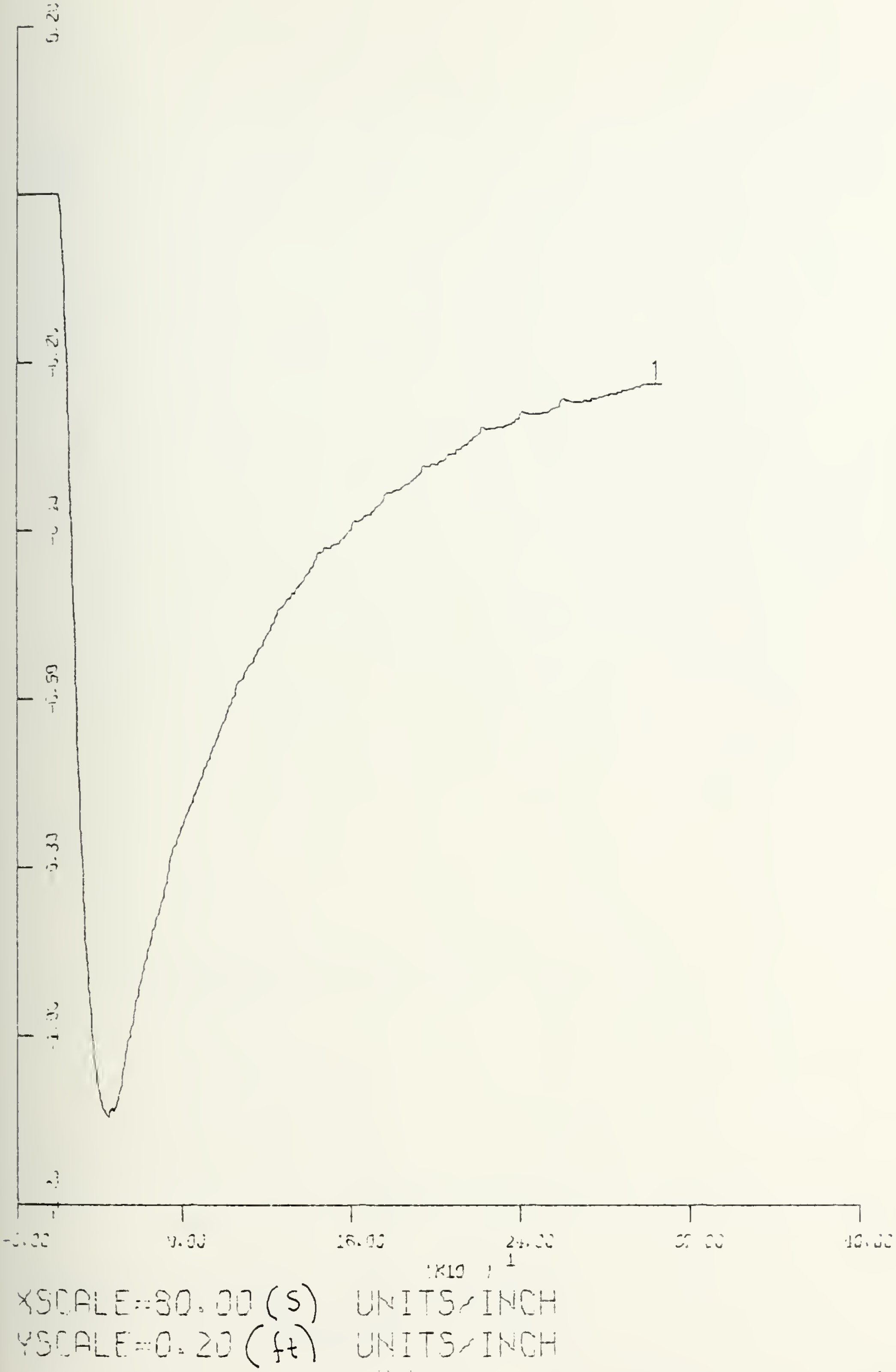


Fig. C-2a. Depth vs. Time. Stern planes only submarine, not "in trim" with feedback controller (SOA=-0.0436, SOB=-3.49, SOC=0.0523, SOD=360) and zero pitch and depth ordered.

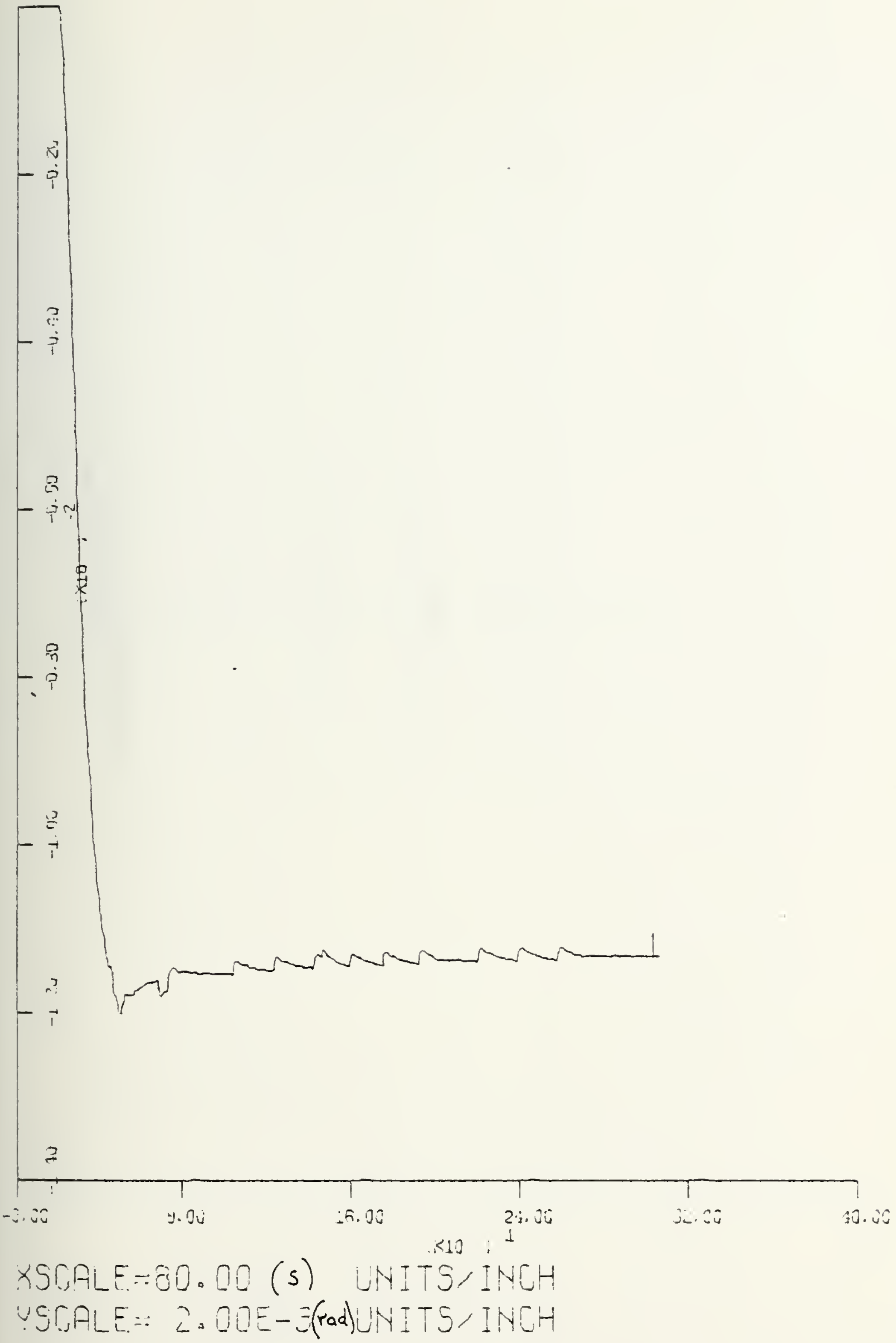
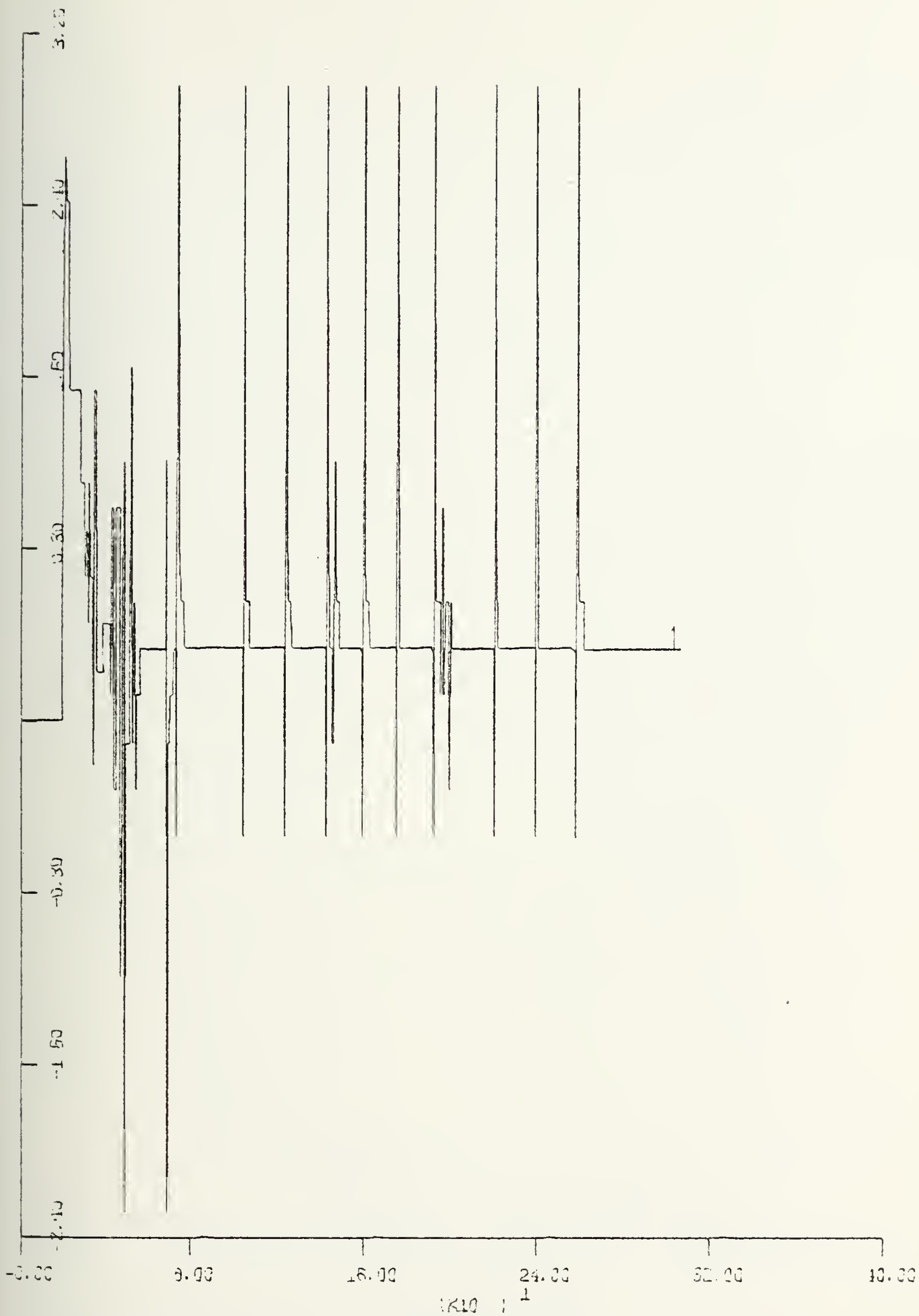


Fig. C-2b. Pitch vs. Time. Stern planes only submarine, not "in trim" with feedback controller (SOA=-0.0436, SOB=-3.49, SOC=0.0523, SOC=360) and zero pitch and depth ordered.



XSCALE=80.00 (s) UNITS/INCH
YSCALE=0.80 (deg) UNITS/INCH

Fig. C-2c. Stern Plane Angle vs. Time. Stern planes only submarine, not "in trim" with feedback controller (SOA=-0.0436, SOB=-3.49, SOC=0.0523, SOD=360) and zero pitch and depth ordered.

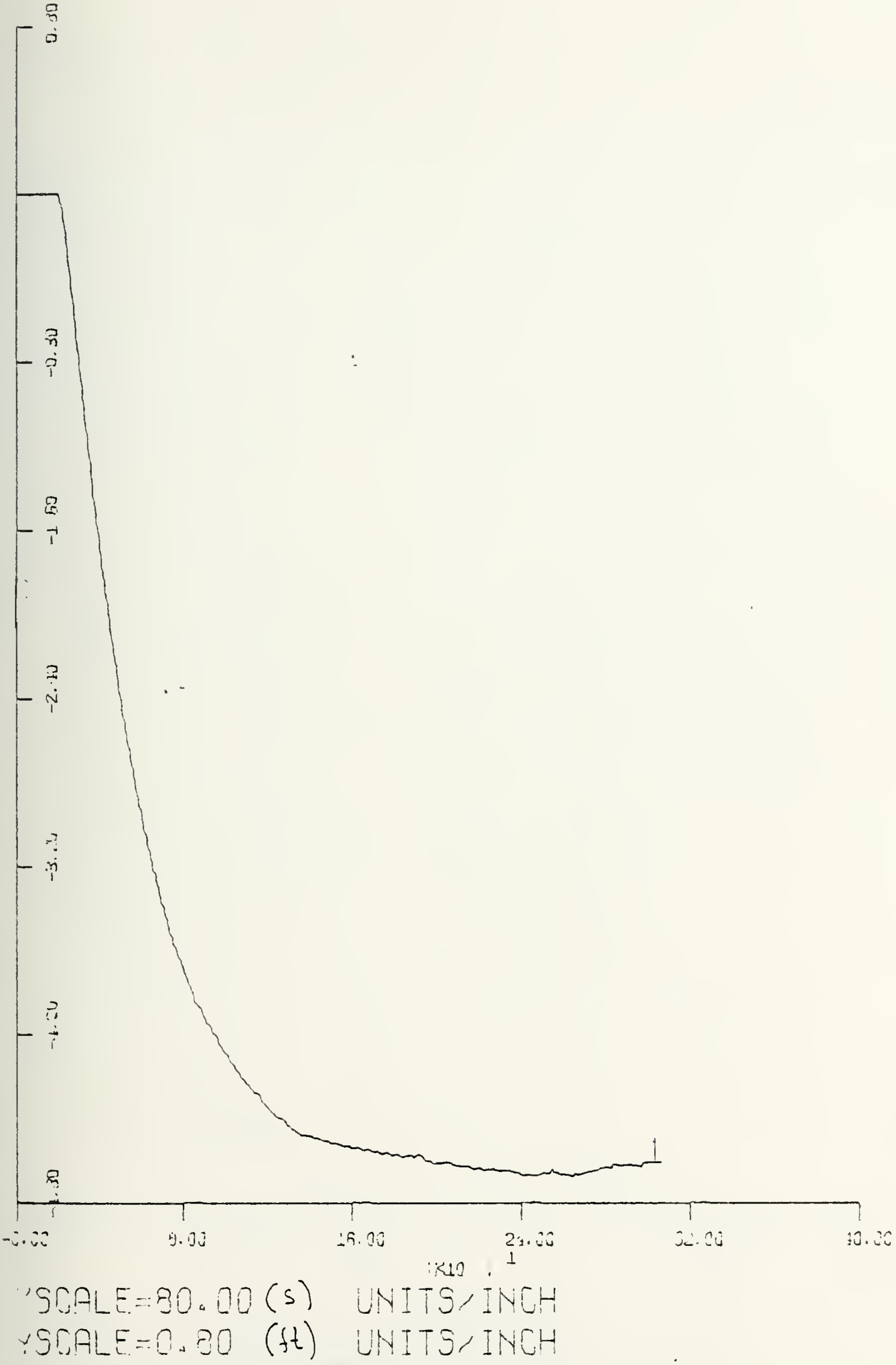


Fig. C-3a. Depth vs. Time. Stern planes only submarine, not "in trim" with feedback controller (SOA=-0.0656, SOB=-1.47, SOC=26.2, SOD=184.1) and zero pitch and depth ordered.

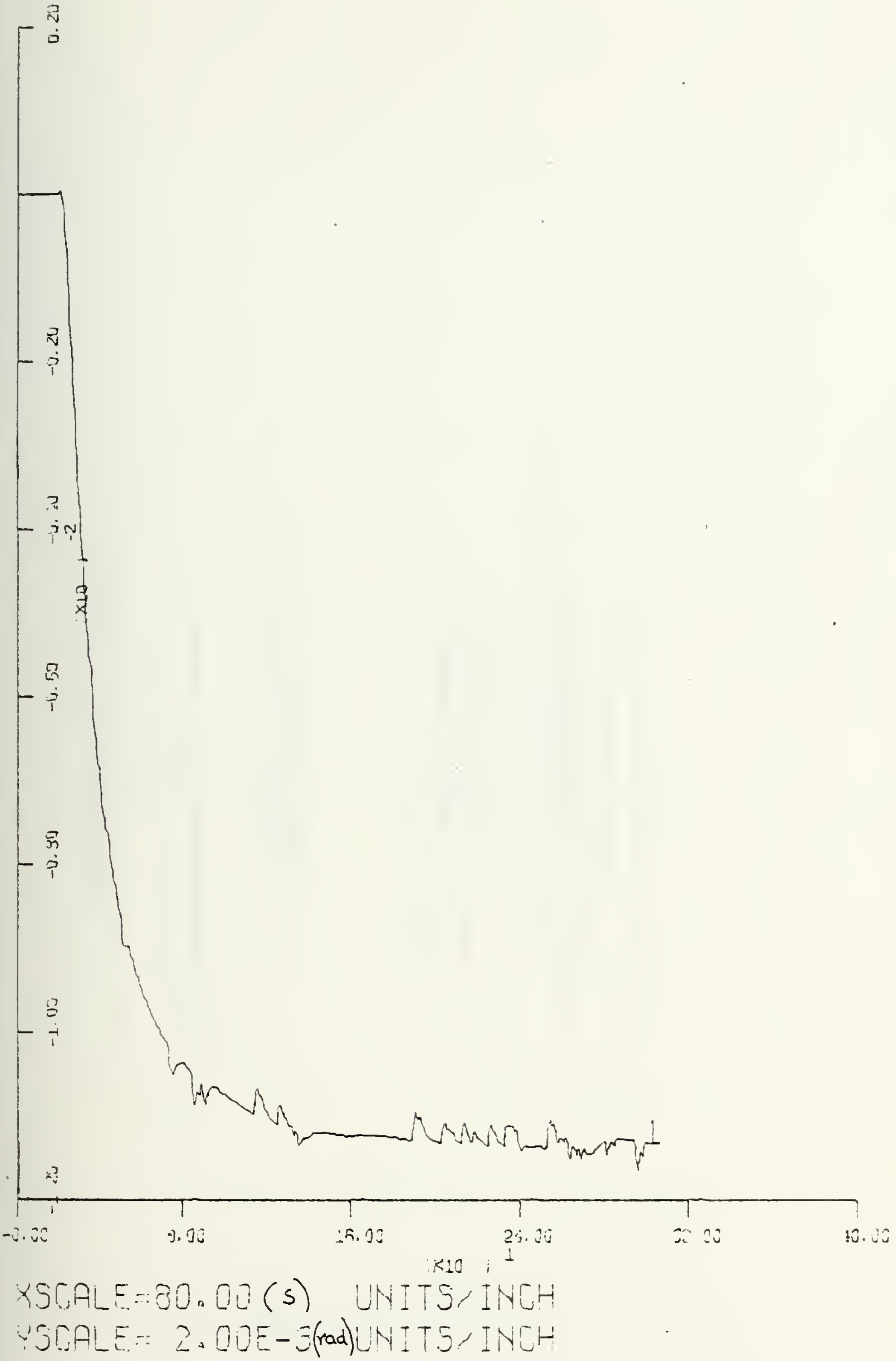


Fig. C-3b. Pitch vs. Time. Stern planes only submarine, not "in trim" with feedback controller (SOA=-0.0656, SOB=-1.47, SOC=26.2, SOD=184.1) and zero pitch and depth ordered.

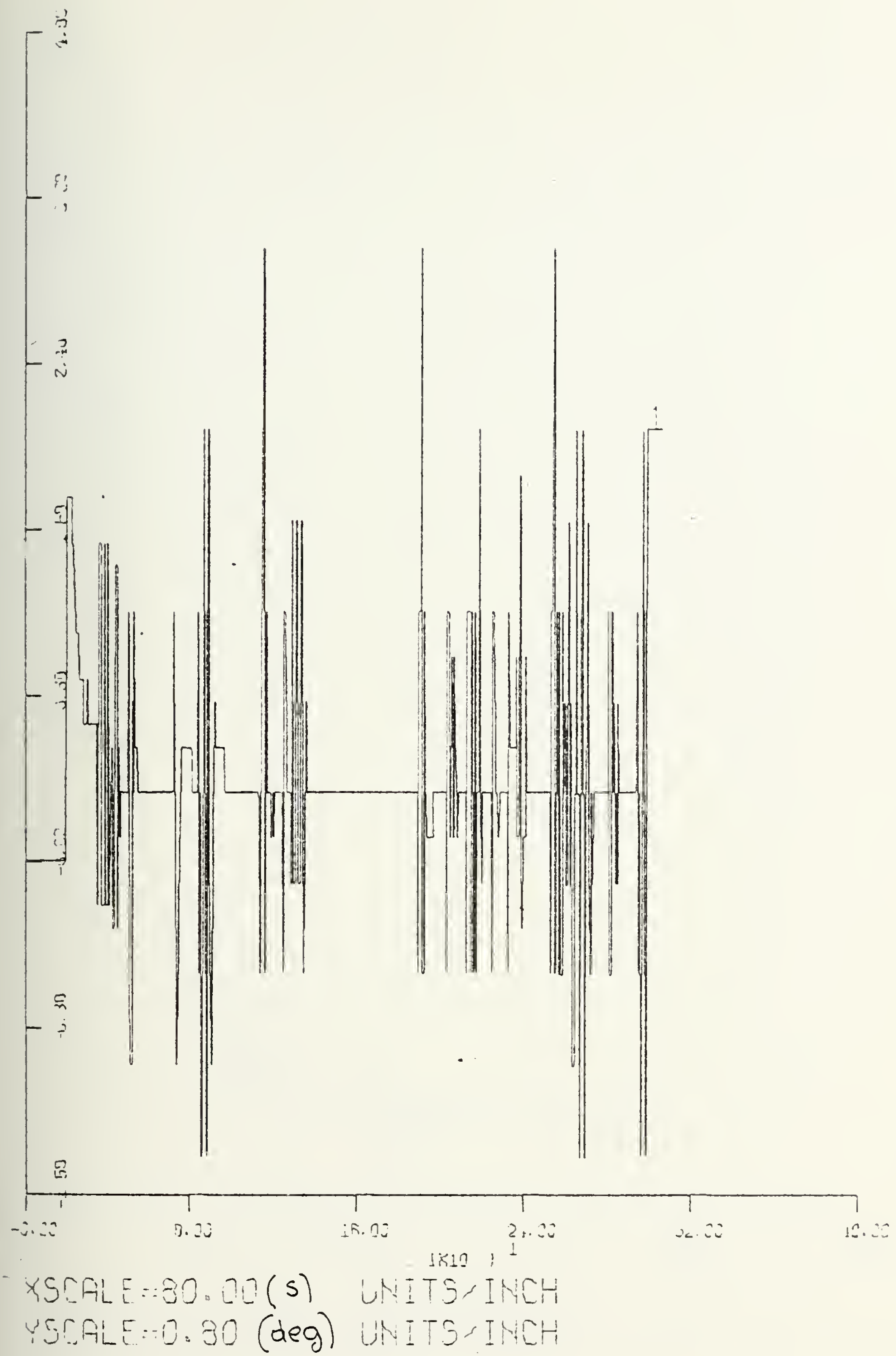


Fig. C-3c. Stern Plane Angle vs. Time. Stern planes only submarine, not "in trim" with feedback controller (SOA =-0.0656, SOB=-1.47, SOC=26.2, SOD=184.1) and zero pitch and depth ordered.

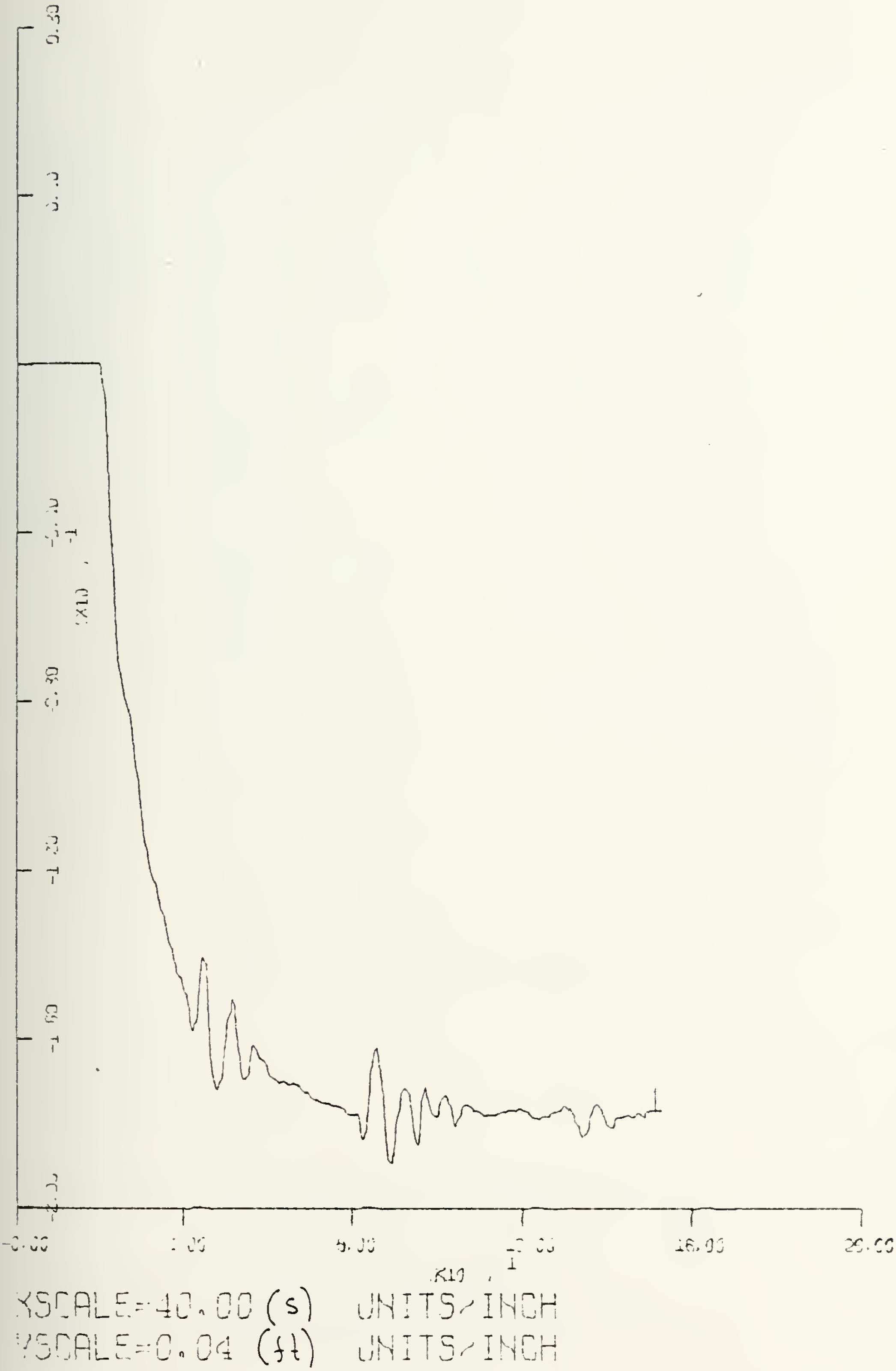


Fig. C-4a. Depth vs. Time. Stern planes only submarine, "in-trim" with feedback controller (SOA=-0.5, SOB=-7.03, SOC=32.5, SOD=360) and zero pitch and depth ordered.

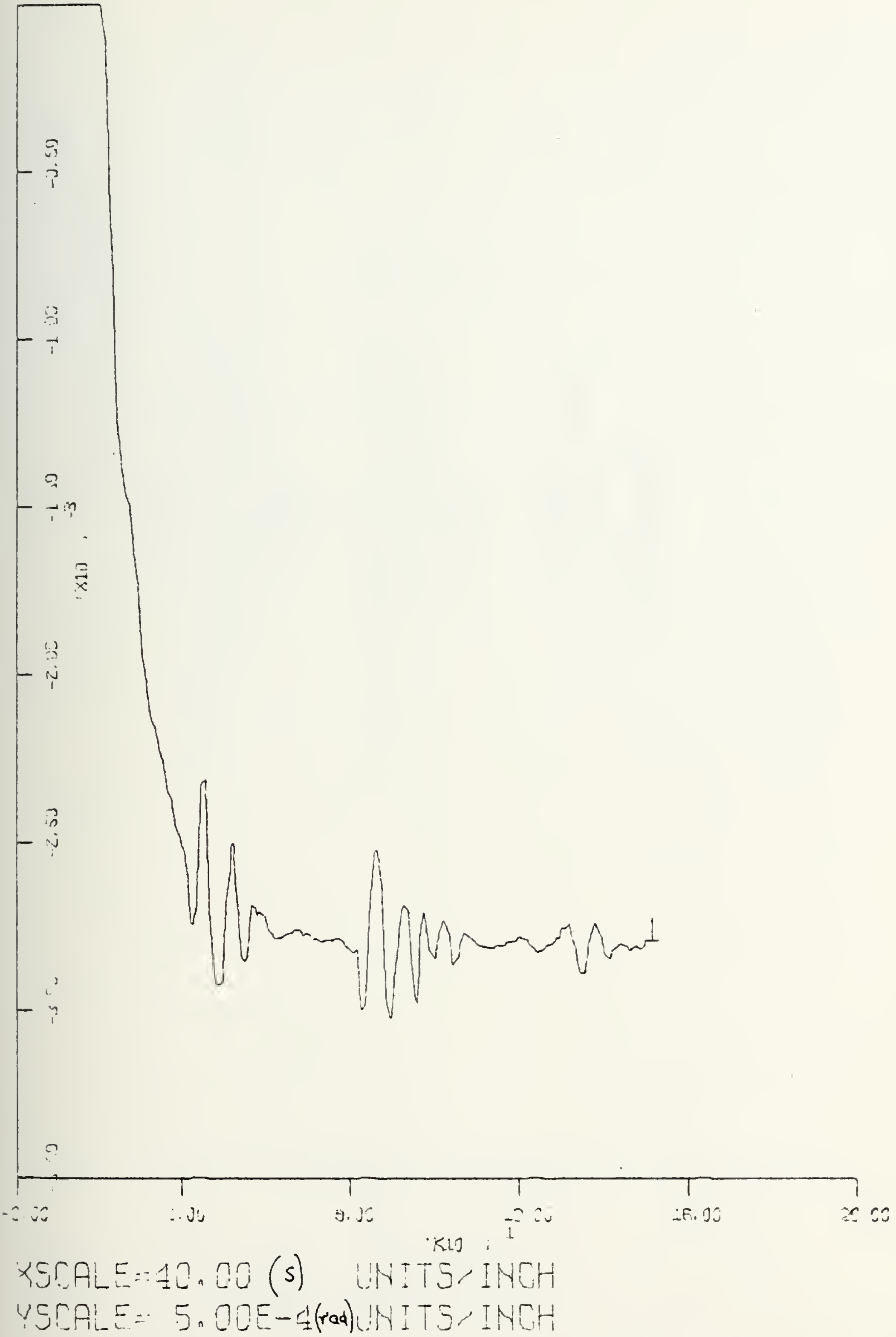


Fig. C-4b. Pitch vs. Time. Stern planes only submarine, "in trim" with feedback controller (SOA=-0.5, SOB=-7.03, SOC=32.5, SOD=360) and zero pitch and depth ordered.

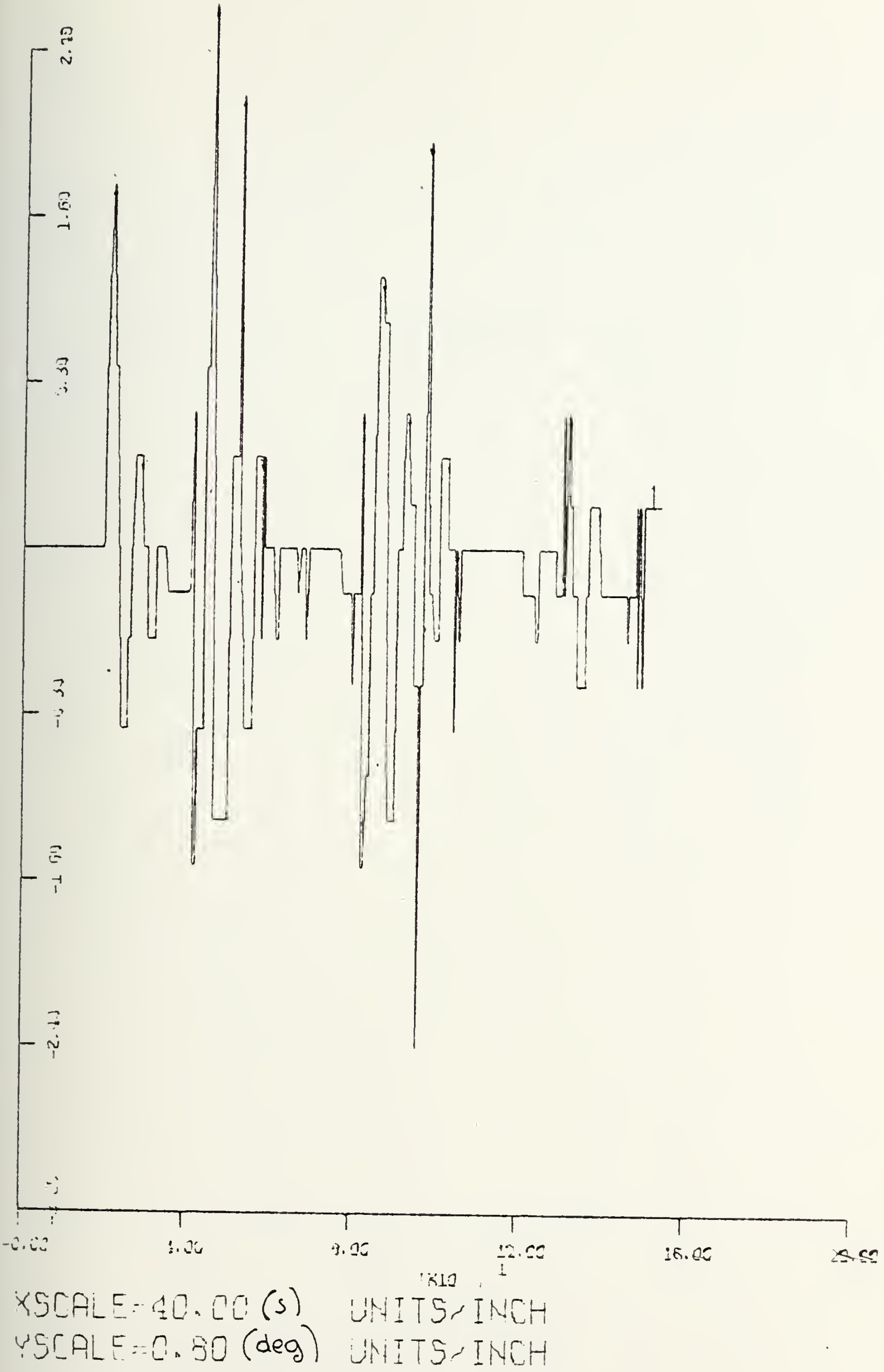


Fig. C-4c. Stern Plane Angle vs. Time. Stern planes only submarine, "in trim" with feedback controller (SOA=-0.5, SOB=-7.03, SOC=32.5, SOD=360) and zero pitch and depth ordered.

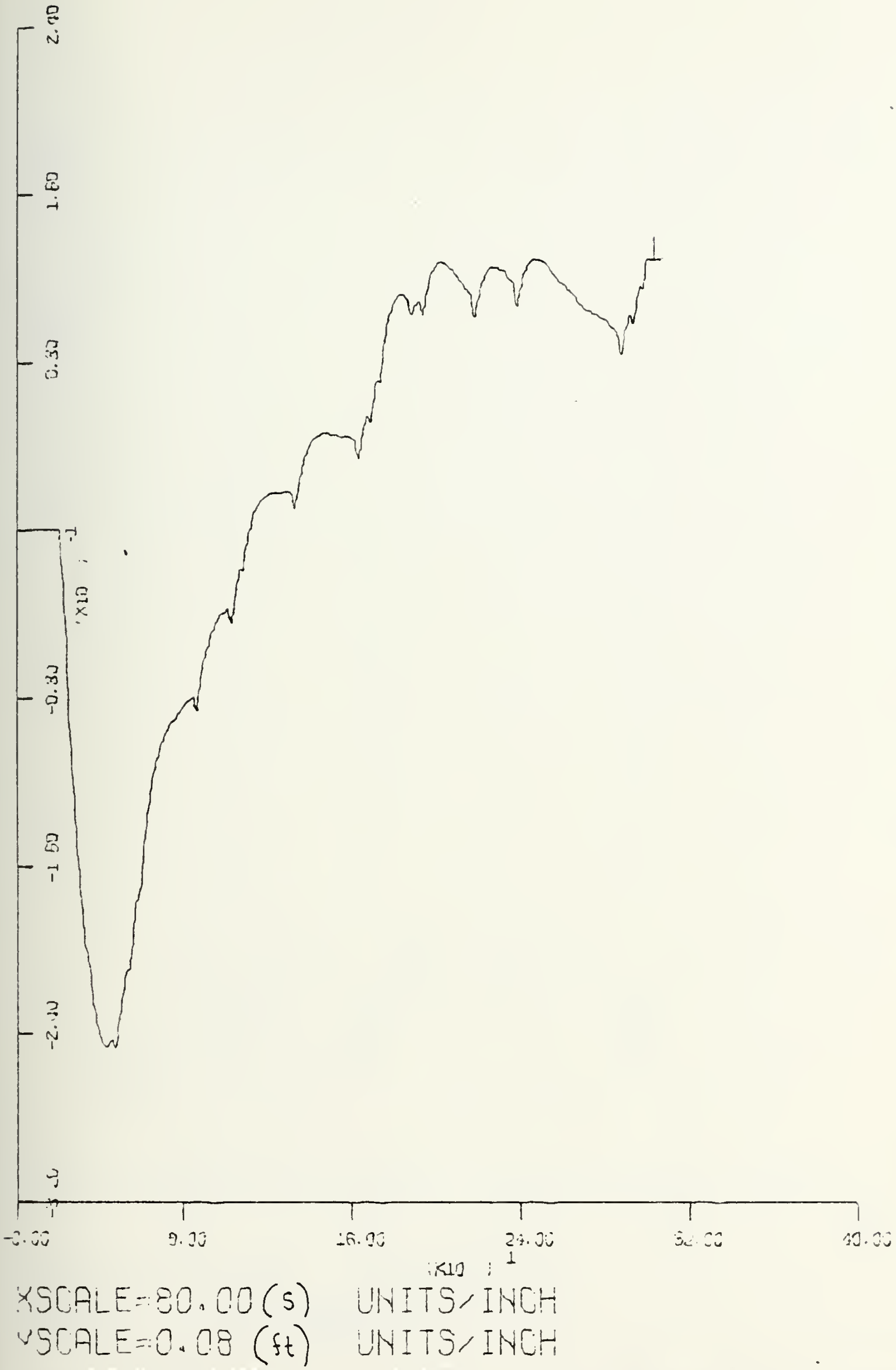


Fig. C-5a. Depth vs. Time. Stern planes only submarine, "in trim" with feedback controller (SOA=-0.0436, SOB=-3.49, SOC=0.0523, SOD=360) and zero pitch and depth ordered.

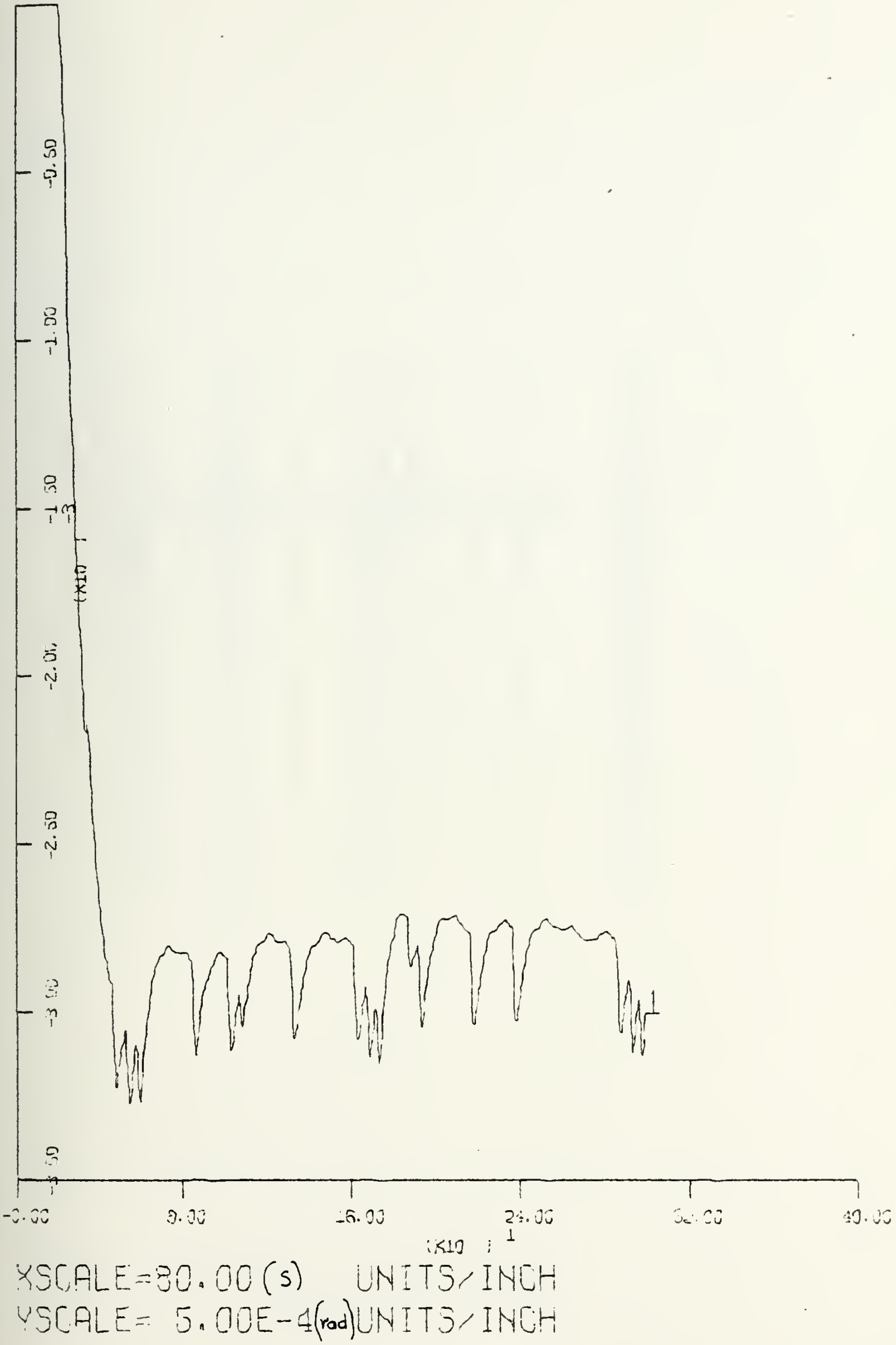
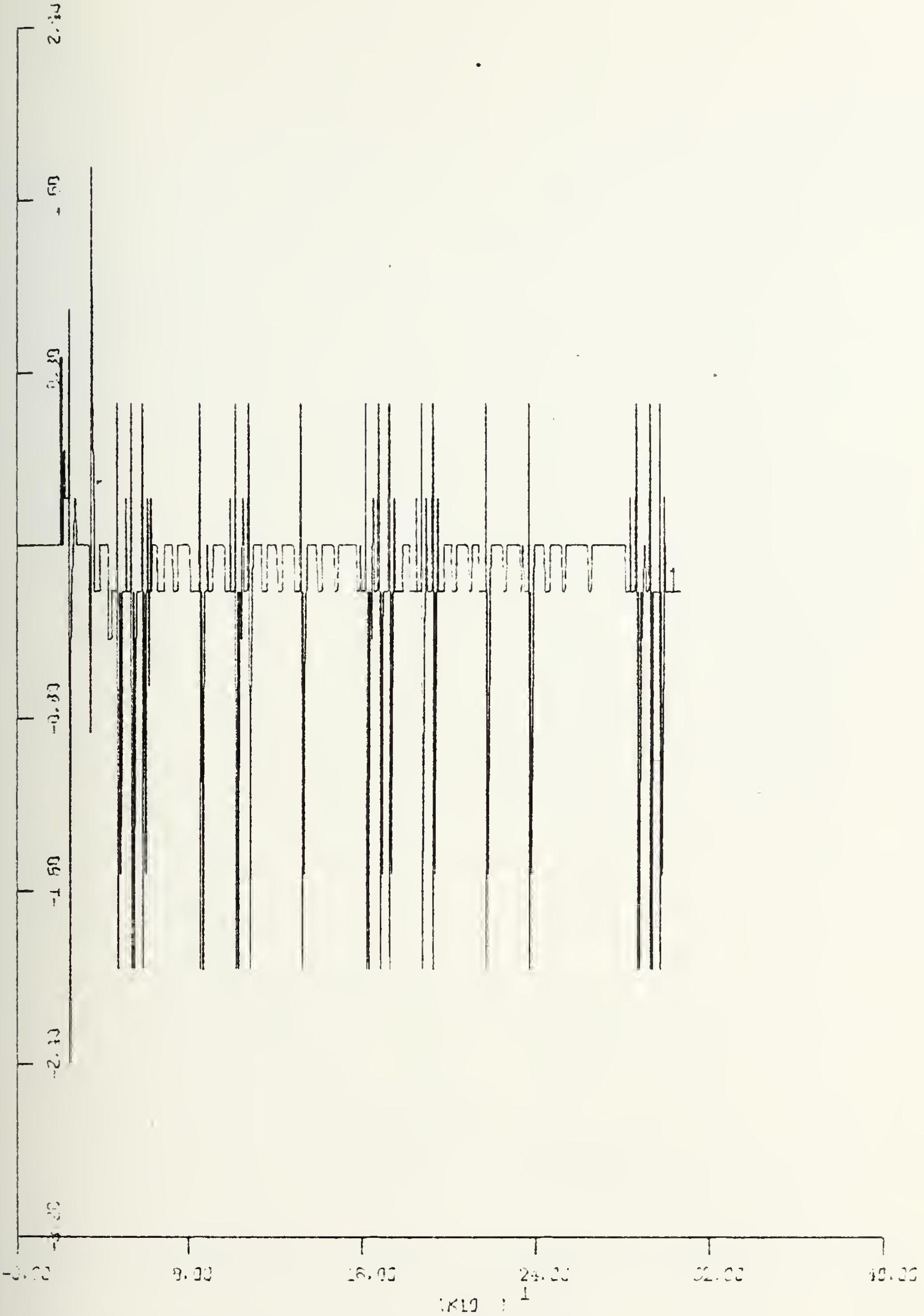


Fig. C-5b. Pitch vs. Time. Stern planes only submarine, "in trim" with feedback controller (SOA=-0.0436, SOB=-3.49, SOC=0.0523, SOD=360) and zero pitch and depth ordered.



XSCALE=80.00 (s) UNITS/INCH
YSCALE=0.80 (deg) UNITS/INCH

Fig. C-5c. Stern Plane Angle vs. Time. Stern planes only submarine, "in trim" with feedback controller (SOA=-0.0436, SOB=-3.49, SOC=0.0523, SOD=360) and zero pitch and depth ordered.

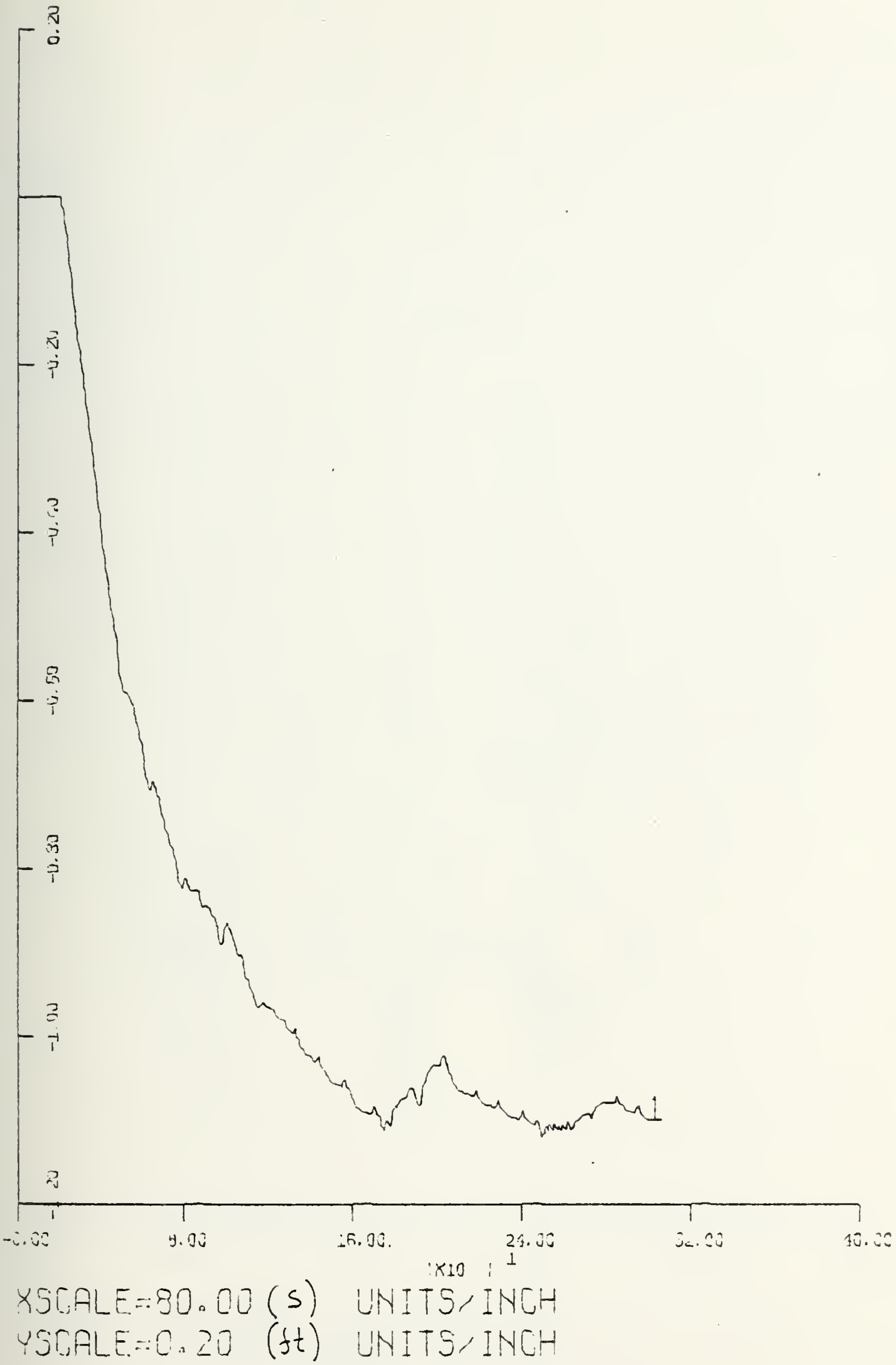


Fig. C-6a. Depth vs. Time. Stern planes only submarine, "in trim" with feedback controller (SOA=-0.0656, SOB=-1.47, SOC=26.2, SOD=184.1) and zero pitch and depth ordered.

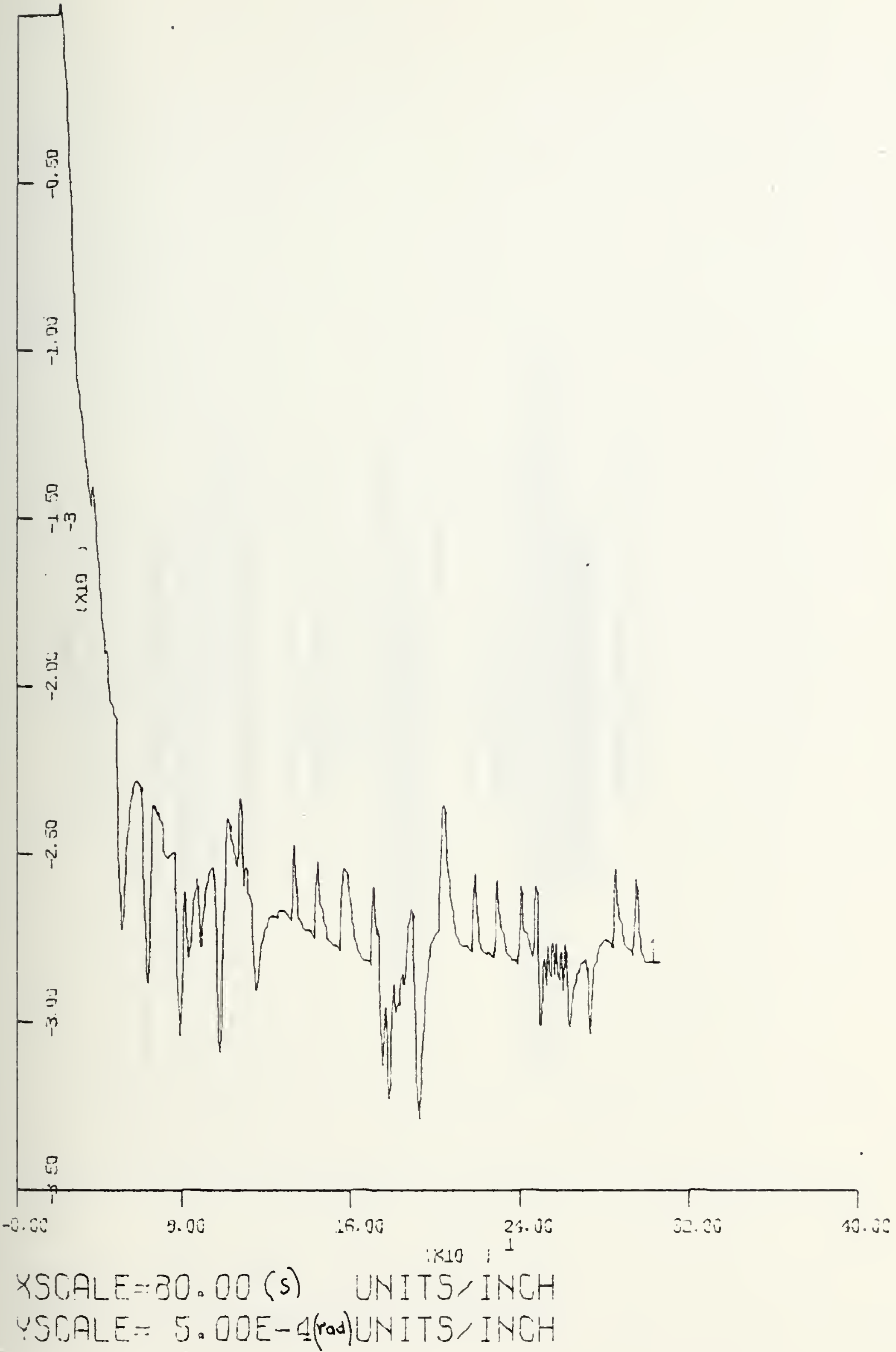


Fig. C-6b. Pitch vs. Time. Stern planes only submarine, "in trim" with feedback controller (SOA=-0.0656, SOB=-1.47, SOC=26.2, SOD=184.1) and zero pitch and depth ordered.

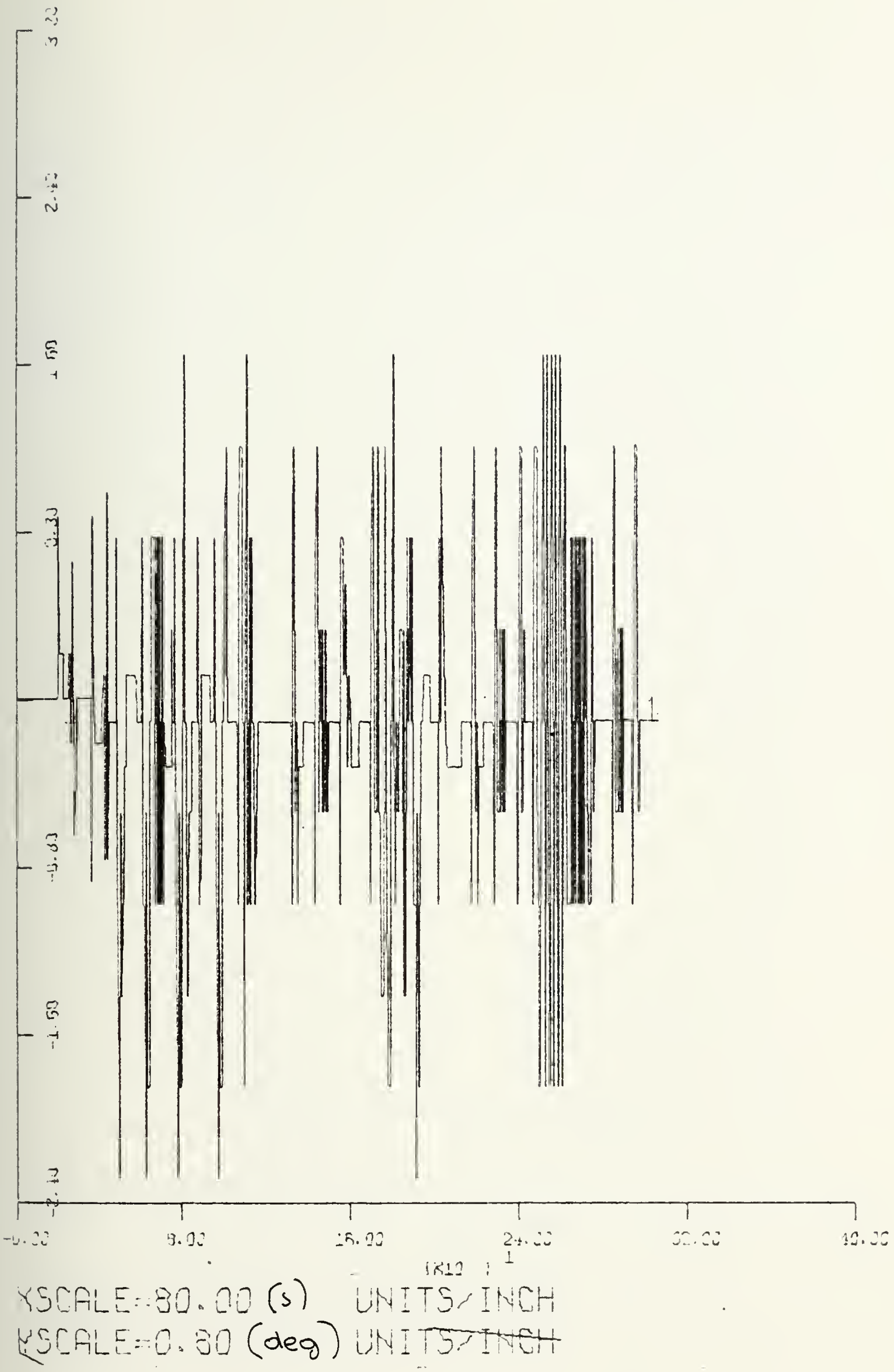


Fig. C-6c. Stern Plane Angle vs. Time. Stern planes only submarine, "in trim" with feedback controller (SOA=-0.0656, SOB=-1.47, SOC=26.2, SOD=184.1) and zero pitch and depth ordered.

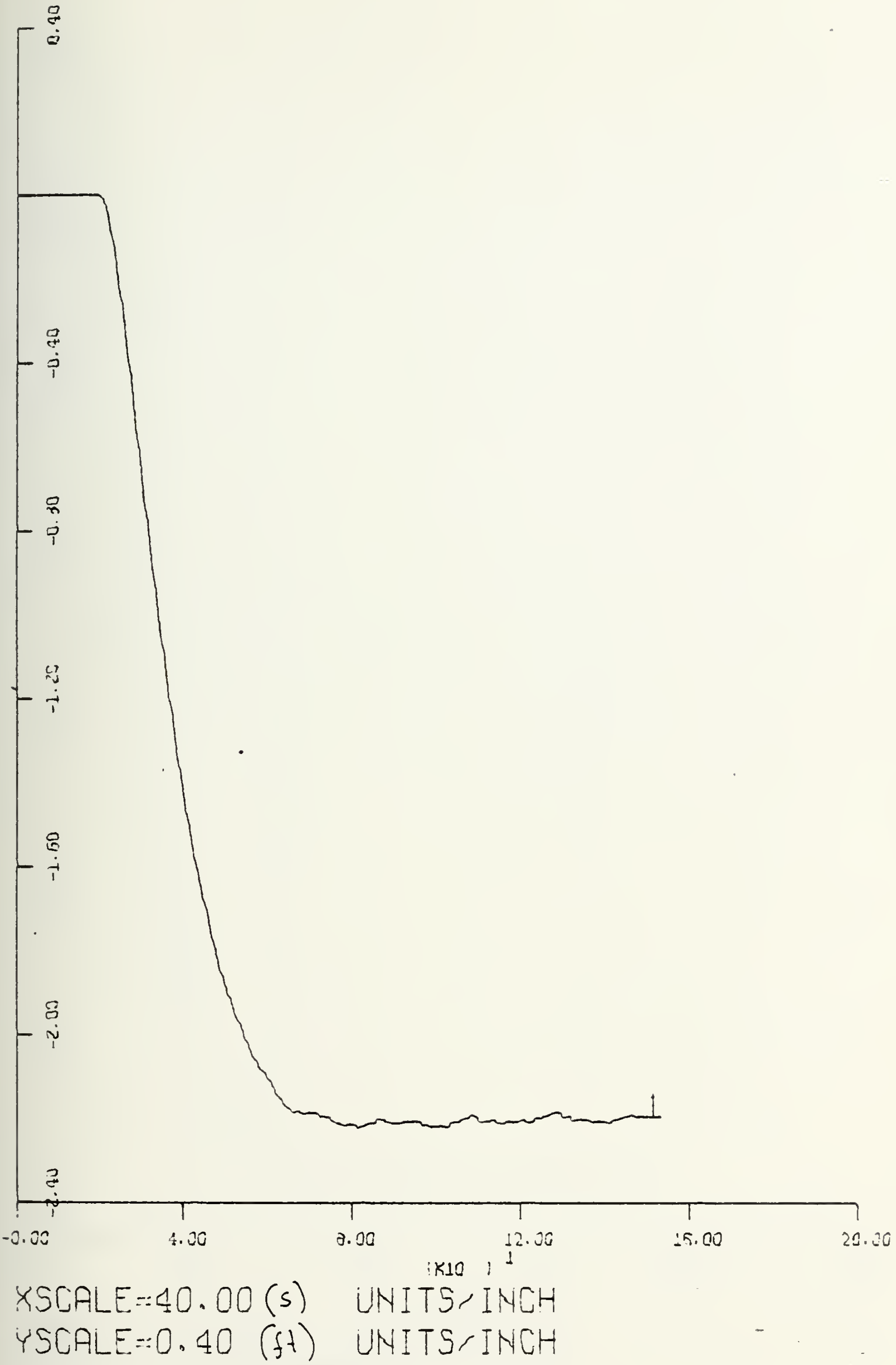


Fig. C-7a. Depth vs. Time. Stern planes only submarine, not "in trim" with SOPOC (B=164, C=0.001, E=0.001) and zero pitch and depth ordered.

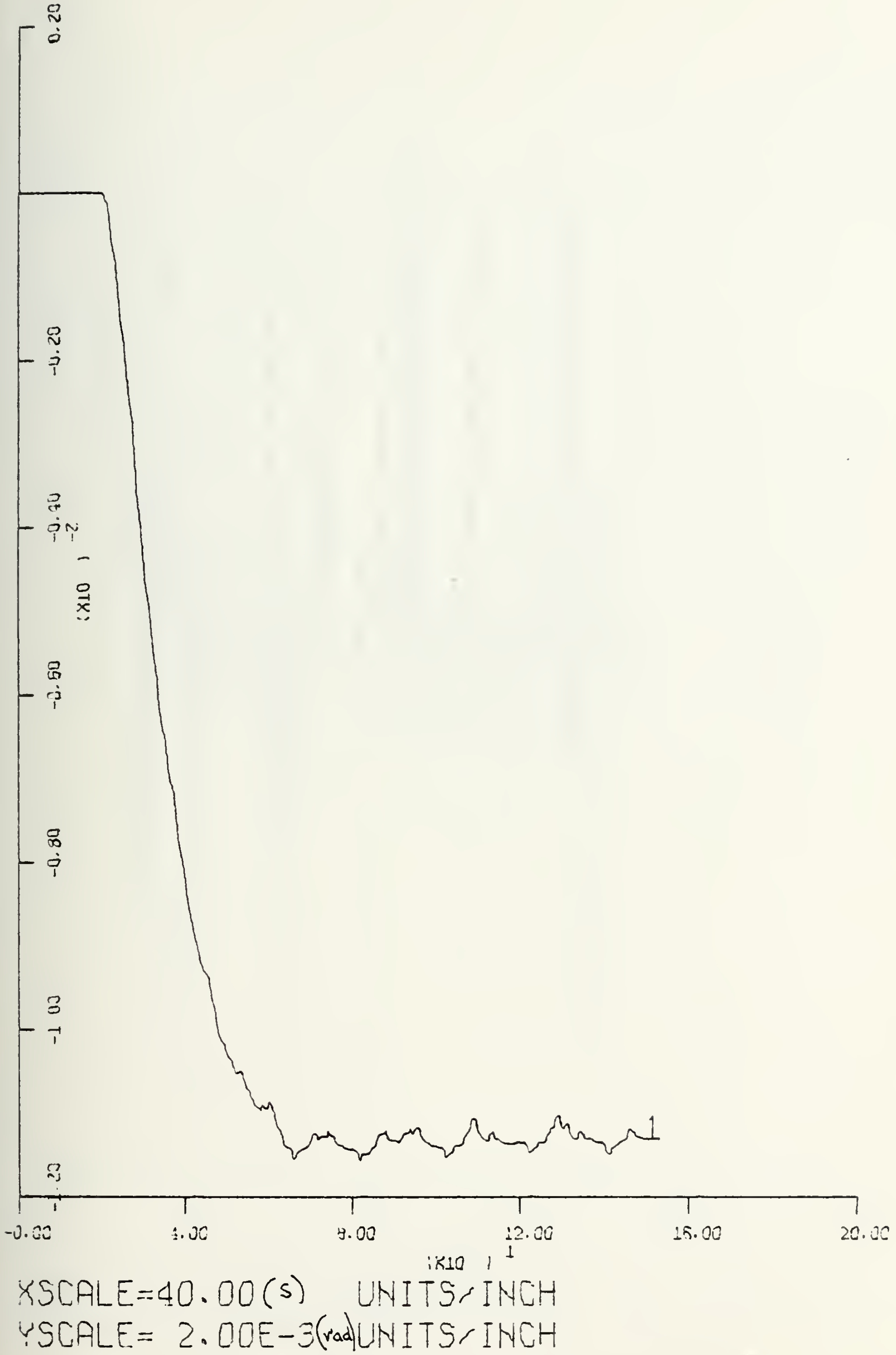


Fig. C-7b. Pitch vs. Time. Stern planes only submarine, not "in trim" with SOPOC (B=164, C=0.001, E=0.001) and zero pitch and depth ordered.

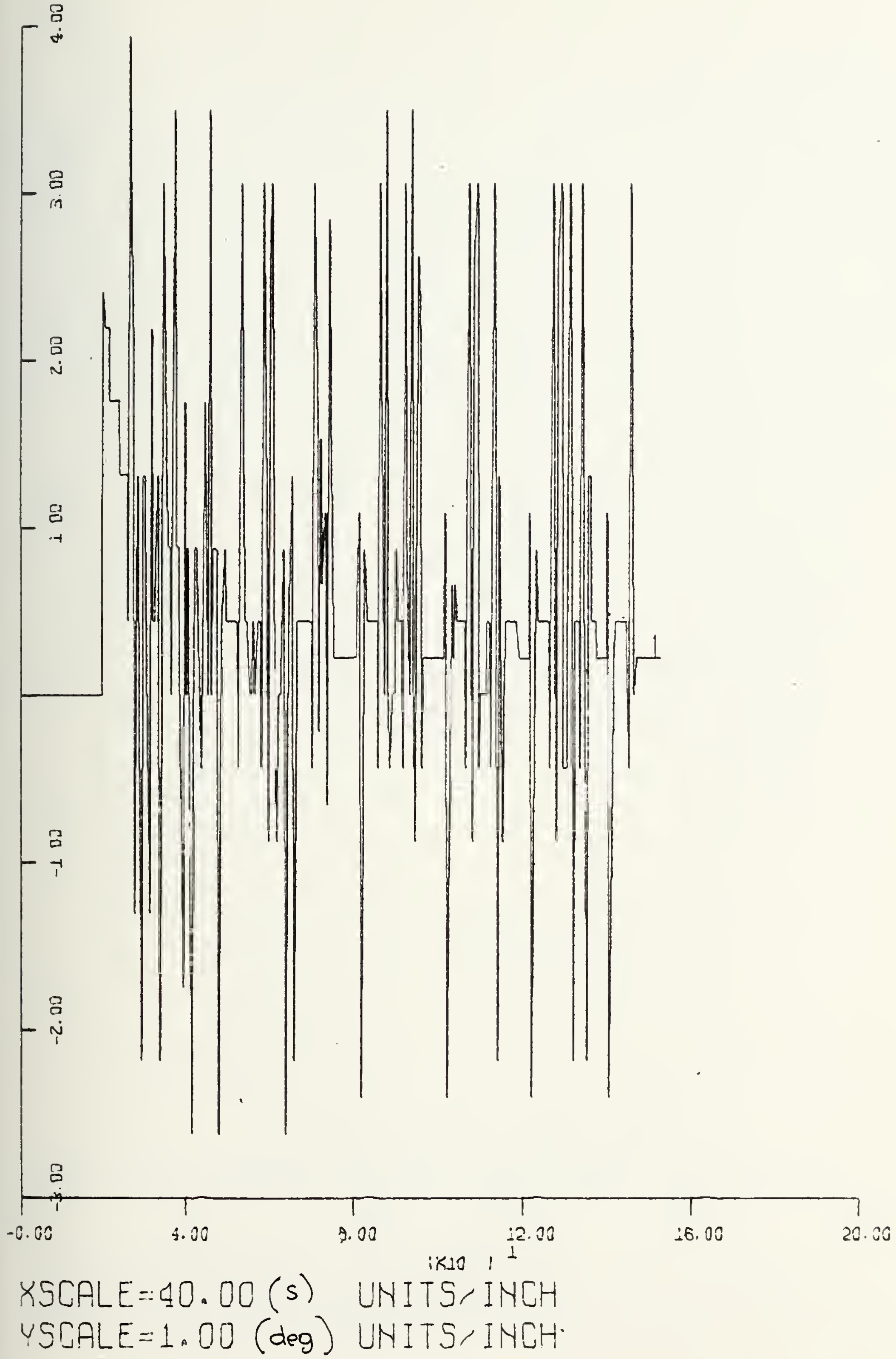


Fig. C-7c. Stern Plane Angle vs. Time. Stern planes only submarine, not "in trim" with SOPOC (B=164, C=0.001, E=0.001) and zero pitch and depth ordered.

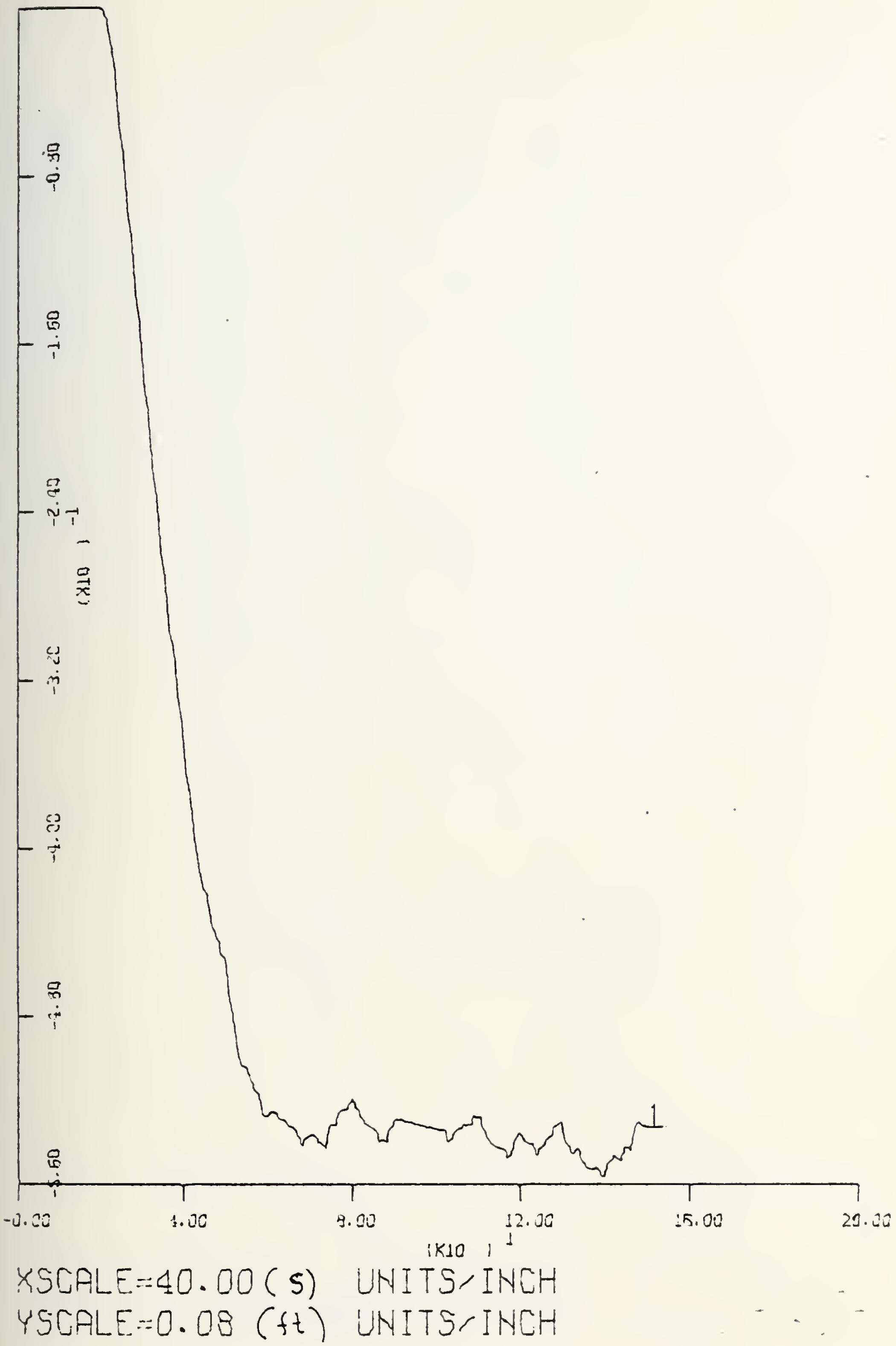


Fig. C-8a. Depth vs. Time. Stern planes only submarine, "in trim" with SOPOC (B=164, C=0.001, E=0.001) and zero pitch and depth ordered.

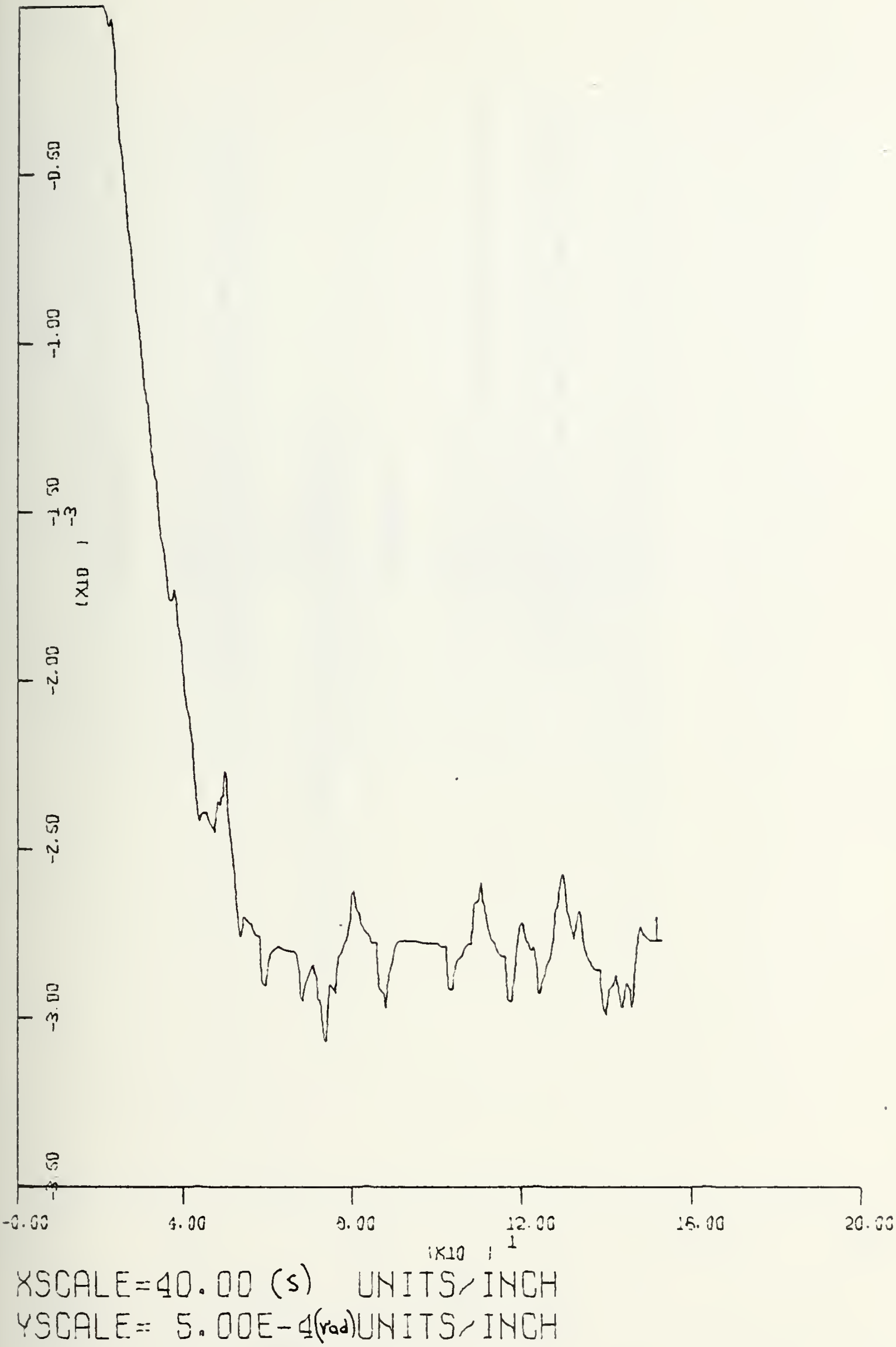


Fig. C-8b. Pitch vs. Time. Stern planes only submarine, "in trim" with SOPOC (B=164, C=0.001, E=0.001) and zero pitch and depth ordered.

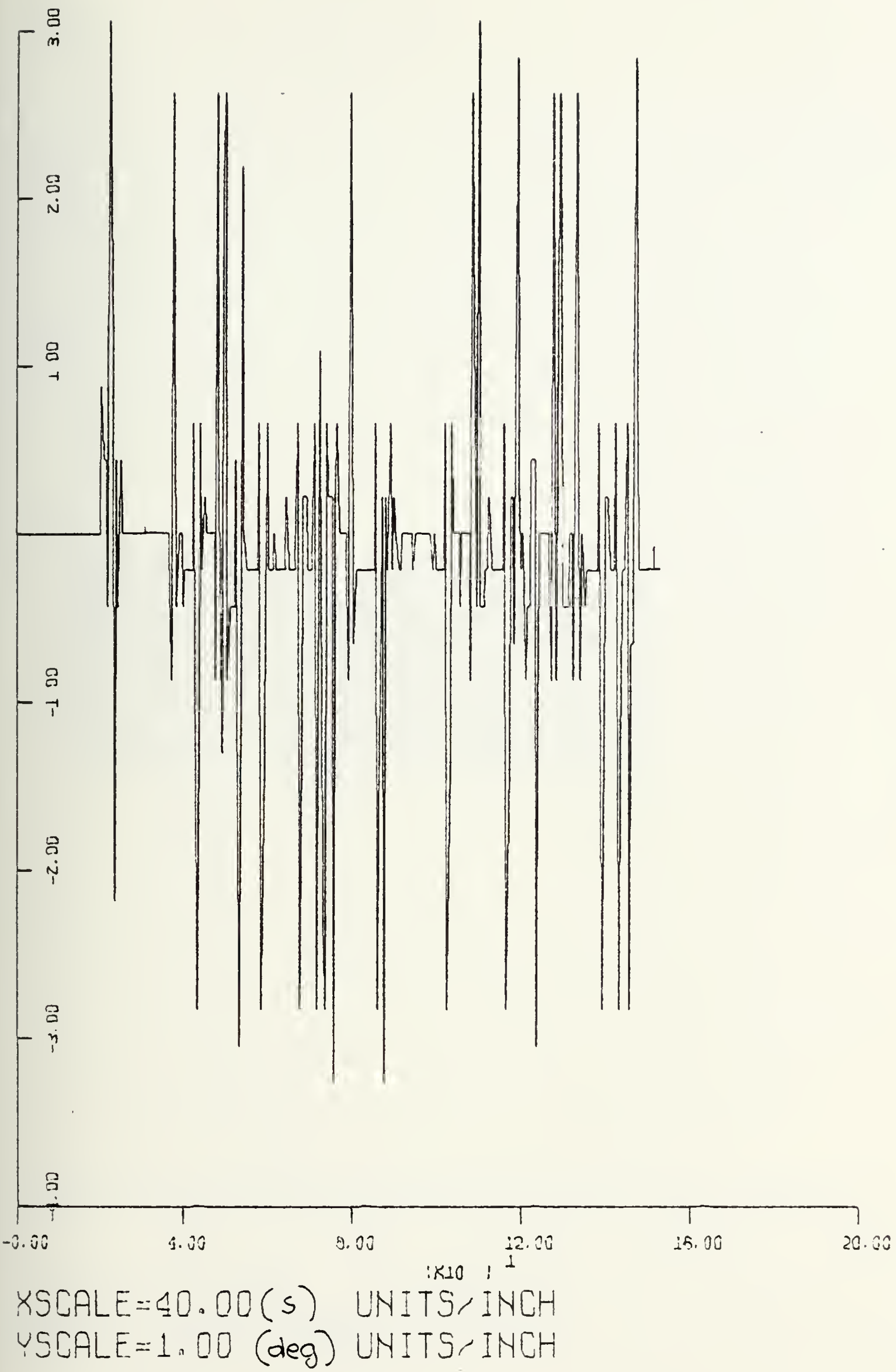


Fig. C-8c. Stern Plane Angle vs. Time. Stern planes only submarine, "in trim" with SOPOC (B=164, C=0.001, E=0.001) and zero pitch and depth ordered.

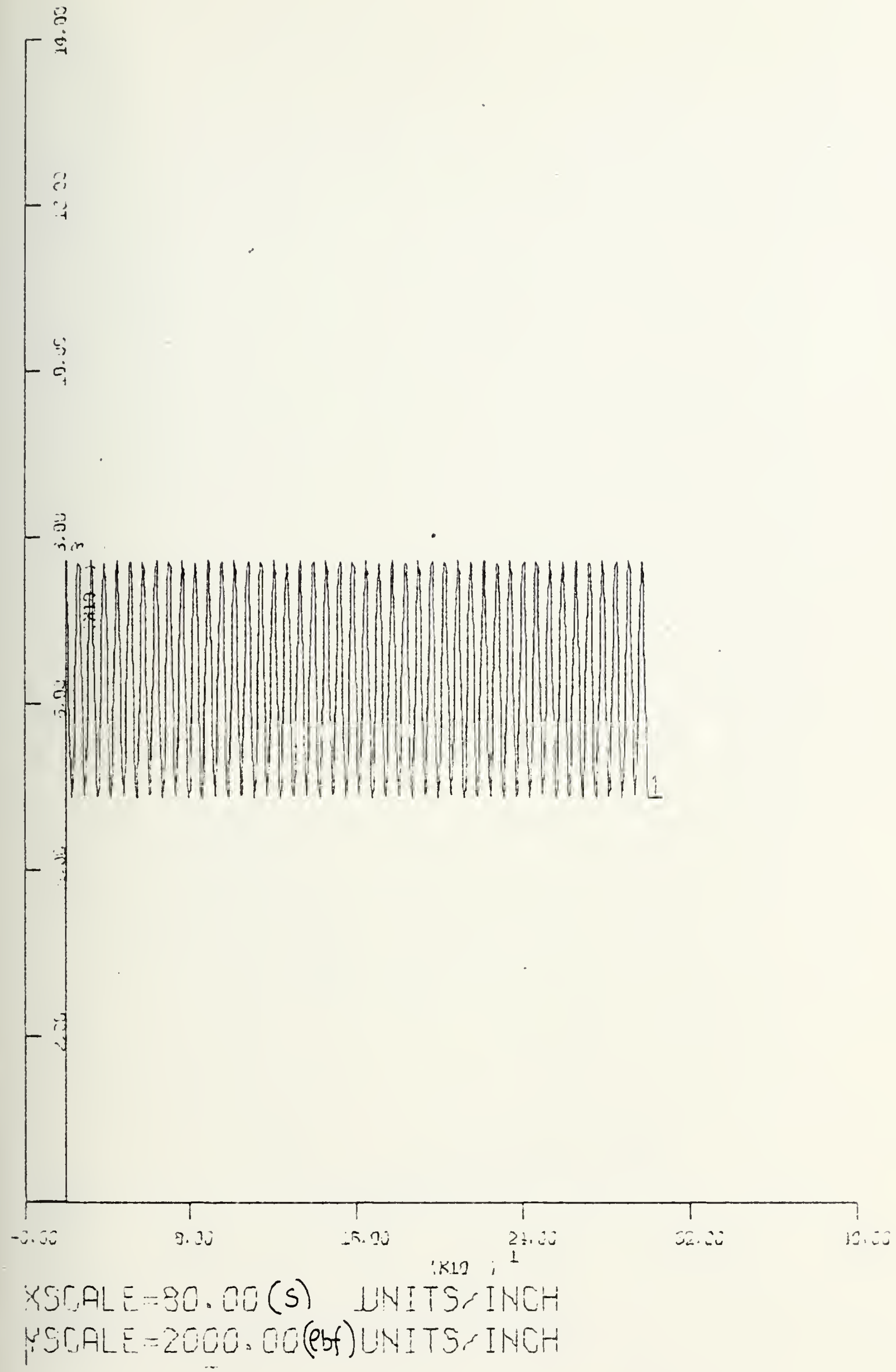


Fig. C-9a. Sinusoidal force at AU

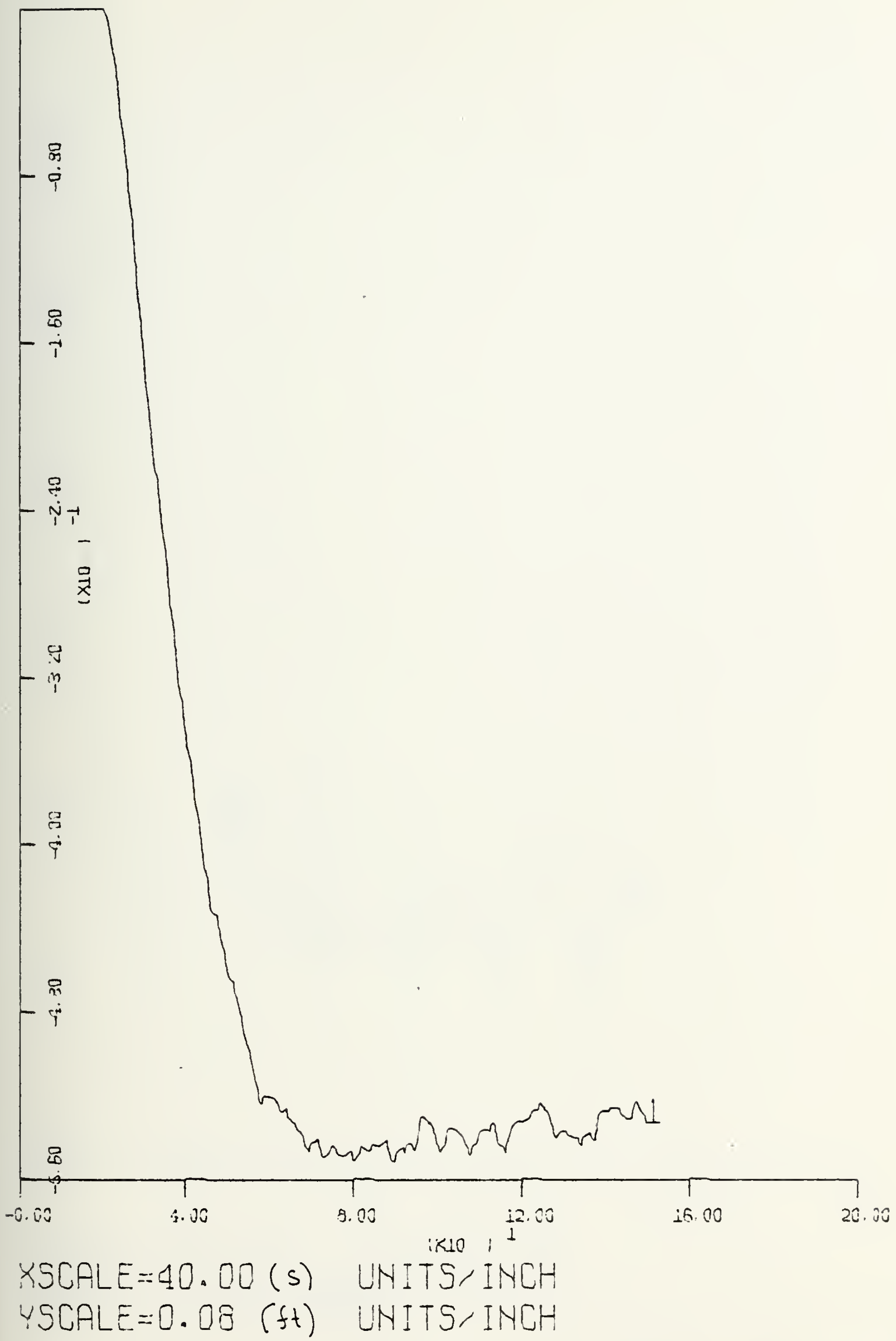


Fig. C-9b. Depth vs. Time. Response to sinusoidal force at AU. Stern planes only submarine, "in trim" with SOPOC (B=164, C=0.001, E=0.001) and zero pitch and depth ordered.

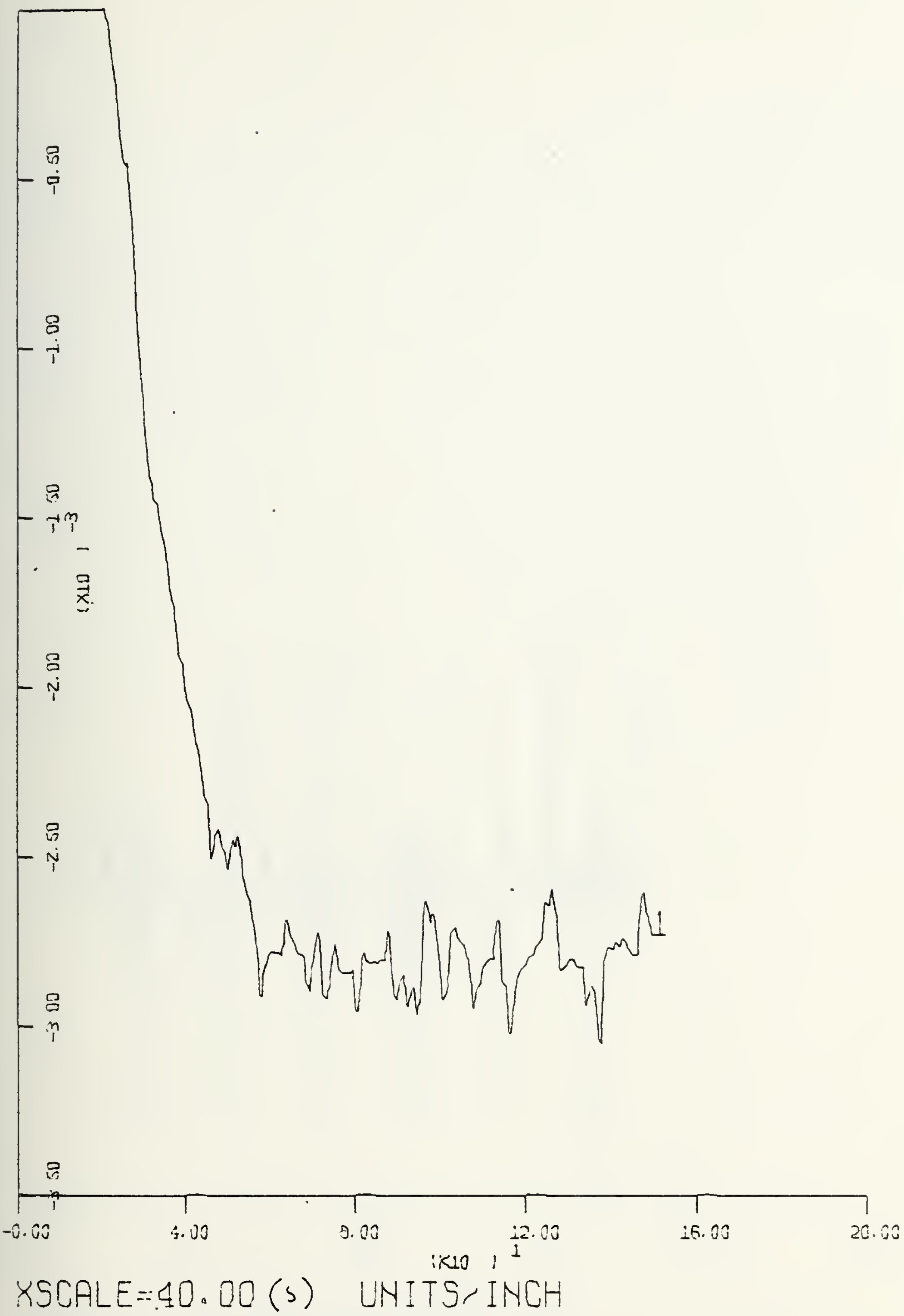


Fig. C-9c. Pitch vs. Time. Response to sinusoidal force at AU. Stern planes only submarine, "in trim" with SOPOC (B=164, C=0.001, E=0.001) and zero pitch and depth ordered.

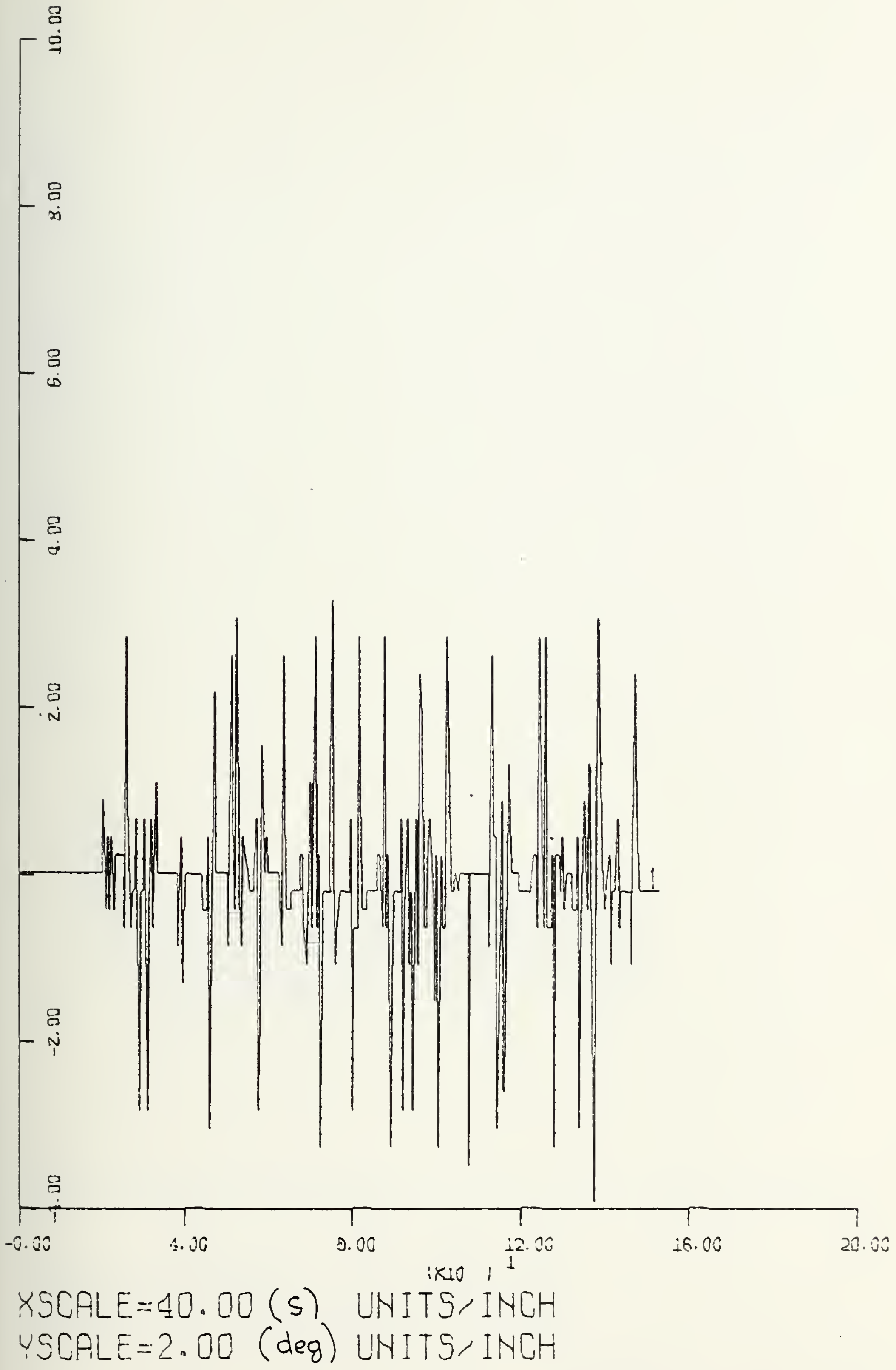


Fig. C-9d. Stern Plane Angle vs. Time. Response to sinusoidal force at AU. Stern planes only submarine, "in trim" with SOPOC (B=164, C=0.001, E=0.001) and zero pitch and depth ordered.

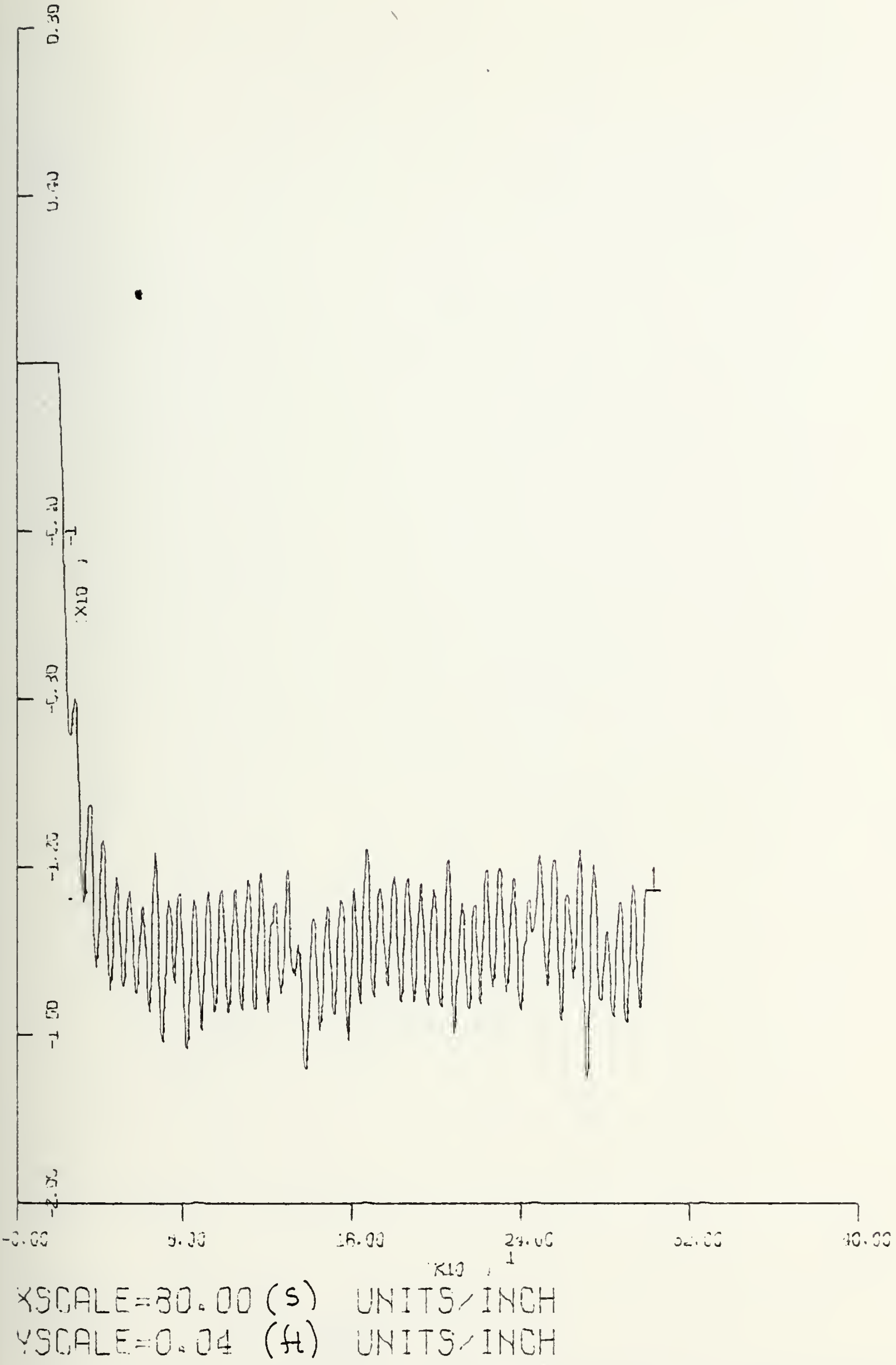


Fig. C-10a. Depth vs. Time. Response to sinusoidal force at AU. Stern planes only submarine, "in trim" with feedback controller (SOA=-0.5, SOB=-7.03, SOC=32.5, SOD=360) and zero pitch and depth ordered.

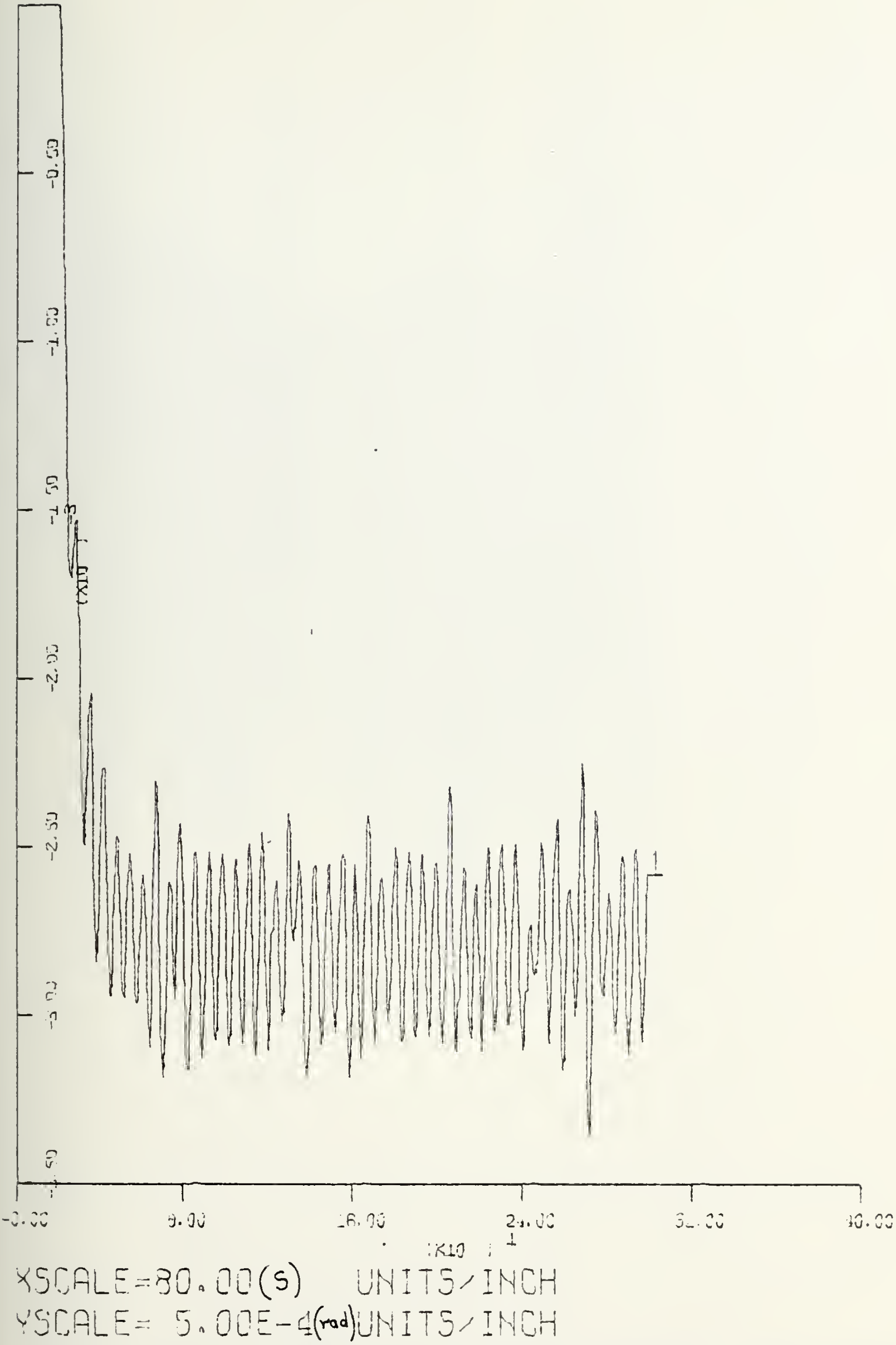


Fig. C-10b. Pitch vs. Time. Response to sinusoidal force at AU. Stern planes only submarine, "in trim" with feedback controller (SOA=-0.5, SOB=-7.03, SOC=32.5, SOD=360) and zero pitch and depth ordered.

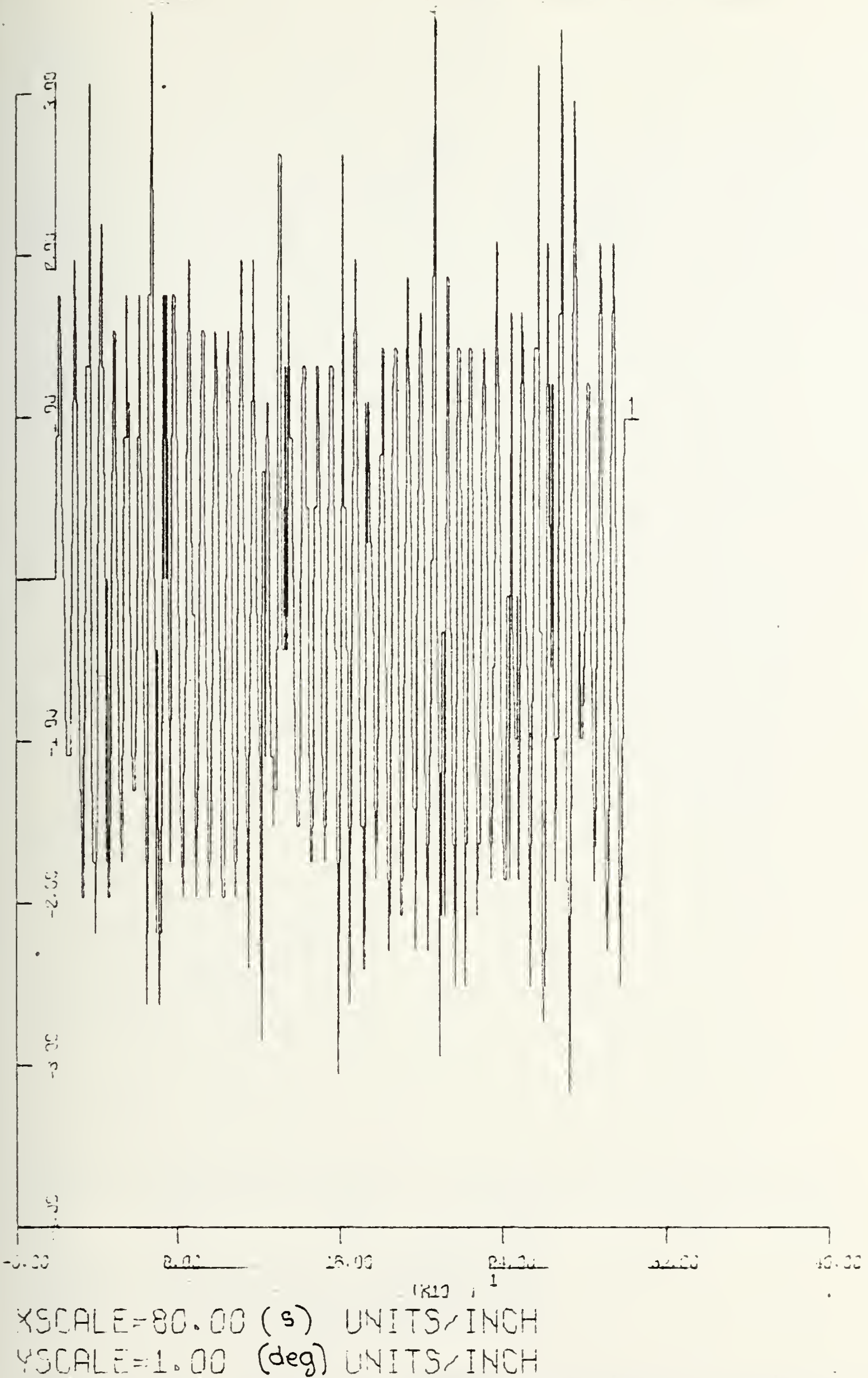
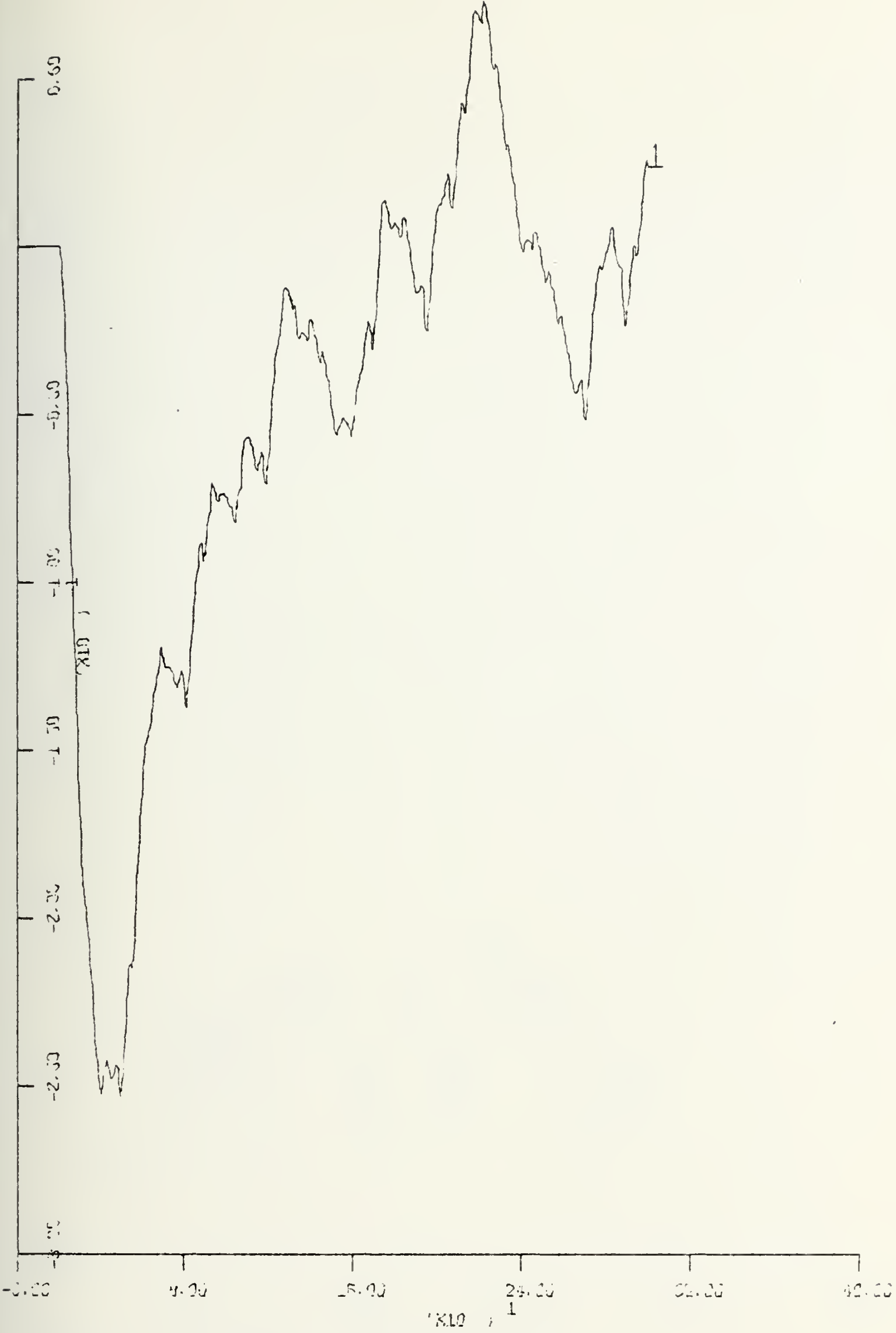


Fig. C-10c. Stern Plane Angle vs. Time. Response to sinusoidal force at AU. Stern planes only submarine, "in trim" with feedback controller (SOA=-0.5, SOB=-7.03, SOC=32.5, SOD=360) and zero pitch and depth ordered.



XSCALE=80.00 (s) UNITS/INCH
YSCALE=0.05 (ft) UNITS/INCH

Fig. C-11a. Depth vs. Time. Response to sinusoidal force at AU. Stern planes only submarine, "in trim" with feedback controller (SOA=-0.0436, SOB=-3.49, SOC=0.0523, SOD=360) and zero pitch and depth ordered.

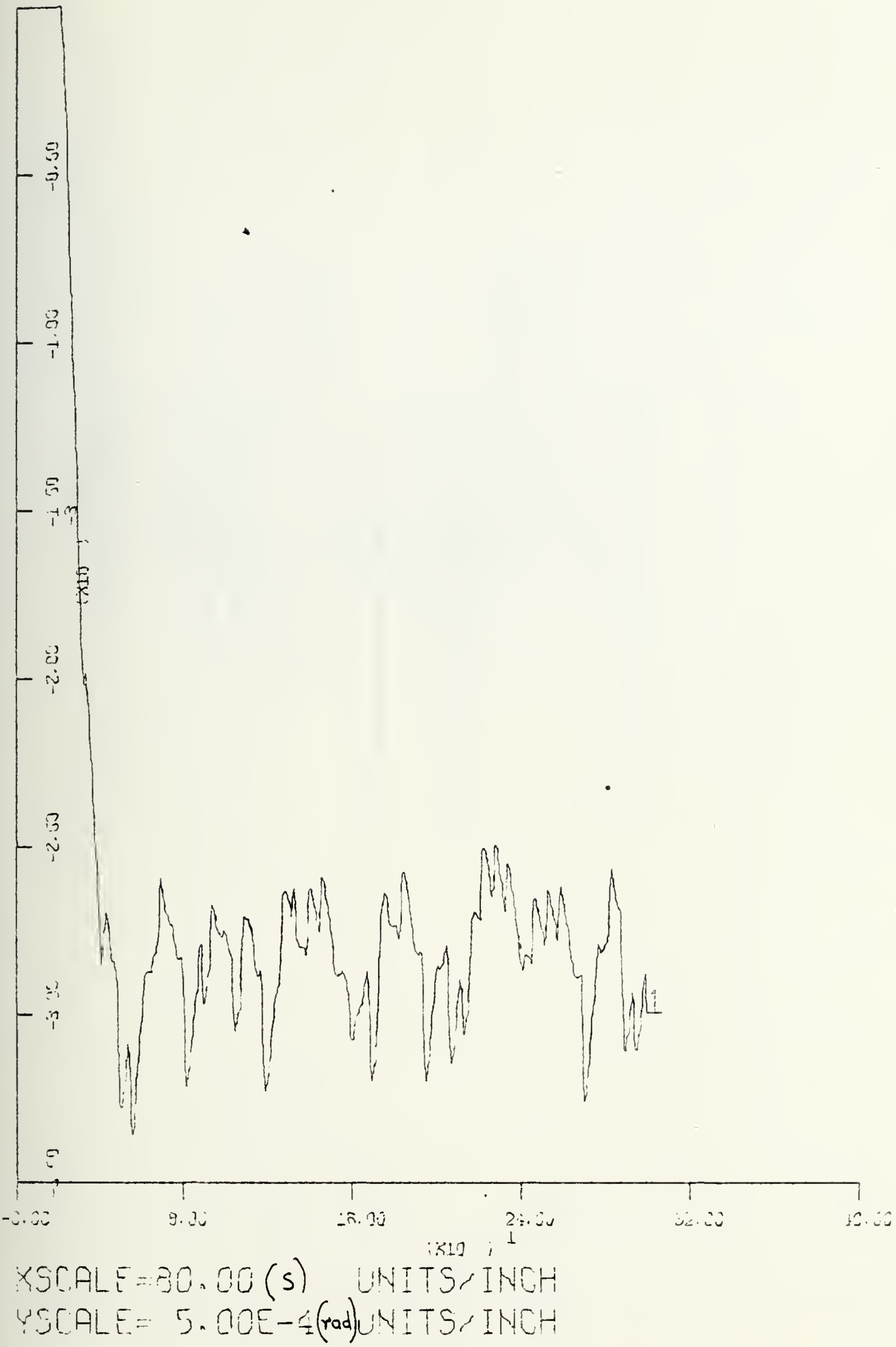


Fig. C-11b. Pitch vs. Time. Response to sinusoidal force at AU. Stern planes only submarine, "in trim" with feedback controller (SOA=-0.0436, SOB=-3.49, SOC=0.0523, SOD=360) and zero pitch and depth ordered.

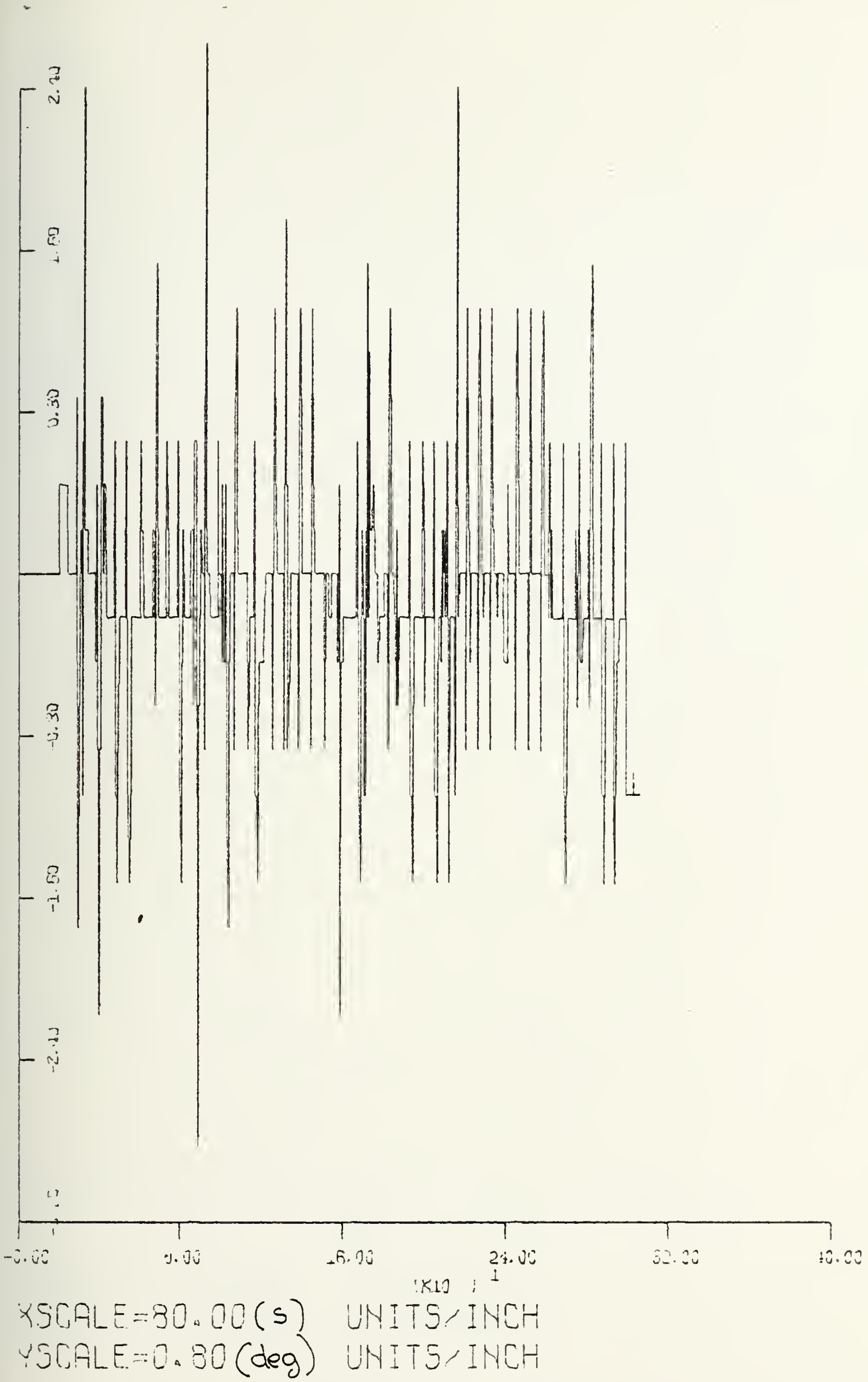


Fig. C-11c. Stern Plane Angle vs. Time. Response to sinusoidal force at AU. Stern planes only submarine; "in trim" with feedback controller (SOA=-0.0436, SOB=-3.49, SOC=0.0523, SOD=360) and zero pitch and depth ordered.

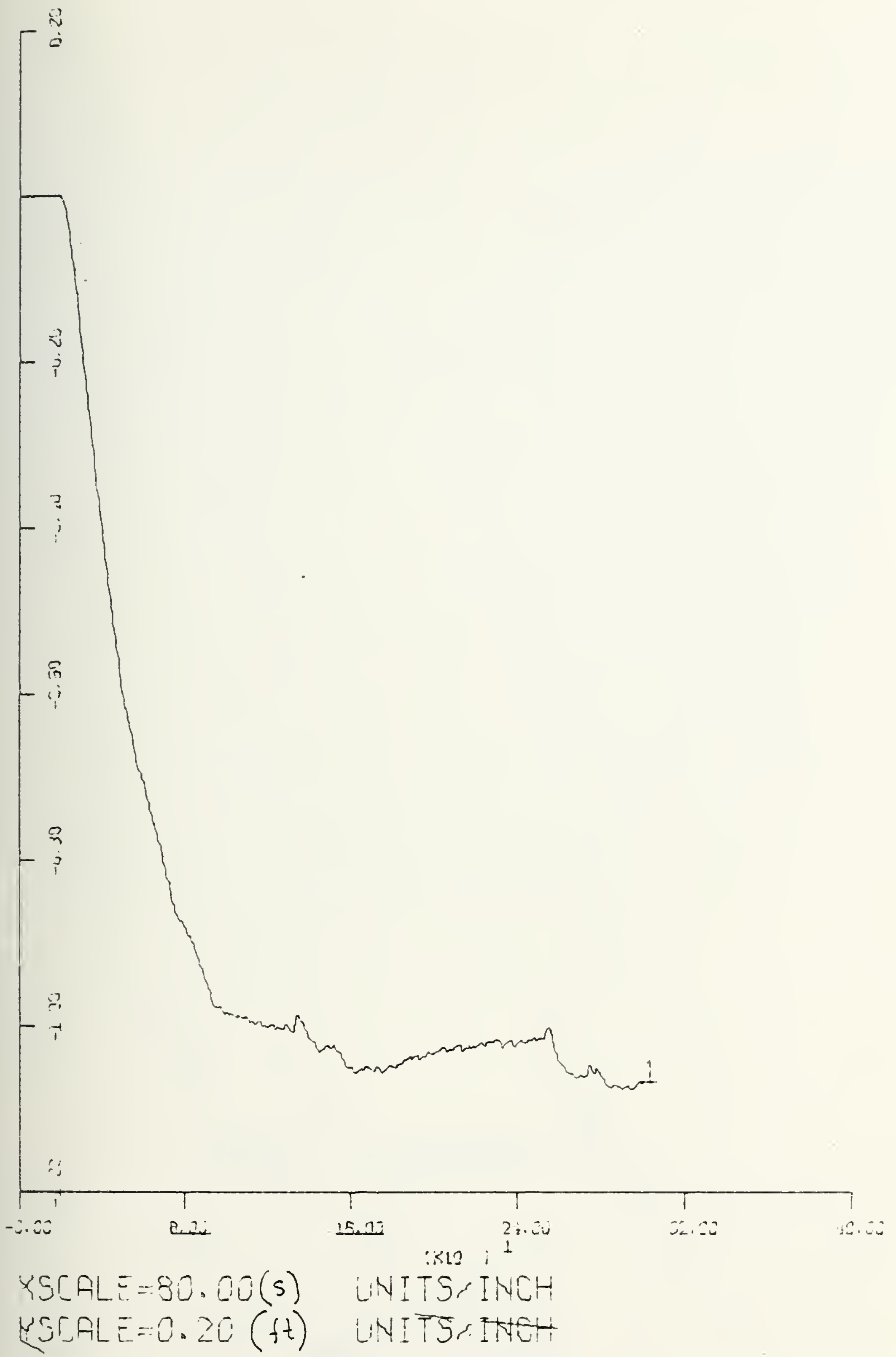


Fig. C-12a. Depth vs. Time. Response to sinusoidal force at AU. Stern planes only submarine, "in trim" with feedback controller (SOA=-0.0656, SOB=-1.47, SOC=26.2, SOD=184.1) and zero pitch and depth ordered.

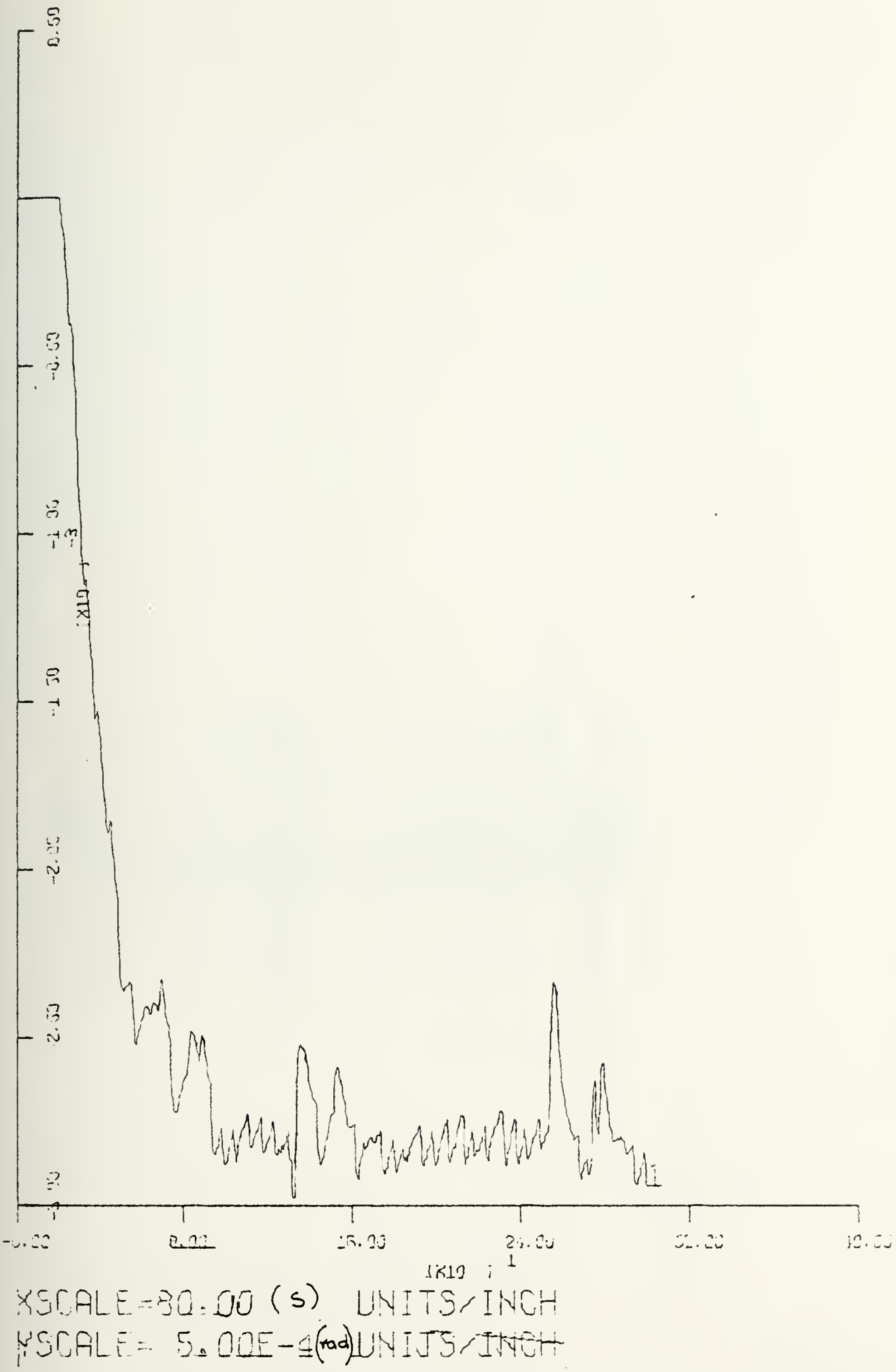


Fig. C-12b. Pitch vs. Time. Response to sinusoidal force at AU. Stern planes only submarine, "in trim" with feedback controller (SOA=-0.0656, SOB=-1.47, SOC=26.2, SOD=184.1) and zero pitch and depth ordered.

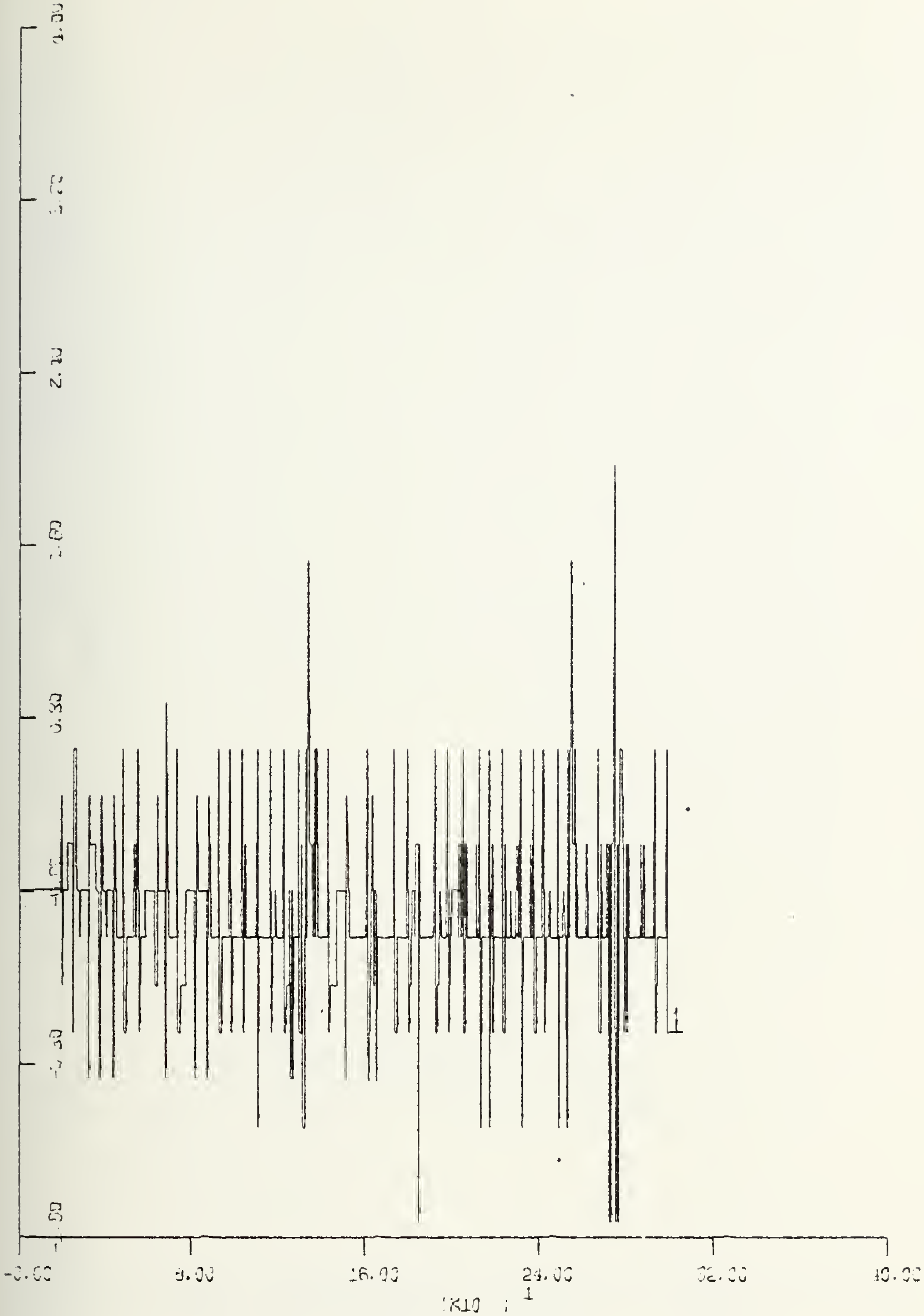


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Fig. C-12c. Stern Plane Angle vs. Time. Response to sinusoidal force at AU. Stern planes only submarine, "in trim" with feedback controller (SOA=-0.0656, SOB=-1.47, SOC=26.2, SOD=184.1) and zero pitch and depth ordered.

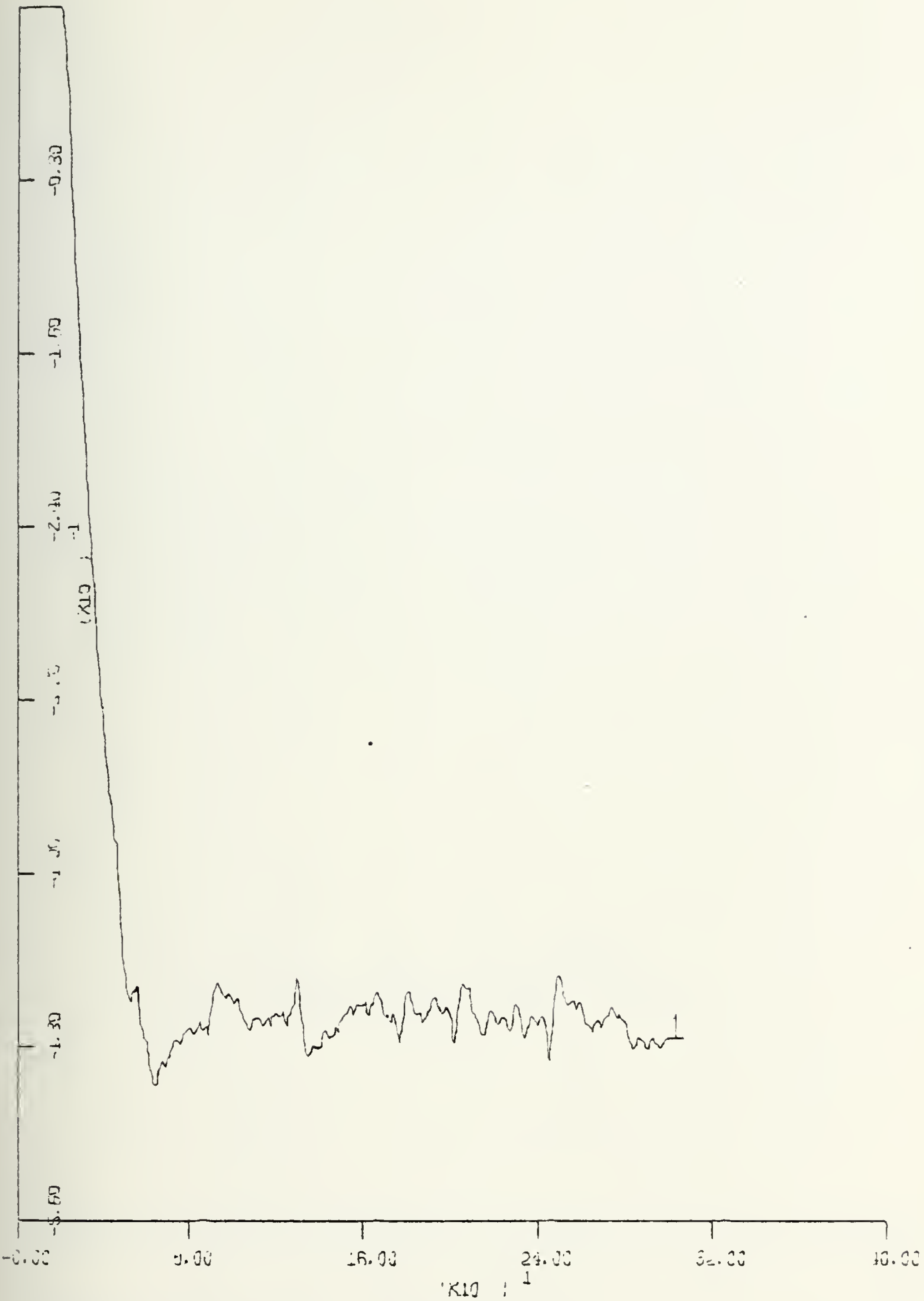

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Fig. C-13a. Depth vs. Time. Response to sinusoidal force at AU. Stern planes only submarine, "in trim" with SOPOC (B=800, C=10, E=1) and zero pitch and depth ordered.

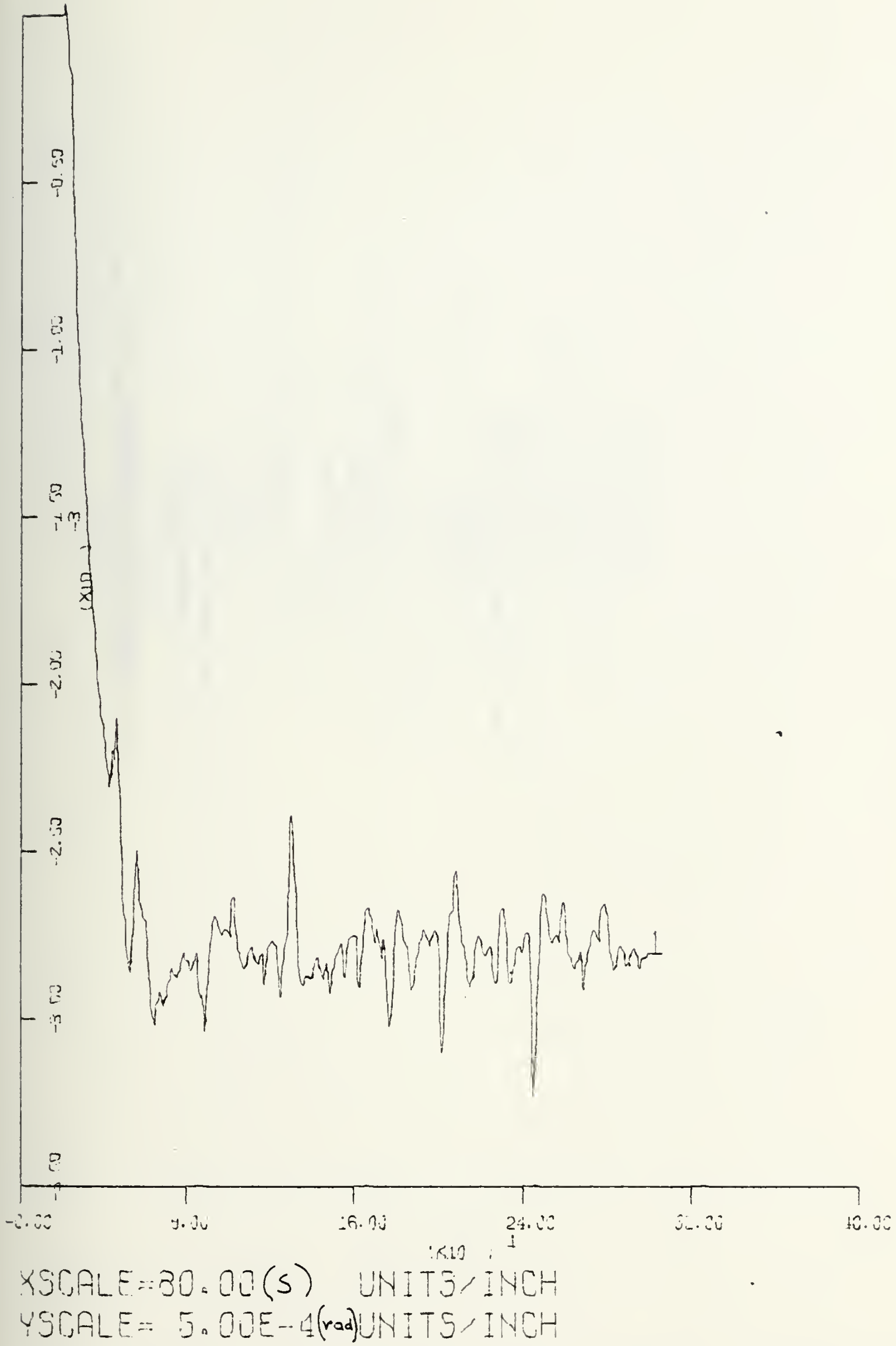


Fig. C-13b. Pitch vs. Time. Response to sinusoidal input at AU. Stern planes only submarine, "in trim" with SOPOC (B=800, C=10, E=1) and zero pitch and depth ordered.

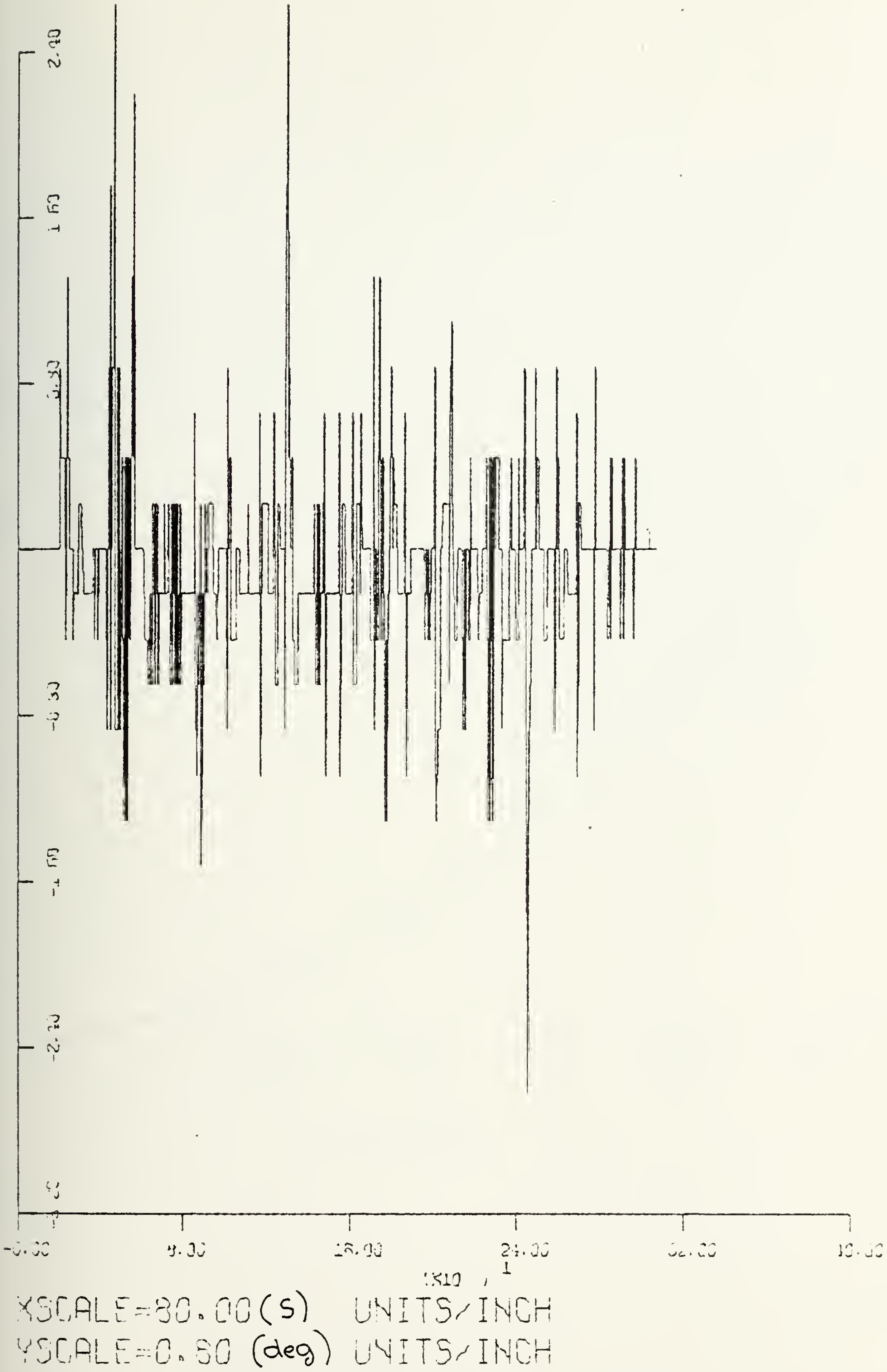


Fig. C-13c. Stern Plane Angle vs. Time. Response to sinusoidal input at AU. Stern planes only submarine, "in trim" with SOPOC (B=800, C=10, E=1) and zero pitch and depth ordered.

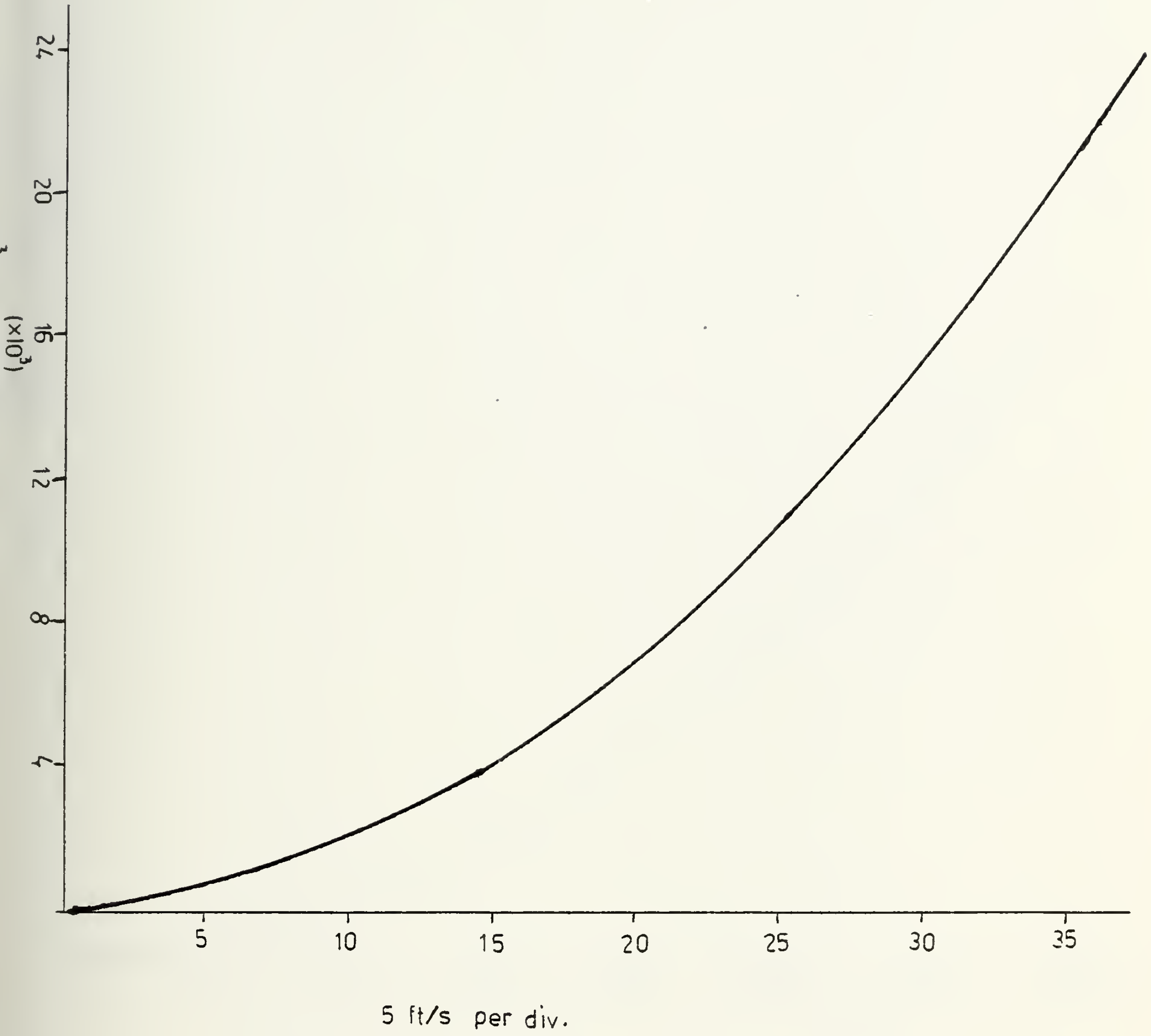


Fig. D-1. Ballast Vs. Speed

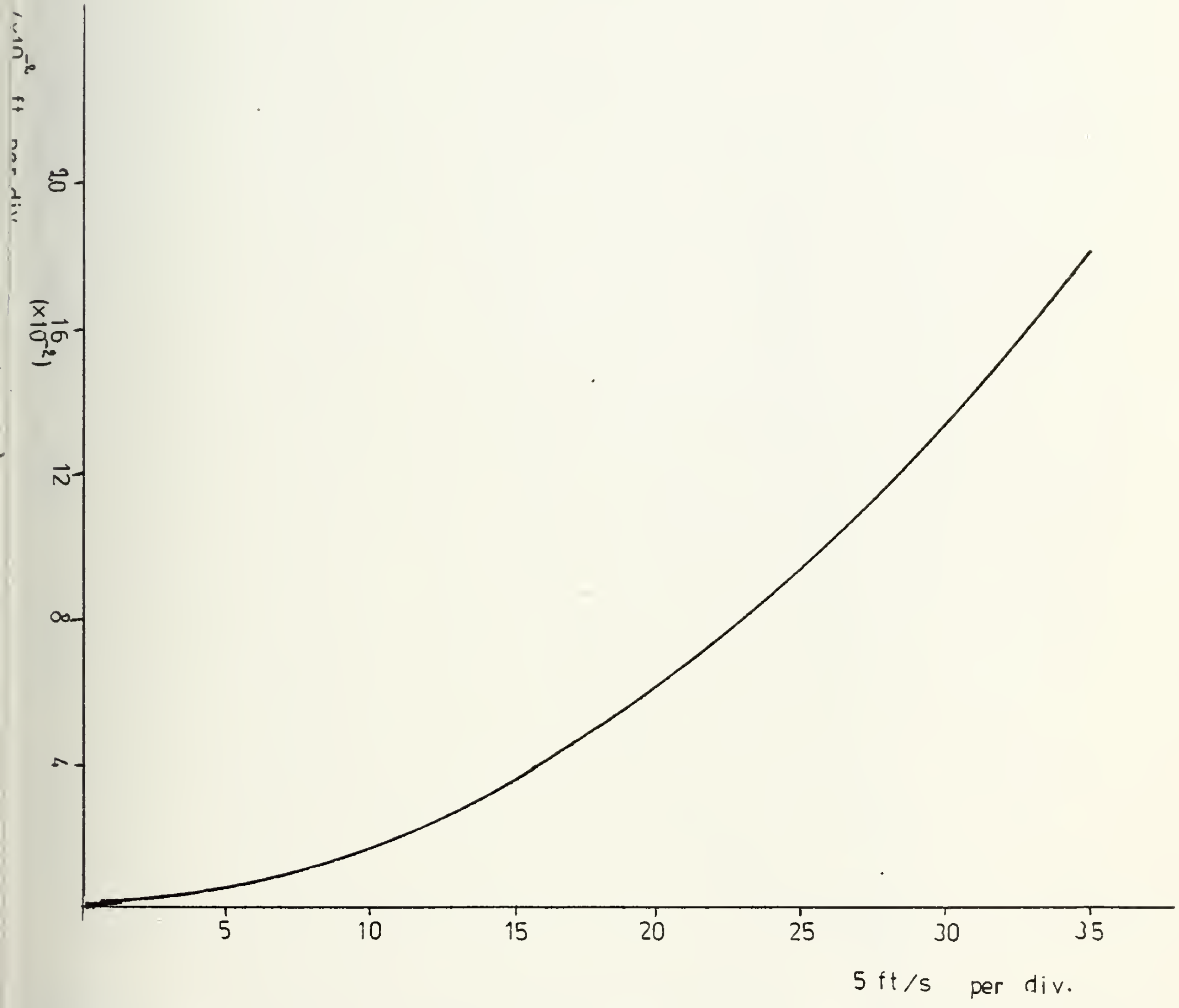
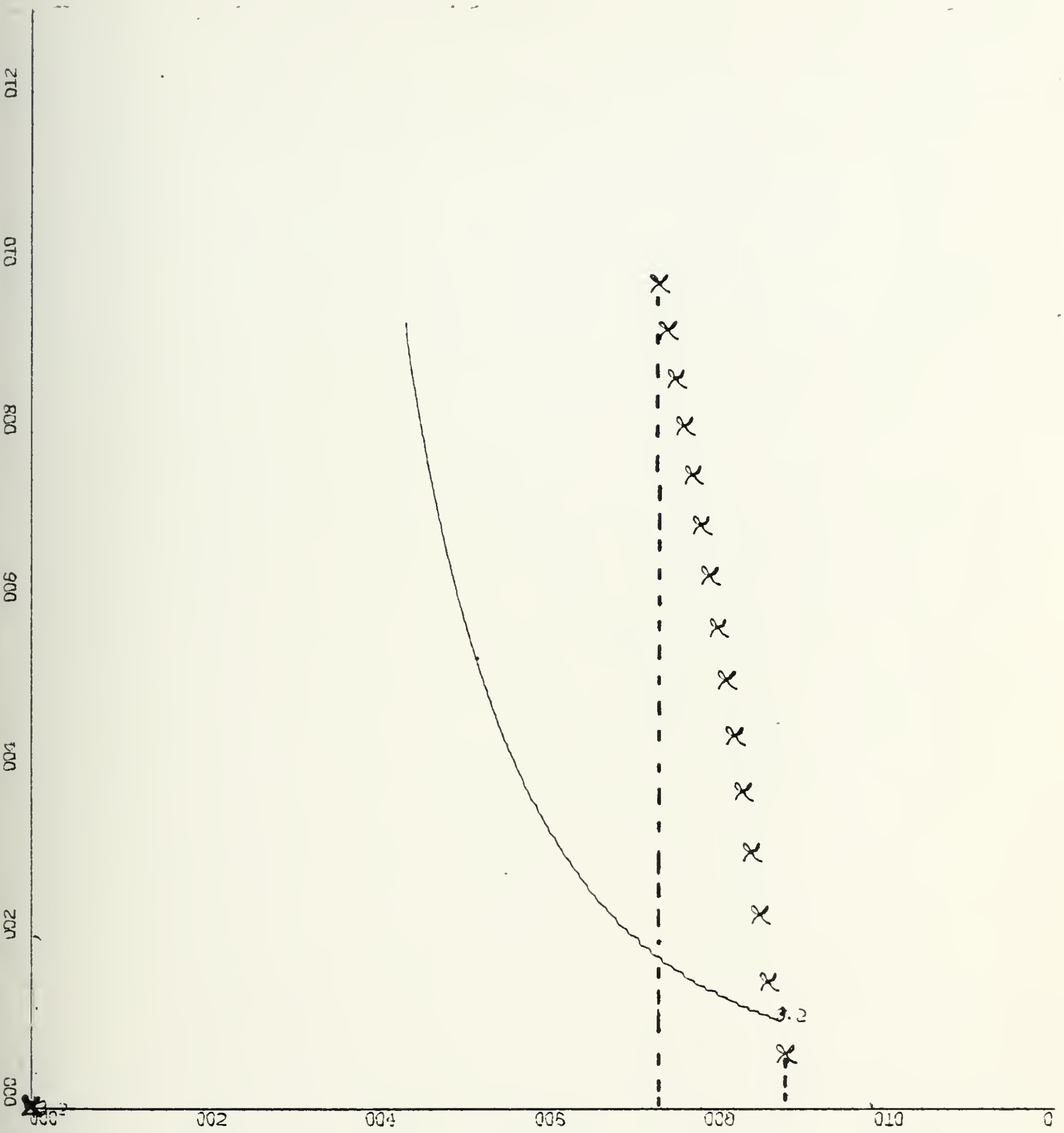
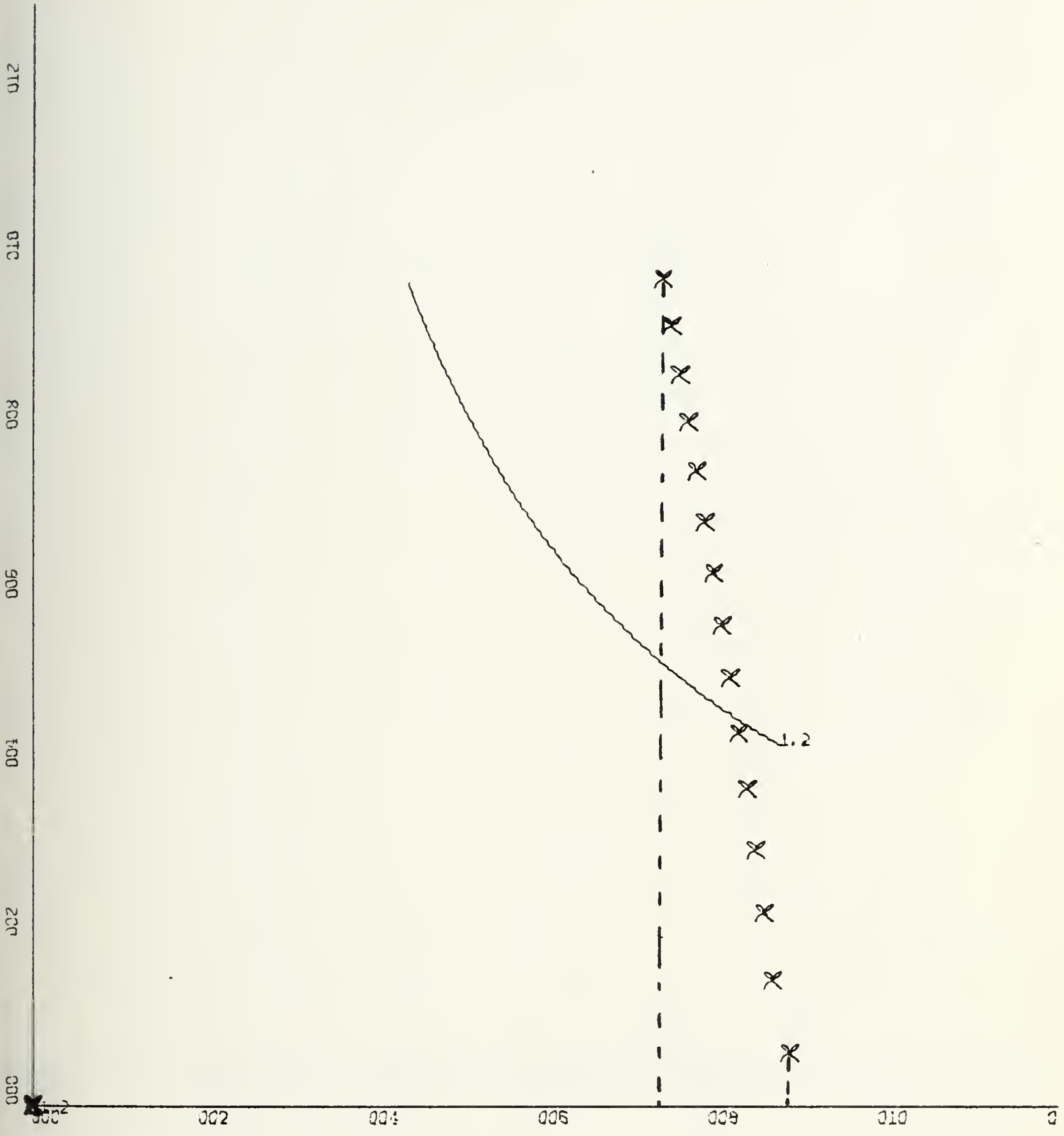


Fig. D-2. X_G vs. Speed



X-SCALE=2.00E-01 UNITS INCH.
Y-SCALE=2.00E-01 UNITS INCH.

Fig. E-1. K vs. dfcom. and X=3.2



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Y-SCALE=2.00E-01 UNITS INCH.

Fig. E-2. K vs. dfcom and X=1.2

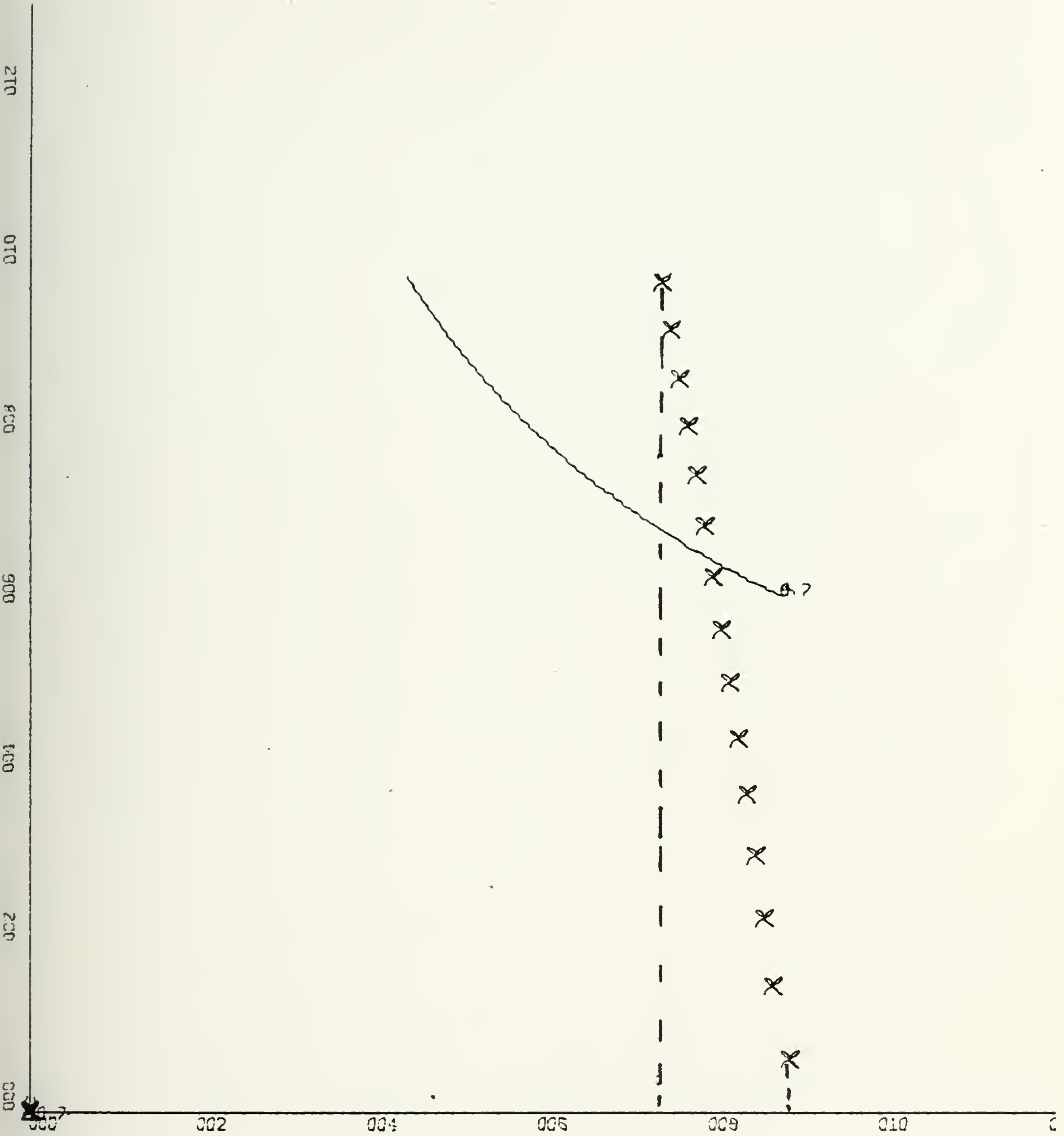

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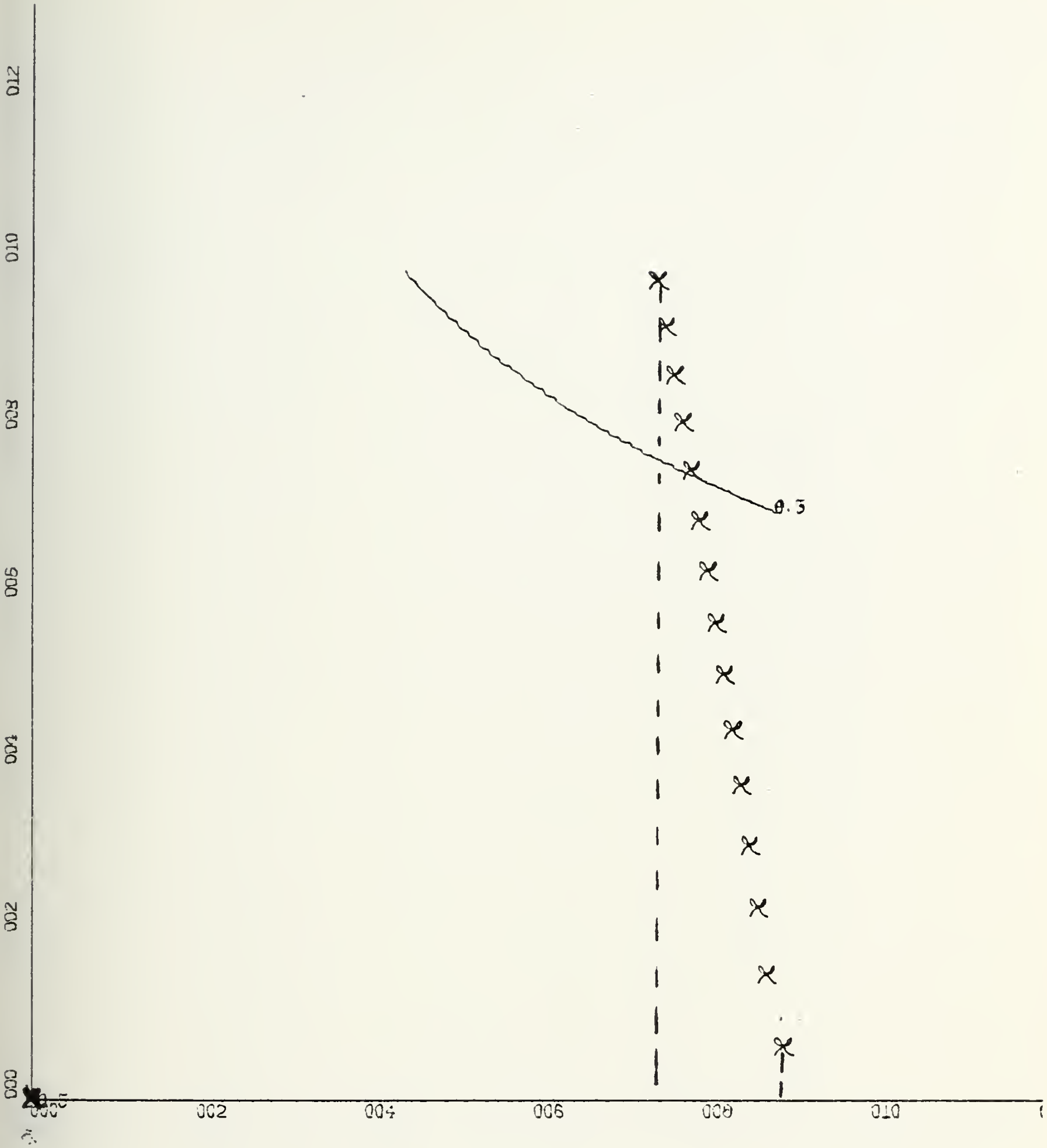
Fig. E-3. K vs. dfcom and X=0.7



X-SCALE=2.00E-01 UNITS INCH.
Y-SCALE=2.00E-01 UNITS INCH.

Fig. E-4. K vs. dfcom and X=0.5

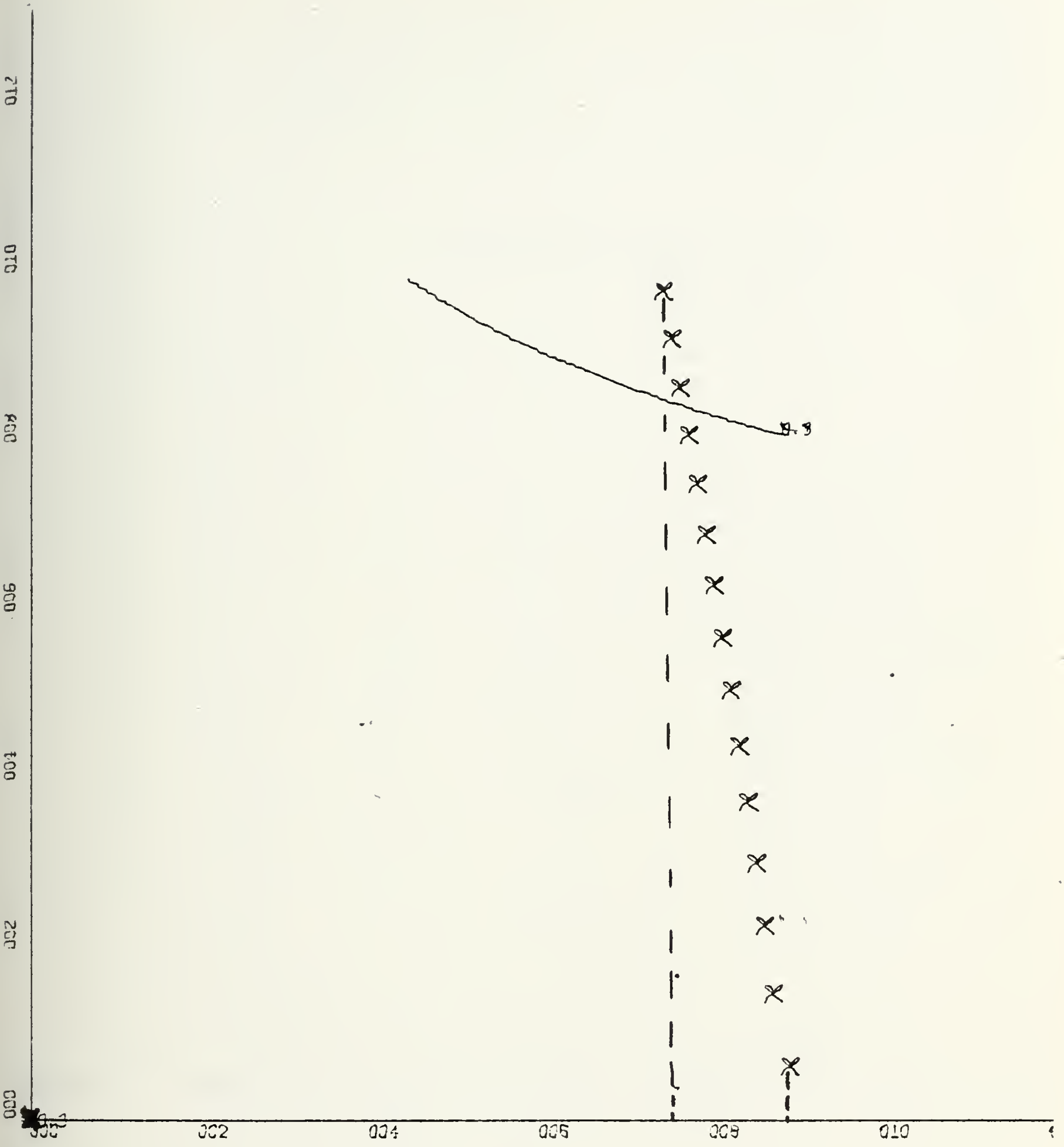


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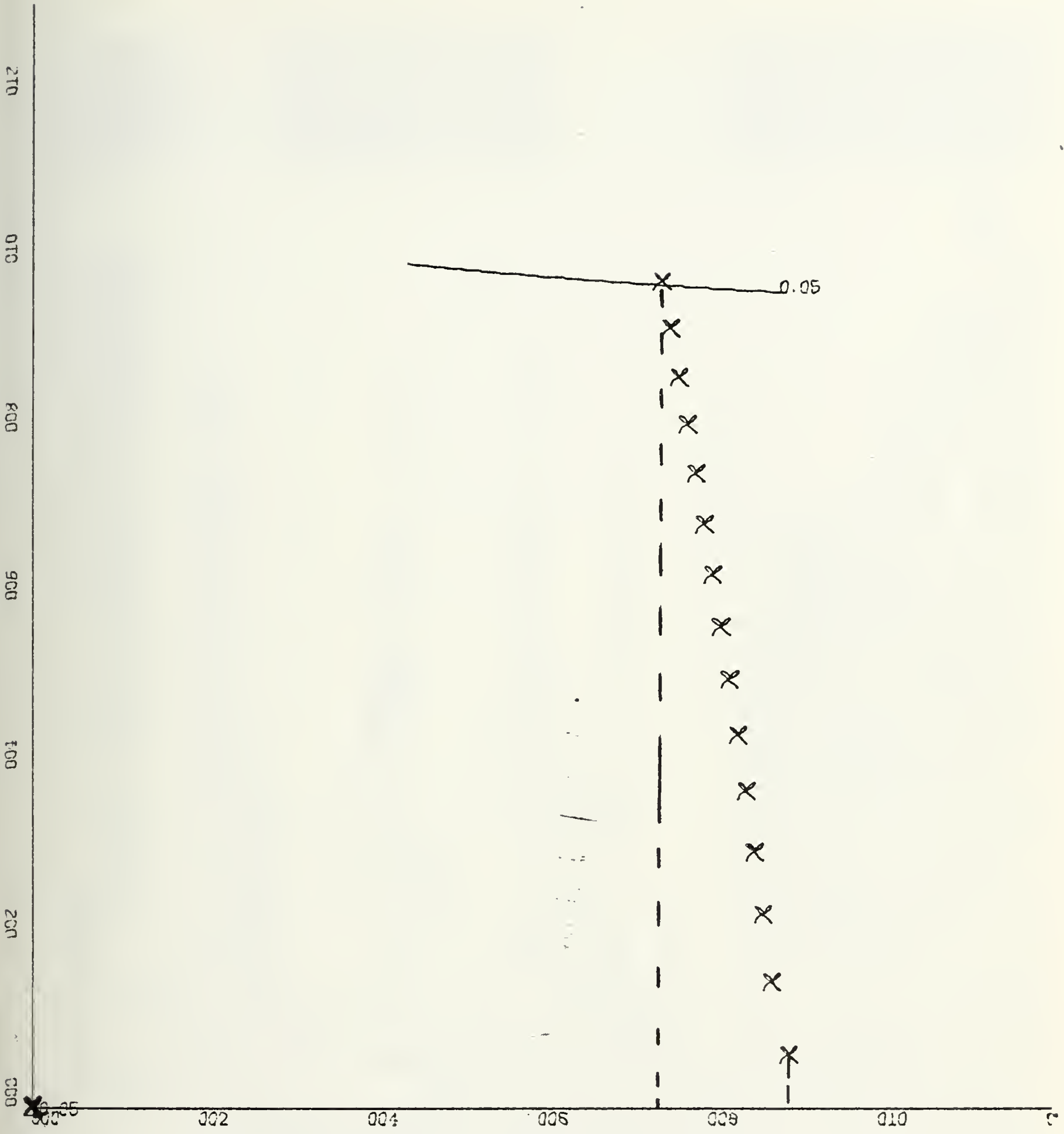
Fig. E-5. K vs. dfcom and X=0.3



X-SCALE=2.00E-01 UNITS INCH.
Y-SCALE=2.00E-01 UNITS INCH.

Fig. E-6. K vs. dfcom and X=0.05

```
// EXEC DSL
//DSL.INPUT DD *
*                MAIN PROGRAM
* OPTIMAL GAINS ARE CALCULATED UNTIL THE TIME REQUIREMENTS OF AN IF     SACS0020
* STATEMENT ARE  MET. THE TIME REQUIREMENT IS SELECTED TO ENSURE A      SACS0030
* STEADY STATE CONDITION IS REACHED. THE STEADY STATE GAINS ARE USED    SACS0040
* IN THE DEPTH CONTROLLER. THE SIMULATION DOES NOT START UNTIL THE      SACS0050
* STEADY STATE IS ACHIEVED.                                             SACS0060
*    THE WEIGHTING FACTOR IN THE GAIN EQUATION FOR DEPTH ERROR IS THE   SACS0070
*PARAMETER E.THE WEIGHT FOR PITCH ERROR IS D.THE WEIGHT FOR
* DEPTH RATE ERROR IS A.THE WEIGHT FOR PITCH RATE ERROR IS B.
* THE CONTROL INPUT IS WEIGHTED WITH C.
* PARAMETERS ARE ASSIGNED AS FOLLOWS                                    SACS0100
* MAX PLANE RATE---PLRT                                                 SACS0110
* PUMP RATE---KPR                                                       SACS0120
* INITIAL TRIM TANK LEVELS---AT,FT,AU                                   SACS0130
* DEADZONE IN THE VERTICAL VELOCITY CHANNEL---DZ1                       SACS0140
* DEADZONE IN THE ROTATIONAL VELOCITY CHANNEL---DY1                     SACS0150
* THE PITCH ERROR AND DEPTH ERROR ARE LIMITED IN THE PROGRAM.           SACS0160
* THE PUMP IS OPERATED AS FOLLOWS                                       SACS0170
* START PUMP---PUMP=1                                                   SACS0180
* PUMP BETWEEN FT AND AT---FTAT=1,AUSE=0                                SACS0190
* PUMP AUX---AUSE=1,FTAT=0                                              SACS0200
* MAXIMUM AMOUNT TO BE PUMPED---LIMID=WEIGHT/415**3                     SACS0210
* DIRECTION---DIRCT=1.0 PUMPS FROM FT TO AT OR FROM SEA TO AUX          SACS0220
* TO REVERSE DIRECTION OF PUMP---DIRCT=-1.0                             SACS0230
* TO ORDER DEPTH USE PARAMETER ZODR.
* TO  ORDER PITCH USE PARAMETER PORD.
* IF THE TRACKING CONTROLLER IS USED,THE PARAMETERS ARE:
* DCHNG FOR  DEPTH CHANGE
* RATE  FOR DEPTH RATE CHANGE
PARAM C=10.,E=1.,D=3000.
PARAM C1=10.,C2=10.
PARAM DEL=.18901E-16,COFAA=.212502E-14,COFAB=0.0,COFAC=0.0              SACS0250
PARAM COFAD=0.0,COFAE=0.0,COFAF=0.0,COFBA=0.0                           SACS0260
PARAM COFBB=.153152E-14,COFBC=0.0,COFBD=-.186106E-10                    SACS0270
PARAM COFBE=0.0,COFBF=.17543E-12,COFCA=0.0,COFCB=0.0                    SACS0280
PARAM COFCC=.11665E-14,COFCD=0.0,COFCE=-.999506E-13                     SACS0290
PARAM COFCF=0.0,COFDA=0.0,COFDB=-.905797E-16,COFDC=0.0                  SACS0300
PARAM COFDD=.294191E-11,COFDE=0.0,COFDF=-.224359E-13                    SACS0310
PARAM COFEA=0.0,COFEB=0.0,COFEC=-.58035E-18,COFED=0.0                   SACS0320
PARAM COFEE=.19562E-13,COFEF=0.0,COFFA=0.0,COFFB=.162929E-17            SACS0330
PARAM COFFC=0.0,COFFD=-.318591E-13,COFFE=0.0,COFFF=.179521E-13          SACS0340
PARAM LC=415.0,UC=25.33,ML=.0087445,A1=-.001,A2=-.00095,A3=.00195       SACS0350
PARAM IX=7.3114E-06,IY=5.6867E-04,IZ=5.6867E-04                         SACS0360
PARAM XUDOT=-.00015,XVR=.011,XWQ=-.0075,XVV=.0065,XWW=.002,XDRDR=-.0028 SACS0370
PARAM XDSDS=-.0025,XDBDB=-.0026,XQQ=-.0002,XRR=-.00009,XRP=.00025       SACS0380
PARAM YVDOT=-.011,YWP=.0075,YV=-.021,Y1V1V=-.06,YR=.003,YV1R1=-.0073    SACS0390
```

```
PARAM YP=-.0007,YRDOT=.00009,YPDOT=-.0003,YDR=.0062,YPQ=.0002           SACS0400
PARAM YWV=-.065                                                         SACS0410
PARAM ZWDOT=-.0075,ZVP=-.007,ZS=-.0001,ZW=-.011,ZW1W1=-.03,ZVV=.065     SACS0420
PARAM ZQ=-.0045,ZW1Q1=-.006,ZVR=-.008,ZRR=-.0015,ZDS=-.005,ZDB=-.0025   SACS0430
PARAM ZQDOT=-.0002,Z1W1=0.0,ZWW=0.0,ZRP=-.0009                          SACS0440
PARAM KPDOT=-3.0E-06,KQR=-.0001,KRDOT=-7.0E-06,K1P1P=-8.0E-07,KV=-.0007 SACS0450
PARAM K1V1V=-.0009,KP=-3.5E-05,KR=-4.0E-05,KVDOT=-.00025,KVW=.0035      SACS0460
PARAM KDR=7.0E-05,KWP=2.5E-04                                           SACS0470
PARAM MQDOT=-.0004,MRP=.00015,MS=4.0E-05,MW=.003,M1W1W=-.005,MVV=.015   SACS0480
PARAM MQ=-.0025,M1W1Q=-.002,MVR=-.004,MRR=-.00055,MWDOT=-.0002          SACS0490
PARAM MCS=-.0025,MDB=.0005,M1W1=0.0,MVP=.0009                           SACS0500
PARAM NRDOT=-5.0E-04,NPQ=-4.0E-04,NPDOT=-7.0E-06                        SACS0510
PARAM NV=-.0075,N1V1V=.014                                              SACS0520
PARAM NR=-.003,N1V1R=-.0045,NP=-2.0E-06,NVDOT=.0003,NDR=-.003           SACS0530
PARAM NWV=.015,NWP=-.0002                                               SACS0540
PARAM BZB=1.011413E-03                                                  SACS0550
PARAM PLRT=.1222                                                        SACS0560
PARAM KPR=1.8E-06                                                       SACS0570
INCON YADOT=0.0,RODOT=0.0,PIDOT=0.0                                     SACS0580
* FOR TRIM CONDITION AT 15 KNOTS(25.33 FT/SEC) USE APPROXIMATELY
*  AU=8.8E-05,AT=-6.4E-05,FT=6.4E-05.
* TO SIMULATE STEP FORCE AT AU,AT,FT,CHANGE ABOVE QUANTITIES
* AS DESIRED.
* TO SIMULATE PULSE FORCE,RETURN TO THE INITIAL TRIM VALUES
* AT THE DESIRED TIME.
*
INCON AU=8.8E-05,AT=-6.4E-05,FT=-2.144E-03
*
INCON  DS=0.0,DB=0.0,DR=0.0                                             SACS0600
PARAM DZ1=.04                                                           SACS0610
PARAM DY1=.0008                                                         SACS0620
INTGER PUMP,FTAT,AUSE                                                   SACS0630
INTGER PUPLS,PUMIN                                                      SACS0640
INTGER DBTMP,DBTMM,DSTMP,DSTMM                                          SACS0650
INTGER PUFTA,PUAUX                                                      SACS0660
INTGER NPLOT                                                            SACS0670
CONST NPLOT=1                                                           SACS0680
CONTRL FINTIM=200.0,DELT=.01,DELS=.5
*
* FOR USE IN STERN ONLY PLANES SUBMARINE.
*
INITIAL                                                                 SACS0700
      CI=1.0/C                                                          SACS0710
      UC2=UC**2                                                         SACS0720
      LC2=LC**2                                                         SACS0730
      LC3=LC2*LC                                                        SACS0740
      IZX=IZ-IX                                                         SACS0750
      IYX=IY-IX                                                         SACS0760
```

```
      IZY=IZ-IY                                                         SACS0770
      ACC=ML-ZWDOT                                                      SACS0780
      ACE=MWDOT/LC                                                      SACS0790
      AEC=ZQDOT*LC                                                      SACS0800
      AEE=IY-MQDOT                                                      SACS0810
      INV=1.0/(ACC*AEE-ZQDOT*MWDOT)                                     SACS0820
      A11=(AEE*ZW+AEC*MW/LC)*INV*UC/LC                                  SACS0830
      A12=(ACE*ZW+ACC*MW/LC)*INV*UC/LC                                  SACS0840
      A21=(AEE*ZQ+AEC*MQ/LC)*INV*UC                                     SACS0850
      A22=(ACE*ZQ+ACC*MQ/LC)*INV*UC                                     SACS0860
      B11=(AEE*ZDS+AEC*MDS/LC)*INV*UC2/LC                               SACS0870
      B12=(ACE*ZDS+ACC*MDS/LC)*INV*UC2/LC                               SACS0880
      B1=A11/UC                                                         SACS0910
      B2=A21/UC                                                         SACS0920
      B3=A12/UC                                                         SACS0930
      B4=A22/UC                                                         SACS0940
      B5=B11/UC2                                                        SACS0950
      B7=B12/UC2                                                        SACS0970
      DY2=-DY1                                                          SACS0990
      DZ2=-DZ1                                                          SACS1000
*
* FOR USE IN BOTH PLANES SUBMARINE(UNBOUNDED,BOUNDED,TRACKING CONTROLLER)
*
INITIAL                                                                 SACS0700
      CI=1.0/C                                                          SACS0710
      CI1=1.0/C1
      CI2=1.0/C2
      UC2=UC**2                                                         SACS0720
      LC2=LC**2                                                         SACS0730
      LC3=LC2*LC                                                        SACS0740
      IZX=IZ-IX                                                         SACS0750
      IYX=IY-IX                                                         SACS0760
      IZY=IZ-IY                                                         SACS0770
      ACC=ML-ZWDOT                                                      SACS0780
      ACE=MWDOT/LC                                                      SACS0790
      AEC=ZQDOT*LC                                                      SACS0800
      AEE=IY-MQDOT                                                      SACS0810
      INV=1.0/(ACC*AEE-ZQDOT*MWDOT)                                     SACS0820
      A11=(AEE*ZW+AEC*MW/LC)*INV*UC/LC                                  SACS0830
      A12=(ACE*ZW+ACC*MW/LC)*INV*UC/LC                                  SACS0840
      A21=(AEE*ZQ+AEC*MQ/LC)*INV*UC
      A22=(ACE*ZQ+ACC*MQ/LC)*INV*UC
      B11=(AEE*ZDS+AEC*MDS/LC)*INV*UC2/LC                               SACS0870
      B12=(ACE*ZDS+ACC*MDS/LC)*INV*UC2/LC                               SACS0880
      B21=(AEE*ZDB+AEC*MDB/LC)*INV*UC2/LC                               SACS0890
      B22=(ACE*ZDB+ACC*MDB/LC)*INV*UC2/LC                               SACS0900
      B1=A11/UC                                                         SACS0910
      B2=A21/UC                                                         SACS0920
```

```
      B3=A12/UC                                                  SACS0930
      B4=A22/UC                                                  SACS0940
      B5=B11/UC2                                                 SACS0950
      B6=B21/UC2                                                 SACS0960
      B7=B12/UC2                                                 SACS0970
      B8=B22/UC2                                                 SACS0980
      DY2=-DY1                                                   SACS0990
      DZ2=-DZ1                                                   SACS1000
*
* FOR USE IN STERN ONLY PLANES SUBMARINE
*
DERIVATIVE                                                       SACS1010
NOSORT                                                           SACS1020
      DEPTH=INTGRL(0.0,ZODOT)                                    SACS1030
*
* CHANGE VALUE OF I.C IN INTEGRATOR IF UC IS ALTERED
*
      U=INTGRL(25.33,UDOT)                                       SACS1040
      V=INTGRL(0.,VDOT)                                          SACS1050
      W=INTGRL(0.,WDOT)                                          SACS1060
      P=INTGRL(0.,PDOT)                                          SACS1070
      Q=INTGRL(0.,QDOT)                                          SACS1080
      R=INTGRL(0.,RDOT)                                          SACS1090
      RCLL=INTGRL(0.0,RODOT)                                     SACS1100
      PITCH=INTGRL(0.0,PIDOT)                                    SACS1110
      YAW=INTGRL(0.0,YADOT)                                      SACS1120
      Q1=INTGRL(0.0,QD1)                                         SACS1130
      THTA=INTGRL(0.0,Q1)
      W1=INTGRL(0.0,WD1)                                         SACS1140
      K11=INTGRL(0.0,KD11)                                       SACS1150
      K12=INTGRL(0.0,KD12)                                       SACS1160
      K13=INTGRL(0.0,KD13)                                       SACS1170
      K14=INTGRL(0.0,KD14)                                       SACS1180
      K22=INTGRL(0.0,KD22)                                       SACS1190
      K23=INTGRL(0.0,KD23)                                       SACS1200
      K24=INTGRL(0.0,KD24)                                       SACS1210
      K33=INTGRL(0.0,KD33)                                       SACS1220
      K34=INTGRL(0.0,KD34)                                       SACS1230
      K44=INTGRL(0.0,KD44)                                       SACS1240
      F11=B11*K12+B12*K14                                        SACS1250
      F21=B11*K22+B12*K24                                        SACS1270
      F31=B11*K23+B12*K34                                        SACS1290
      F41=B11*K24+B12*K44                                        SACS1310
      KD11=E-CI*(F11**2)
      KD12=K11+K12*A11+K14*A12-CI*(F21*F11)
      KD13=-CI*(F31*F11)-UC*K11
      KD14=K12*A21+K13+K14*A22-CI*(F41*F11)
      KD22=2.0*K12+2.0*K22*A11+2.0*K24*A12-CI*(F21**2)+A
```

```
      KD23=K13+K23*A11+K34*A12-CI*(F31*F21)-UC*K12
      KD24=K14+K24*A11+K44*A12+K22*A21+K23+K24*A22-CI*...
      (F41*F21)
      KD33=-CI*(F31**2)+D-UC*2.0*K13
      KD34=K23*A21+K33+K34*A22-CI*(F41*F31)-UC*K14
      KD44=2.0*K24*A21+2.0*K34+2.0*K44*A22-CI*(F41**2)+B
      IF(TIME.GE.20.0) GOTO 30                                          SACS1440
      ZODOT=0.0                                                         SACS1450
      UDOT=0.0                                                          SACS1460
      VDOT=0.0                                                          SACS1470
      WDOT=0.0                                                          SACS1480
      PDOT=0.0                                                          SACS1490
      QDOT=0.0                                                          SACS1500
      RDOT=0.0                                                          SACS1510
      RODOT=0.0                                                         SACS1520
      PIDOT=0.0                                                         SACS1530
      YADOT=0.0                                                         SACS1540
      GOTO 31                                                           SACS1550
   30 CONTINUE                                                          SACS1560
      ZM1=K12*641.609                                                   SACS1570
      ZM2=K14*641.609                                                   SACS1580
      ZM3=K22*16251.953                                                 SACS1590
      ZM4=K23*641.609                                                   SACS1600
      ZM5=K24*16251.953                                                 SACS1610
      ZM6=K34*641.609                                                   SACS1620
      ZM7=K44*16251.953                                                 SACS1630
      Y11=-(B5*ZM1+B7*ZM2)*CI                                           SACS1640
      Y21=-(B5*ZM3+B7*ZM5)*CI                                           SACS1660
      Y31=-(B5*ZM4+B7*ZM6)*CI                                           SACS1680
      Y41=-(B5*ZM5+B7*ZM7)*CI                                           SACS1700
*
* FOR USE IN BOTH PLANES SUBMARINE(BOUNDED,UNBOUNDED CCNTROLLER)
*
DERIVATIVE                                                              SACS1010
NOSORT                                                                  SACS1020
      DEPTH=INTGRL(0.0,ZODOT)                                           SACS1030
*
* CHANGE VALUE OF I.C IN INTEGRATOR IF UC IS ALTERED
*
      U=INTGRL(25.33,UDOT)                                              SACS1040
      V=INTGRL(0.,VDOT)                                                 SACS1050
      W=INTGRL(0.,WDOT)                                                 SACS1060
      P=INTGRL(0.,PDOT)                                                 SACS1070
      Q=INTGRL(0.,QDOT)                                                 SACS1080
      R=INTGRL(0.,RDOT)                                                 SACS1090
      RCLL=INTGRL(0.0,RODOT)                                            SACS1100
      PITCH=INTGRL(0.0,PIDOT)                                           SACS1110
      YAW=INTGRL(0.0,YADOT)                                             SACS1120
```

```
      Q1=INTGRL(0.0,QD1)                                                SACS1130
      W1=INTGRL(0.0,WD1)                                                SACS1140
      K11=INTGRL(0.0,KD11)                                              SACS1150
      K12=INTGRL(0.0,KD12)                                              SACS1160
      K13=INTGRL(0.0,KD13)                                              SACS1170
      K14=INTGRL(0.0,KD14)                                              SACS1180
      K22=INTGRL(0.0,KD22)                                              SACS1190
      K23=INTGRL(0.0,KD23)                                              SACS1200
      K24=INTGRL(0.0,KD24)                                              SACS1210
      K33=INTGRL(0.0,KD33)                                              SACS1220
      K34=INTGRL(0.0,KD34)                                              SACS1230
      K44=INTGRL(0.0,KD44)                                              SACS1240
      F11=B11*K12+B12*K14                                               SACS1250
      F12=B21*K12+B22*K14                                               SACS1260
      F21=B11*K22+B12*K24                                               SACS1270
      F22=B21*K22+B22*K24                                               SACS1280
      F31=B11*K23+B12*K34                                               SACS1290
      F32=B21*K23+B22*K34                                               SACS1300
      F41=B11*K24+B12*K44                                               SACS1310
      F42=B21*K24+B22*K44                                               SACS1320
      KD11=E-CI1*F11**2-CI2*F12**2
      KD12=K11+A11*K12+A12*K14-CI1*F11*F21-CI2*F12*F22
      KD13=      -UC*K11-CI1*F11*F31-CI2*F12*F32
      KD14=A21*K12+K13+A22*K14-CI1*F11*F41-CI2*F12*F42
      KD22=2.0*(K12+A11*K22+K24*A12)-CI1*F21**2-CI2*F22**2+A
      KD23=K13+A11*K23+K34*A12-UC*K12        -CI1*F21*F31...
      -CI2*F22*F32
      KD24=K14+A11*K24+A12*K44+A21*K22+A22*K24+K23-CI1*F21*F41...
      -CI2*F22*F42
      KD33=-2.0*UC*K13           -CI1*F31**2-CI2*F32**2+D
      KD34=-UC*K14      +A21*K23+K33+A22*K34-CI1*F31*F41-CI2*F32*F42
      KD44=2.0*(A21*K24+K34+A22*K44)-CI1*F41**2-CI2*F42**2+B
      IF(TIME.GE.20.0) GOTO 30                                          SACS1440
      ZODOT=0.0                                                         SACS1450
      UDOT=0.0                                                          SACS1460
      VDOT=0.0                                                          SACS1470
      WDOT=0.0                                                          SACS1480
      PDOT=0.0                                                          SACS1490
      QDOT=0.0                                                          SACS1500
      RDOT=0.0                                                          SACS1510
      RODOT=0.0                                                         SACS1520
      PIDOT=0.0                                                         SACS1530
      YADOT=0.0                                                         SACS1540
      GOTO 31                                                           SACS1550
   30 CONTINUE                                                          SACS1560
      ZM1=K12*641.609                                                   SACS1570
      ZM2=K14*641.609                                                   SACS1580
      ZM3=K22*16251.953                                                 SACS1590
```

```
      ZM4=K23*641.609                                                   SACS1600
      ZM5=K24*16251.953                                                 SACS1610
      ZM6=K34*641.609                                                   SACS1620
      ZM7=K44*16251.953                                                 SACS1630
      Y11=-(B5*ZM1+B7*ZM2)*CI                                           SACS1640
      Y12=-(B6*ZM1+B8*ZM2)*CI                                           SACS1650
      Y21=-(B5*ZM3+B7*ZM5)*CI                                           SACS1660
      Y22=-(B6*ZM3+B8*ZM5)*CI                                           SACS1670
      Y31=-(B5*ZM4+B7*ZM6)*CI                                           SACS1680
      Y32=-(B6*ZM4+B8*ZM6)*CI                                           SACS1690
      Y41=-(B5*ZM5+B7*ZM7)*CI                                           SACS1700
      Y42=-(B6*ZM5+B8*ZM7)*CI                                           SACS1710
*
* FOR USE IN BOTH PLANES SUBMARINE(TRACKING CONTROLLER)
*
DERIVATIVE
NOSORT
      DEPTH=INTGRL(0.0,ZODOT)
*
* CHANGE VALUE OF I.C IN INTEGRATOR IF UC IS ALTERED
*
      U=INTGRL(25.33,UDOT)
      V=INTGRL(0.,VDOT)
      W=INTGRL(0.,WDOT)
      P=INTGRL(0.,PDOT)
      Q=INTGRL(0.,QDOT)
      R=INTGRL(0.,RDOT)
      ROLL=INTGRL(0.0,RODOT)
      PITCH=INTGRL(0.0,PIDOT)
      YAW=INTGRL(0.0,YADOT)
      Q1=INTGRL(0.0,QD1)
      W1=INTGRL(0.0,WD1)
      K11=INTGRL(0.0,KD11)
      K12=INTGRL(0.0,KD12)
      K13=INTGRL(0.0,KD13)
      K14=INTGRL(0.0,KD14)
      K22=INTGRL(0.0,KD22)
      K23=INTGRL(0.0,KD23)
      K24=INTGRL(0.0,KD24)
      K33=INTGRL(0.0,KD33)
      K34=INTGRL(0.0,KD34)
      K44=INTGRL(0.0,KD44)
      S1=INTGRL(0.0,SK1)
      S2=INTGRL(0.0,SK2)
      S3=INTGRL(0.0,SK3)
      S4=INTGRL(0.0,SK4)
      F11=B11*K12+B12*K14
      F12=B21*K12+B22*K14
```

```
      F21=B11*K22+B12*K24
      F22=B21*K22+B22*K24
      F31=B11*K23+B12*K34
      F32=B21*K23+B22*K34
      F41=B11*K24+B12*K44
      F42=B21*K24+B22*K44
      KC11=E-CI1*F11**2-CI2*F12**2
      KD12=K11+A11*K12+A12*K14-CI1*F11*F21-CI2*F12*F22
      KD13=      -UC*K11-CI1*F11*F31-CI2*F12*F32
      KD14=A21*K12+K13+A22*K14-CI1*F11*F41-CI2*F12*F42
      KD22=2.0*(K12+A11*K22+K24*A12)-CI1*F21**2-CI2*F22**2+A
      KD23=K13+A11*K23+K34*A12-UC*K12          -CI1*F21*F31...
      -CI2*F22*F32
      KD24=K14+A11*K24+A12*K44+A21*K22+A22*K24+K23-CI1*F21*F41...
      -CI2*F22*F42
      KD33=-2.0*UC*K13              -CI1*F31**2-CI2*F32**2+D
      KD34=-UC*K14         +A21*K23+K33+A22*K34-CI1*F31*F41-CI2*F32*F42
      KD44=2.0*(A21*K24+K34+A22*K44)-CI1*F41**2-CI2*F42**2+B
      SK1=-(CI1*B11*F11+CI2*B21*F12)*S2-(CI1*B12*F11+CI2*B22*F12)*S4...
      -E*ZODR
      SK2=S1+(A11-CI1*B11*F21-CI2*B21*F22)*S2+(A12-CI1*B12*F21...
      -CI2*B22*F22)*S4
      SK3=      -(CI1*B11*F31+CI2*B21*F32)*S2+(   -CI1*B12*F31...
      -CI2*B22*F32)*S4-D*PORD-UC*S1
      SK4=(A21-CI1*B11*F41-CI2*B21*F42)*S2+S3+(A22-CI1*B12*F41...
      -CI2*B22*F42)*S4
      IF(TIME.GE.20.0) GOTO 30
      ZODOT=0.0
      UDOT=0.0
      VDOT=0.0
      WDOT=0.0
      PDOT=0.0
      QDOT=0.0
      RDOT=0.0
      RODOT=0.0
      PIDOT=0.0
      YADOT=0.0
      GOTO 31
   30 CONTINUE
      ZM1=K12*641.609
      ZM2=K14*641.609
      ZM3=K22*16251.953
      ZM4=K23*641.609
      ZM5=K24*16251.953
      ZM6=K34*641.609
      ZM7=K44*16251.953
      Y11=-(B5*ZM1+B7*ZM2)*CI
      Y12=-(B6*ZM1+B8*ZM2)*CI
```

```
      Y21=-(B5*ZM3+B7*ZM5)*CI
      Y22=-(B6*ZM3+B8*ZM5)*CI
      Y31=-(B5*ZM4+B7*ZM6)*CI
      Y32=-(B6*ZM4+B8*ZM6)*CI
      Y41=-(B5*ZM5+B7*ZM7)*CI
      Y42=-(B6*ZM5+B8*ZM7)*CI
*
* STERN ONLY PLANES OPTIMAL CONTROLLER
*
*AUTOMATIC CONTROLLER FOR THE STERN PLANES ONLY
      PERR=PITCH-PORD                                                   SACS1730
      ZOER=DEPTH-ZODR                                                   SACS1740
      DSAD=Y11*ZOER+Y21*ZODOT/U+Y31*PERR+Y41*PIDOT/U                    SACS1790
      IF (DSAD.GE.0.436) DSAD=0.436
      IF (DSAD.LE.-0.436) DSAD=-0.436
      DSOD=DSAD
*
*INITIAL BOTH PLANES OPTIMAL UNBOUNDED CONTROLLER.
*
* AUTOMATIC CONTROLLER FOR THE FAIRWATER AND STERN PLANES               SACS1720
      PERR=PITCH-PORD
      ZOER=DEPTH-ZODR
      IF(PERR.LT.-.174) PERR=-.174
      IF(PERR.GT..174) PERR=.174
      IF(ZOER.LT.-2.0) ZOER=-2.0
      IF(ZOER.GT.2.0) ZOER=2.0
      DSAD=Y11*ZOER+Y21*ZODOT/U+Y31*PERR+Y41*PIDOT/U
      DBAD=Y12*ZOER+Y22*ZODOT/U+Y32*PERR+Y42*PIDOT/U
      DSOD=REALPL(0.0,.1,DSAD)
      DBOD=REALPL(0.0,.1,DBAD)
*
* OPTIMAL BOTH PLANES BOUNDED CONTROLLER WITHOUT LIMITERS AND
* PLANE NOISE SUPPRESSORS
*
* AUTOMATIC CONTROLLER FOR THE FAIRWATER AND STERN PLANES
      PERR=PITCH-PORD                                                   SACS1730
      ZOER=DEPTH-ZODR                                                   SACS1740
      P2=K12*ZOER+K22*ZODOT+K23*PERR+K24*PIDOT
      P4=K14*ZOER+K24*ZODOT+K34*PERR+K44*PIDOT
      REGS=(B11*P2+B12*P4)/C1
      ABRGS=ABS(REGS)
      IF(ABRGS.LE.0.436) DSAD=-REGS
      IF(REGS.LT.-0.436) DSAD=0.436
      IF(REGS.GT.0.436) DSAD=-0.436
      REGB=(B21*P2+B22*P4)/C2
      ABRGB=ABS(REGB)
      IF(ABRGB.LE.0.436) DBAD=-REGB
      IF(REGB.GT.0.436) DBAD=0.436
```

```
      IF(REGB.LT.-0.436) DBAD=-0.436
      DBOD=DBAD
      DSOD=DSAD
*
* BOTH PLANES TRACKING CONTROLLER.
*
* AUTOMATIC CONTROLLER FOR THE FAIRWATER AND STERN PLANES
      PI1=-0.052
      PI2=0.052
      ADCG=ABS(DCHNG)
      DY=20.+ADCG/RATE
      XD=(DCHNG)*PI2/100.0
      Y1=RAMP(20.)
      Y2=RAMP(DY)
      ZODR=RATE*(Y1-Y2)*DCHNG/ADCG
      PORD=LIMIT(PI1,PI2,XD)
      PERR=PITCH-PORD
      ZOER=DEPTH-ZODR
      P2=K12*DEPTH+K22*ZODOT+K23*PITCH+K24*PIDOT+S2
      P4=K14*DEPTH+K24*ZODOT+K34*PITCH+K44*PIDOT+S4
      REGS=(B11*P2+B12*P4)/C1
      ABRGS=ABS(REGS)
      IF(REGS.LT.-0.436) DSAD=0.436
      IF(REGS.GT.0.436) DSAD=-0.436
      IF(ABRGS.LE.0.436) DSAD=-REGS
      REGB=(B21*P2+B22*P4)/C2
      ABRGB=ABS(REGB)
      IF(REGB.GT.0.436) DBAD=-0.436
      IF(REGB.LT.-0.436) DBAD=0.436
      IF(ABRGB.LE.0.436) DBAD=-REGB
      DBOD=DBAD
      DSOD=DSAD
*
* COMBINED MODE CONTROLLER.
*
* AUTOMATIC CONTROLLER FOR THE FAIRWATER AND STERN PLANES
      PERR=PITCH-PORD
      ZOER=DEPTH-ZODR
      P2=K12*ZOER+K22*ZODOT+K23*PERR+K24*PIDOT
      P4=K14*ZOER+K24*ZODOT+K34*PERR+K44*PIDOT
      REGS=(B11*P2+B12*P4)/C1
      ABRGS=ABS(REGS)
      IF(REGS.LT.-0.436) DSAD=0.436
      IF(REGS.GT.0.436) DSAD=-0.436
      IF(ABRGS.LE.0.436) DSAD=-REGS
      REGB=(B21*P2+B22*P4)/C2
      ABRGB=ABS(REGB)
      DBAD2=-REGB
```

```
      DFM=1.0
      IF(DBAD2.LT.0.0) DFM=-0.436
      IF(DBAD2.GT.0.0) DFM=0.436
      IF(DBAD2.EQ.0.0) DBAD2=DFM
      XKI=DFM/DBAD2
      IF(XKI.LT.1.0) GO TO 111
      IF(ABRGB.LE.0.436) DBAD=-REGB
      DBOD=DBAD
      DSOD=DSAD
      GC TO 112
  111 DS1=-REGS
      XK=XKI**0.5
*
* HERE GAINS OF S.O.P.O.C OR S.O.P.C ARE INSERTED.
*
      Y11=-0.98777
      Y21=668.35*25.33
      Y31=1857.2
      Y41=-29929.8*25.33
      DS2=Y11*ZOER+Y21*ZODOT/UC+Y31*PERR+Y41*PIDOT/UC
      XK1=(DS1-DS2)**2+DS2
      DSAD=XK*DS1+(1-XK)*DS2
      DBAD=DFM
      IF(DSAD.GE.0.436) DSAD=0.436
      IF(DSAD.LE.-0.436) DSAD=-0.436
      DBOD=DBAD
      DSOD=DSAD
*
* EQUIVALENT CONTROLLER FOR B=800.,C=10.,E=1.
*
* AUTOMATIC CONTROLLER FOR THE FAIRWATER AND STERN PLANES
      PERR=PITCH-PORD
      ZOER=DEPTH-ZODR
      P2=K12*ZOER+K22*ZODOT+K23*PERR+K24*PIDOT
      P4=K14*ZOER+K24*ZODOT+K34*PERR+K44*PIDOT
      REGS=(B11*P2+B12*P4)/C1
      ABRGS=ABS(REGS)
      IF(REGS.LT.-0.436) DSAD=0.436
      IF(REGS.GT.0.436) DSAD=-0.436
      IF(ABRGS.LE.0.436) DSAD=-REGS
      REGB=(B21*P2+B22*P4)/C2
      ABRGB=ABS(REGB)
      DBAD2=-REGB
      DFM=1.0
      IF(DBAD2.LT.0.0) DFM=-0.436
      IF(DBAD2.GT.0.0) DFM=0.436
      IF(DBAD2.EQ.0.0) DBAD2=DFM
      XKI=DFM/DBAD2
```

```
      IF(XKI.LT.1.0) GO TO 111
      IF(ABRGB.LE.0.436) DBAD=-REGB
      DBOD=DBAD
      DSOD=DSAD
      GO TO 112
  111 XK=XKI**(-1.0)
      CSAD=-REGS*(1.25*XK-0.25)
      CBAD=DFM
      IF(DSAD.GE.0.436) DSAD=0.436
      IF(DSAD.LE.-0.436) DSAD=-0.436
      DBOD=DBAD
      DSOD=DSAD
*
* FOR USE IN BOTH PLANES SUBMARINE(UNBOUNDED,BOUNCED,TRACKING CONTROLLER)
*
* PLANE ANGLE GENERATOR
  112 DSER=DSOD-DS
      DBER=DBOD-DB                                                      SACS1850
      IF(DBER.EQ.0.0) GOTO 12                                           SACS1860
      IF(DBER.LT.0.0) GOTO 17                                           SACS1870
      IF(DBTMP.EQ.1) GOTO 18                                            SACS1880
      DBTMM=0                                                           SACS1890
      DBTMP=1                                                           SACS1900
      DBC=DB                                                            SACS1910
      CBT=TIME                                                          SACS1920
   18 DBA=(TIME-DBT)*PLRT                                               SACS1930
      DB=DBC+DBA                                                        SACS1940
      GCTO 11                                                           SACS1950
   17 IF(DBTMM.EQ.1) GOTO 19                                            SACS1960
      DBTMP=0                                                           SACS1970
      DBTMM=1                                                           SACS1980
      CBC=DB                                                            SACS1990
      DBT=TIME                                                          SACS2000
   19 DBA=(DBT-TIME)*PLRT                                               SACS2010
      DB=DBC+DBA                                                        SACS2020
      GOTO 11                                                           SACS2030
   12 DBTMM=0                                                           SACS2040
      DBTMP=0                                                           SACS2050
   11 CCNTINUE                                                          SACS2060
      IF(DSER.EQ.0.0) GOTO 22                                           SACS2070
      IF(DSER.LT.0.0) GOTO 27                                           SACS2080
      IF(DSTMP.EQ.1) GOTO 28                                            SACS2090
      DSTMM=0                                                           SACS2100
      DSTMP=1                                                           SACS2110
      DSC=DS                                                            SACS2120
      DST=TIME                                                          SACS2130
   28 DSA=(TIME-DST)*PLRT                                               SACS2140
      DS=DSC+DSA                                                        SACS2150
```

```
      GOTO 21                                                           SACS2160
   27 IF(DSTMM.EQ.1) GOTO 29                                            SACS2170
      DSTMP=0                                                           SACS2180
      DSTMM=1                                                           SACS2190
      DSC=DS                                                            SACS2200
      DST=TIME                                                          SACS2210
   29 DSA=(DST-TIME)*PLRT                                               SACS2220
      DS=DSC+DSA                                                        SACS2230
      GOTO 21                                                           SACS2240
   22 DSTMM=0                                                           SACS2250
      DSTMP=0                                                           SACS2260
   21 CONTINUE                                                          SACS2270
*
* FOR USE IN STERN ONLY PLANES SUBMARINE
*
* PLANE ANGLE GENERATOR
      DSER=DSOD-DS                                                      SACS1840
      IF(DSER.EQ.0.0) GOTO 22                                           SACS2070
      IF(DSER.LT.0.0) GOTO 27                                           SACS2080
      IF(DSTMP.EQ.1) GOTO 28                                            SACS2090
      DSTMM=0                                                           SACS2100
      DSTMP=1                                                           SACS2110
      DSC=DS                                                            SACS2120
      DST=TIME                                                          SACS2130
   28 DSA=(TIME-DST)*PLRT                                               SACS2140
      DS=DSC+DSA                                                        SACS2150
      GOTO 21                                                           SACS2160
   27 IF(DSTMM.EQ.1) GOTO 29                                            SACS2170
      DSTMP=0                                                           SACS2180
      DSTMM=1                                                           SACS2190
      DSC=DS                                                            SACS2200
      DST=TIME                                                          SACS2210
   29 DSA=(DST-TIME)*PLRT                                               SACS2220
      DS=DSC+DSA                                                        SACS2230
      GOTO 21                                                           SACS2240
   22 DSTMM=0                                                           SACS2250
      DSTMP=0                                                           SACS2260
   21 CONTINUE                                                          SACS2270
*
* FROM THAT POINT THE PROGRAM IS UNCHANGED FOR BOTH SUBMARINES
* AND ALL CONTROLLERS
*
* PUMP EQUATIONS                                                        SACS2280
      IF(PUMP.EQ.1) GOTO 1                                              SACS2290
      GOTO 9                                                            SACS2300
    1 IF(DIRCT.EQ.-1.0) GOTO 3                                          SACS2310
      IF(PUPLS.EQ.1) GOTO 2                                             SACS2320
      FWT=FT                                                            SACS2330
```

```
      AFT=AT                                                            SACS2340
      ALX=AU                                                            SACS2350
      WW=TIME                                                           SACS2360
      PUPLS=1                                                           SACS2370
      PUMIN=0                                                           SACS2380
      COUP=COUP+1.0                                                     SACS2390
    2 MAXI=(TIME-WW)*KPR                                                SACS2400
      IF(MAXI.GT.LIMID) GOTO 6                                          SACS2410
      IF(FTAT.NE.1) GOTO 8                                              SACS2420
      GOTO 7                                                            SACS2430
    3 IF(PUMIN.EQ.1) GOTO 4                                             SACS2440
      COUDN=COUDN+1.0                                                   SACS2450
      FWT=FT                                                            SACS2460
      AFT=AT                                                            SACS2470
      AUX=AU                                                            SACS2480
      WW=TIME                                                           SACS2490
      PUPLS=0                                                           SACS2500
      PUMIN=1                                                           SACS2510
    4 MAXI=(WW-TIME)*KPR                                                SACS2520
      MAXIM=-MAXI                                                       SACS2530
      IF(MAXIM.GE.LIMID) GOTO 6                                         SACS2540
      IF(FTAT.NE.1) GOTO 8                                              SACS2550
    7 FT=FWT-MAXI                                                       SACS2560
      AT=AFT+MAXI                                                       SACS2570
      GCTO 9                                                            SACS2580
    8 IF(AUSE.NE.1) GOTO 6                                              SACS2590
      AU=AUX+MAXI                                                       SACS2600
      GOTO 9                                                            SACS2610
    6 PUMP=0                                                            SACS2620
      FTAT=0                                                            SACS2630
      AUSE=0                                                            SACS2640
      PUPLS=0                                                           SACS2650
      PUMIN=0                                                           SACS2660
    9 CONTINUE                                                          SACS2670
* AUXILIARY EQUATIONS                                                   SACS2680
      ZODOT=-U*SIN(PITCH)+V*COS(PITCH)*SIN(ROLL)...                     SACS2690
      +W*COS(PITCH)*COS(ROLL)                                           SACS2700
      PIDOT=Q*COS(ROLL)-R*SIN(ROLL)                                     SACS2710
      YADOT=(R*COS(ROLL)+Q*SIN(ROLL))/COS(PITCH)                        SACS2720
      RODOT=P+YADOT*SIN(PITCH)                                          SACS2730
      DSGRA=DS*57.296                                                   SACS2740
      DBGRA=DB*57.296                                                   SACS2750
      PITGRA=180.*PITCH/3.14159                                         SACS2760
      RCLGRA=ROLL*57.296                                                SACS2770
      PA1=XDRDR*U*U*DR*DR/LC                                            SACS2780
      PA2=XDSDS*U*U*DS*DS/LC                                            SACS2790
      PA3=XDBDB*U*U*DB*DB/LC                                            SACS2800
      PB1=YDR*U*U*DR/LC                                                 SACS2810
```

```
      PC2=ZDS*U*U*DS/LC                                                 SACS2820
      PC3=ZDB*U*U*DB/LC                                                 SACS2830
      PC1=KDR*U*U*DR/LC2                                                SACS2840
      PE2=MDS*U*U*DS/LC2                                                SACS2850
      PE3=MDB*U*U*DB/LC2                                                SACS2860
      PF1=NDR*U*U*DR/LC2                                                SACS2870
      PA=PA1+PA2+PA3                                                    SACS2880
      PB=PB1                                                            SACS2890
      PC=PC2+PC3                                                        SACS2900
      PD=PD1                                                            SACS2910
      PE=PE2+PE3                                                        SACS2920
      PF=PF1                                                            SACS2930
*NON LINEAR RELATIONS                                                   SACS2940
      ABV=ABS(V)                                                        SACS2950
      ABW=ABS(W)                                                        SACS2960
      ABP=ABS(P)                                                        SACS2970
      ABQ=ABS(Q)                                                        SACS2980
      ABR=ABS(R)                                                        SACS2990
      VVWW=V*V+W*W                                                      SACS3000
      AVW=SQRT(VVWW)                                                    SACS3010
      ABWP=FCNSW(W,-1.0,0.0,1.0)                                        SACS3020
      ABVP=FCNSW(V,-1.0,0.0,1.0)                                        SACS3030
      SA1=+LC*(XQQ*Q**2+XRR*R**2+XRP*R*P)                               SACS3040
      SA2=+(ML*V*R+XVR*V*R+XWQ*W*Q-ML*W*Q)                              SACS3050
      SA3=+(XVV*V**2+XWW*W**2)/LC-SIN(PITCH)*(AT+FT+AU)                 SACS3060
      SA4=+(A1*U**2+A2*U*UC+A3*UC**2)/LC                                SACS3070
      SB1=+LC*YPQ*P*Q                                                   SACS3080
      SB2=+(YWP*W*P+YV1R1*ABR*AVW*ABVP+ML*W*P-ML*U*R)                   SACS3090
      SB3=+(YWV*W*V+Y1V1V*AVW*V)/LC+SIN(ROLL)*COS(PITCH)...             SACS3100
      *(AT+FT+AU)                                                       SACS3110
      SB4=(YR*R+YP*P+YV*V/LC)*U                                         SACS3120
      SC1=LC*R*(ZRR*R+ZRP*P)                                            SACS3130
      SC2=+(ZVP*V*P+ZVR*V*R+ZW1Q1*ABQ*AVW*ABWP+ML*U*Q-ML*P*V)           SACS3140
      SC3=+(ZWW*W**2+ZVV*V**2+ZW1W1*W*AVW+U*Z1W1*ABW+U*U*ZS)/LC         SACS3150
      SC4=ZQ*U*Q+ZW*U*W/LC+COS(PITCH)*COS(ROLL)*(AT+FT+AU)              SACS3160
      SD1=+(KQR*Q*R+K1P1P*ABP*P)-IZY*Q*R                                SACS3170
      SD2=(KWP*W*P-BZB*SIN(ROLL)*COS(PITCH))/LC                         SACS3180
      SD3=+(K1V1V*V*AVW+KVW*V*W+KS*U**2)/LC2                            SACS3190
      SD4=((KP*P+KR*R)/LC+KV*V/LC2)*U                                   SACS3200
      SE1=(MRP*P+MRR*R+IZX*P)*R                                         SACS3210
      SE2=((MVR*R+MVP*P)*V+M1W1Q*AVW*Q-BZB*SIN(PITCH))/LC               SACS3220
      SE3=(MVV*V**2+MWW*W**2+M1W1W*AVW*W+M1W1*U*AVW+U**2*MS)/LC2        SACS3230
      SE4=MQ*U*Q/LC+(MW*U*W-(175.5*FT-219.5*AT)*COS(PITCH)...           SACS3240
      *COS(ROLL))/LC2                                                   SACS3250
      SF1=(NPQ-IYX)*P*Q                                                 SACS3260
      SF2=+(NWP*W*P+N1V1R*AVW*R)/LC                                     SACS3270
      SF3=(NWV*W+N1V1V*AVW)*V/LC2                                       SACS3280
      SF4=(NP*P+NR*R)*U/LC+(NV*U*V+(175.5*FT-219.5*AT)*...              SACS3290
```

```
      COS(PITCH)*SIN(ROLL))/LC2                                         SACS3300
      SA=SA1+SA2+SA3+SA4                                                SACS3310
      SB=SB1+SB2+SB3+SB4                                                SACS3320
      SC=SC1+SC2+SC3+SC4                                                SACS3330
      SD=SD1+SD2+SD3+SD4                                                SACS3340
      SE=SE1+SE2+SE3+SE4                                                SACS3350
      SF=SF1+SF2+SF3+SF4                                                SACS3360
      ZA=SA+PA                                                          SACS3370
      ZB=SB+PB                                                          SACS3380
      ZC=SC+PC                                                          SACS3390
      ZD=SD+PD                                                          SACS3400
      ZE=SE+PE                                                          SACS3410
      ZF=SF+PF                                                          SACS3420
*
* STATEMENT USED FOR PULSE EXCITATION SIMULATION.
*
      IF(TIME.GT.30.0) FT=6.4E-05
* EQUATIONS OF MOTION                                                   SACS3430
      UDOT=(COFAA*ZA+COFAB*ZB+COFAC*ZC+COFAD*ZD+COFAE*ZE+COFAF*ZF)/DEL  SACS3440
      VDOT=(COFBA*ZA+COFBB*ZB+COFBC*ZC+COFBD*ZD+COFBE*ZE+COFBF*ZF)/DEL  SACS3450
      WDOT=(COFCA*ZA+COFCB*ZB+COFCC*ZC+COFCD*ZD+COFCE*ZE+COFCF*ZF)/DEL  SACS3460
      PDOT=(COFDA*ZA+COFDB*ZB+COFDC*ZC+COFDD*ZD+COFDE*ZE+COFDF*ZF)/DEL  SACS3470
      QDOT=(COFEA*ZA+COFEB*ZB+COFEC*ZC+COFED*ZD+COFEE*ZE+COFEF*ZF)/DEL  SACS3480
      RDOT=(COFFA*ZA+COFFB*ZB+COFFC*ZC+COFFD*ZD+COFFE*ZE+COFFF*ZF)/DEL  SACS3490
      HEVY=HEAVY/LC3                                                    SACS3500
      WD1=(B1*W1+B2*Q1)*U+(B5*DS+B6*DB)*U**2+HEVY                       SACS3510
      QD1=(B3*W1+B4*Q1)*U+(B7*DS+B8*DB)*U**2                            SACS3520
*
* THE FOLLOWING TWO VARIABLES  WERE SET EQUAL TO ZERO FOR TRIM
* CONTROLLER OUT OF OPERATION.
*
      VERT=0.0
      PRET=0.0
      PRER=REALPL(0.0,30.0,PRET)                                        SACS3550
      VERR=REALPL(0.0,30.0,VERT)                                        SACS3560
      PUMPR=DEADSP(DZ2,DZ1,VERR)                                        SACS3570
      IF(PUMPR.EQ.0.0) GOTO 40                                          SACS3580
      IF(PUAUX.EQ.1) GOTO 42                                            SACS3590
      PUFTA=0                                                           SACS3600
      PUAUX=1                                                           SACS3610
      PUPLS=0                                                           SACS3620
      PUMIN=0                                                           SACS3630
   42 APUPR=ABS(PUMPR)                                                  SACS3640
      PUMP=1                                                            SACS3650
      LIMID=1.0                                                         SACS3660
      AUSE=1                                                            SACS3670
      FTAT=0                                                            SACS3680
      DIRCT=-PUMPR/APUPR                                                SACS3690
```

```
      GCTO 51                                                           SACS3700
   40 PUAUX=0                                                           SACS3710
      PUMP=0                                                            SACS3720
   41 CONTINUE                                                          SACS3730
      PUMPS=DEADSP(DY2,DY1,PRER)                                        SACS3740
      IF(PUMPS.EQ.0.0) GOTO 50                                          SACS3750
      IF(PUFTA.EQ.1) GOTO 52                                            SACS3760
      PUFTA=1                                                           SACS3770
      PUPLS=0                                                           SACS3780
      PUMIN=0                                                           SACS3790
   52 APUPS=ABS(PUMPS)                                                  SACS3800
      APUPS=ABS(PUMPS)                                                  SACS3810
      PUMP=1                                                            SACS3820
      LIMID=1.0                                                         SACS3830
      FTAT=1                                                            SACS3840
      AUSE=0                                                            SACS3850
      DIRCT=-PUMPS/APUPS                                                SACS3860
      GOTO 51                                                           SACS3870
   50 LIMID=0.0                                                         SACS3880
   51 CONTINUE                                                          SACS3890
      ATT=AT*LC3                                                        SACS3900
      AUT=AU*LC3                                                        SACS3910
      FTT=FT*LC3                                                        SACS3920
   31 CONTINUE                                                          SACS3930
SAMPLE                                                                  SACS3940
PRINT 5.00,AUT,DSGRA,DBGRA,DEPTH,PITCH,REGS,REGB,ZODOT,PIDOT,DS1,XK,...
      Y11,Y21,Y31,Y41,Y12,Y22,Y32,Y42,DSAD,DBAD2,U,DS2,XK1
      CALL DRWG(1,1,TIME,DSGRA)
      CALL DRWG(2,1,TIME,DBGRA)
      CALL DRWG(3,1,TIME,DEPTH)
      CALL DRWG(4,1,TIME,PITCH)
      CALL DRWG(5,1,TIME,FTT)
TERMINAL                                                                SACS4060
      CALL ENDRW(NPLOT)                                                 SACS4070
END                                                                     SACS4080
STOP                                                                    SACS4090
//PLOT.SYSIN DD *
VAS TEST C . M OPT. D . 3000
STERN PLANE ANGLE VS TIME                                                      4
                                        5.0       7.0
VAS TEST C . M OPT. D . 3000
FAIRWATER PLANE ANGLE VS TIME                                                  4
                                        5.0       7.0
VAS TEST C . M OPT. D . 3000
DEPTH VS TIME                                                                  4
                                        5.0       7.0
VAS TEST C . M OPT. D . 3000
PITCH VS TIME                                                                  4
```

```
                                                    5.0       7.0
            VAS TEST C . M OPT. D . 3000                                                   4
            FWD TRIM TANK
                                                    5.0       7.0
```

```
C AUXILIARY PROGRAMS


// EXEC FORTCLG,REGION.GO=126K
//FORT.SYSIN   DD *
C
C PROGRAM 1
C
C REAL EIGENSYSTEM PROBLEM/ SUBMARINE PLANT
      DIMENSION A(4,4),WR(4),WI(4),Z(4,4),ZI(4,4),L(4),M(4)
      DOUBLE PRECISION A,WR,WI,Z,ZI,D
   50 FORMAT(I10)
  100 FORMAT(4D20.13)
  150 FORMAT('0 PROBLEM MATRIX:')
  200 FORMAT(' ',4D30.16)
  250 FORMAT('0  EIGENVALUES:')
  300 FORMAT('0  EIGENVECTORS:')
  400 FORMAT('0 INVERSE EIGENVECTOR MATRIX:')
    4 READ(5,50)N
      IF(N.EQ.0)STOP
      DO 1 J=1,N
      READ(5,100)(A(I,J),I=1,N)
    1 CONTINUE
      WRITE(6,150)
      DO 2 I=1,N
      WRITE(6,200)(A(I,J),J=1,N)
    2 CONTINUE
      CALL EISPAC(4,N ,MATRIX('REAL',A),VALUES(WR,WI),
     1VECTOR(Z),ERROR(IERROR))
C     NORMALIZE EIGENVECTORS
      CALL NORMRL(4,N,WR,WI,Z)
      WRITE (6,250)
      WRITE(6,200)(WR(I),I=1,N)
      WRITE(6,200)(WI(I),I=1,N)
      WRITE(6,300)
      DO 3 I=1,N
      WRITE(6,200)(Z(I,J),J=1,N)
    3 CONTINUE
      WRITE(6,50)IERROR
      CALL DMINV(Z,N,D,L,M)
      WRITE(6,400)
      DO 5 I=1,N
      WRITE(6,200)(Z(I,J),J=1,N)
    5 CONTINUE
      GO TO 4
      END
//LINK.SYSLIB  DD DSN=SYS3.EISPACK,DISP=SHR
```

```
//                  DD DSN=SYS1.FORTLIB,DISP=SHR
//                  DD DSN=SYS1.MPSLIB,DISP=SHR
//GO.EISPACLB       DD DSN=SYS3.EISPACK,DISP=SHR
//GO.SYSIN          DD *


// EXEC FORTCLG,REGION.GO=126K
//FORT.SYSIN        DD *
C
C  PRCGRAM 2
C
C  MATRIX MULTIPLICATION / DECOUPLING THE STATES
      DIMENSION Z(4,4),ZI(4,4),A(4,4),B(4,4),R(4,4)
   10 FORMAT(I2)
   20 FORMAT(4E13.6)
   30 FORMAT('0 ZMATRIX:')
   40 FORMAT('0 ZIMATRIX:')
   50 FORMAT('0 AMATRIX:')
   60 FORMAT('0 BMATRIX:')
   70 FORMAT(' ',4E23.6)
   80 FORMAT('0 ANMATRIX:')
   90 FORMAT('0 BNMATRIX:')
   11 READ(5,10)N
      IF(N.EQ.0)STOP
      DO 1 J=1,N
      READ(5,20)(ZI(I,J),I=1,N)
    1 CONTINUE
      WRITE(6,40)
      DO 2 I=1,N
      WRITE(6,70)(ZI(I,J),J=1,N)
    2 CONTINUE
      DO 3 J=1,N
      READ(5,20)(A(I,J),I=1,N)
    3 CONTINUE
      WRITE(6,50)
      DO 4 I=1,N
      WRITE(6,70)(A(I,J),J=1,N)
    4 CONTINUE
      CALL GMPRD(ZI,A,R,N,N,N)
      DO 5 J=1,N
      READ(5,20)(Z(I,J),I=1,N)
    5 CONTINUE
      WRITE(6,30)
      DO 6 I=1,N
      WRITE(6,70)(Z(I,J),J=1,N)
    6 CONTINUE
      CALL GMPRD(R,Z,A,N,N,N)
```

```
      WRITE(6,80)
      DO 7 I=1,N
      WRITE(6,70)(A(I,J),J=1,N)
    7 CONTINUE
      READ(5,10)L
      DO 8 J=1,L
      READ(5,20)(B(I,J),I=1,N)
    8 CONTINUE
      WRITE(6,60)
      DO 9 I=1,N
      WRITE(6,70)(B(I,J),J=1,L)
    9 CONTINUE
      CALL GMPRD(ZI,B,R,N,N,L)
      WRITE(6,90)
      DO 12 I=1,N
      WRITE(6,70)(R(I,J),J=1,L)
   12 CONTINUE
      GO TO 11
      END
//GO.SYSIN     DD *


C
C PROGRAM 3
C
C C.E FROM FEEDBACK GAINS/STERN PLANES ONLY MODE
   50 FORMAT(I10)
    1 FORMAT(4E20.6)
    2 FORMAT(3E20.6)
    4 READ(5,50)N
      IF(N.EQ.0)STOP
      READ(5,2) AC,AD,U
      FA=-0.98777
      FB=668.35
      FC=1857.21
      FD=-29929.82
      AA=-0.43769E-01
      AB=0.47727E-03
      BA=-0.42783
      BB=-0.94019E-02
      XA=-AD-FB*BA-AA-FD*BB
      XB=   -FC*BB+FB*AD*BA+AA*AD+FD*AA*BB-FB*BB*AC-FA*BA-AB*AC-
     1AB*FD*BA
      XC=                  AA*FC*BB-AB*FC*BA
     1+BA*AD*FA-FA*BB*AC+FA*BB*U
      XD=            FA*U*BA*AB-FA*U*BB*AA
      WRITE(6,1) XA,XB,XC,XD
```

```
      GO TO 4
      END
# DATA



C
C PROGRAM 4
C
C GAINS FROM C.E / STERN PLANES ONLY MODE
      DIMENSION A(4,4),B(4),L(4),M(4),R(4),C(4,4)
      DOUBLE PRECISION A,D
   50 FORMAT(I10)
  100 FORMAT(4D20.13)
  200 FORMAT(' ',4D30.16)
  250 FORMAT('  GAINS:')
    4 READ(5,50)N
      IF(N.EQ.0)STOP
      READ(5,100) AC,AD,U
      READ(5,100) XA,XB,XC,XD
      WRITE (6,100) XA,XB,XC,XD
      AA=-0.43769E-01
      AB=0.47727E-03
      BA=-0.42783
      BB=-0.94019E-02
      A(1,1)=0.0
      A(2,1)=-BA
      A(3,1)=BA*AD-BB*AC+BB*U
      A(4,1)=      U*BA*AB-U*BB*AA
      A(1,2)=-BA
      A(2,2)=AD*BA-BB*AC
      A(3,2)=0.0
      A(4,2)=0.0
      A(1,3)=0.0
      A(2,3)=-BB
      A(3,3)=AA*BB-AB*BA
      A(4,3)=0.0
      A(1,4)=-BB
      A(2,4)=AA*BB-AB*BA
      A(3,4)=0.0
      A(4,4)=0.0
      B(1)=XA+AD+AA
      B(2)=  -AA*AD+AB*AC+XB
      B(3)=XC
      B(4)=XD
      CALL DMINV(A,N,D,L,M)
      DO 1 I=1,N
      DO 1 J=1,N
```

```
      C(I,J)=A(I,J)
    1 CONTINUE
      CALL GMPRD(C,B,R,N,N,1)
      WRITE(6,250)
      WRITE(6,200) R
      GO TO 4
      END
# DATA



// EXEC FORTCLG,REGION.GO=126K
//FORT.SYSIN    DD *
C
C PROGRAM 5
C
C SOLUTION OF A SYSTEM OF 14 LINEAR ALGEBRAIC EQUATIONS
      DIMENSION A(14,14),B(14)
      DOUBLE PRECISION A,B
   50 FORMAT(I10)
  100 FORMAT(4D20.13)
  150 FORMAT('0 PROBLEM MATRIX:')
  200 FORMAT(' ',4D30.16)
  250 FORMAT('  GAINS:')
    4 READ(5,50)N
      IF(N.EQ.0)STOP
      DO 1 I=1,N
      READ(5,100)(A(I,J),J=1,N)
    1 CONTINUE
      WRITE(6,150)
      DO 2 I=1,N
      WRITE(6,200)(A(I,J),J=1,N)
    2 CONTINUE
      READ(5,100) B
      WRITE(6,200) B
      CALL  DSIMQ(A,B,N,KS)
      WRITE(6,50) KS
      WRITE(6,250)
      WRITE(6,200)  B
      GO TO 4
      END
//GO.SYSIN      DD *
         14
0.0                 0.0                 0.0                 0.0
0.0                 0.0                 0.0                  0.0
0.0                 0.0                 1.0                 0.0
0.0                 0.0
1.0                 -0.043769           0.0                 0.477267        E-03
```

```
0.0                  0.0                   0.0                   0.0
0.0                  0.0                   0.0                   0.0
0.0                  0.0
-25.33               0.0                   0.0                  0.0
0.0                  0.0                   0.0                  0.0
0.0                  0.0                   0.0                   0.0
0.0                  0.0
0.0                       -6.2278           1.0                  -0.15443
0.0                  0.0                   0.0                   0.0
0.0                  0.0                   0.0                   0.0
0.0                  0.0
0.0                         1.0            0.0                  0.0
-0.43769         E-01 0.0                   0.477267        E-03 0.0
0.0                  0.0                   0.0                   0.0
0.5                  0.0
0.0                  -25.33                1.0                  0.0
0.0                  -0.43769         E-00.0                   0.0
0.477267         E-03      0.0             0.0                  0.0
0.0                  0.0
0.0                   0.0                    0.0                1.0
-6.2278              1.0                   -.198199             0.0
0.0                  0.477267         E-03 0.0                  0.0
0.0                  0.0
0.0                  0.0                   -25.33                0.0
0.0                  0.0                   0.0                   0.0
0.0                       0.0              0.0                  0.5
0.0                  0.0
0.0                  0.0                   0.0                  -25.33
0.0                  -6.2278               0.0                  1.0
-0.15443             0.0                      0.0                0.0
0.0                  0.0
0.0                  0.0                   0.0                   0.0
0.0                  0.0                   -6.2278              0.0
1.0                  -0.15443              0.0                   0.0
0.0                  0.5
0.0                  -0.42783              0.0                  -0.94019         E-02
0.0                  0.0                   0.0                   0.0
0.0                  0.0                   0.0                   0.0
0.0                  0.0
0.0                  0.0                   0.0                   0.0
-0.42783             0.0                   -0.94019         E-02                0.0
0.0                  0.0                   0.0                   0.0
0.0                  0.0
0.0                  0.0                   0.0                   0.0
0.0                  -0.42783              0.0                   0.0
-0.94019         E-02          0.0          0.0                    0.0
0.0                  0.0
0.0                  0.0                   0.0                  0.0
```

```
0.0                   0.0                -0.42783            0.0
0.0                  -0.94019        E-02 0.0                 0.0
0.0                   0.0
0.1                   0.5265             -16.9847            -50.1888
1.3861               -89.4272            -264.2521           1444.382
8524.3141            12594.432            0.31623            1.665
-53.71               -158.71
0


// EXEC FORTCLG,REGION.GO=126K
//FORT.SYSIN    DD *
C
C  PROGRAM 6
C
C  PROGRAM TO FIND THE COMBINED MODE,CLOSED LOOP C.E,GIVEN THE FEEDBACK GAINS IN
C  DSI AND DF1.THE CHARACTERISTIC EQUATION WILL BE OF FOURTH DEGREE,I.E:
C                S**4+AS**3+BS+D
  100 FORMAT(4E12.5)
  200 FORMAT(4E16.5)
C READ INPUT DATA
      READ (5,100) FSA,FSB,FSC,FSD
      READ (5,100) FBA,FBB,FBC,FBD
      READ (5,100) UC
      AA=-0.43769E-01
      AB=0.47727E-03
      AC=-6.2278
      AD=-0.15443
      BA=-0.42783
      BB=-0.94019E-02
      BC=-2.4839E-01
      BD=2.0465E-03
      A=-AA-AD-BA*FSB-BC*FBB-BB*FSD-BD*FBD
      B=AA*AD-AC*AB+BA*AD*FSB+BC*AD*FBB+
     1AA*BB*FSD+BC*BB*FBB*FSD+AA*BD*FBD+
     1BA*BD*FSB*FBD-AB*BA*FSD-AB*BC*FBD-
     1AC*BB*FSB-BC*BB*FBD*FSB-AC*BD*FBB-
     1BA*BD*FSD*FBB-BA*FSA-BC*FBA-BB*FSC-BD*FBC
      C1 =FSA*(AD*BA+UC*BB-AC*BB)
      C2=FBA*(AD*BC-AC*BD+UC*BD)
      C3=FSC*(AA*BB-AB*BA)
      C4=FBC*(AA*BD-AB*BC)
      C5=(BA*BD-BB*BC)*(FBD*FSA-FSD*FBA+FSB*FBC-FBB*FSC)
      C=C1+C2+C3+C4+C5
      D=(BA*BD-BB*BC)*(FSA*FBC-FSC*FBA+UC*FSA*FBB-UC*FSB*FBA)
     1+FSA*UC*(AB*BA-AA*BB)+FBA*UC*(AB*BC-AA*BD)
      WRITE (6,200) FSA,FSB,FSC,FSD
```

```
      WRITE (6,200) FBA,FBB,FBC,FBD
      WRITE(6,200) UC
      WRITE (6,200)A,B,C,D
      STOP
      END
//GO.SYSIN     DD *


// EXEC FORTCLG,REGION.GO=126K
//FORT.SYSIN   DD *
C
C PROGRAM 7
C
C PROGRAM TO FIND FEEDBACK GAINS IN THE ORDER TO BOW PLANES,GIVEN
C THE FOURTH ORDER C.E  S**4+X*S**3+Y*S**2+Z*S+W,AND THE GAINS
C FCR THE ORDER TO STERN PLANES
      DIMENSION A(4,4),B(4)
  100 FORMAT(4E12.5)
  200 FORMAT(4E16.5)
  300 FORMAT(I2)
C READ INPUT DATA
      READ (5,100) FSA,FSB,FSC,FSD
      READ (5,100) X,Y,Z,W
      READ (5,100) UC
      N=4
      AA=-0.43769E-01
      AB=0.47727E-03
      AC=-6.2278
      AD=-0.15443
      BA=-0.42783
      BB=-0.94019E-02
      BC=-2.4839E-01
      BD=2.0465E-03
      A(1,1)=0.0
      A(2,1)=-BC
      A(3,1)=UC*BD+AD*BC+BB*BC*FSD-AC*BD-BA*BD*FSD
      A(4,1)=BB*BC*FSC-BA*BD*FSC+UC*AB*BC-UC*AA*BD+UC*BB*BC*FSB-UC*BA
     1*BD*FSB
      A(1,2)=-BC
      A(2,2)=BC*AD+BC*BB*FSD-AC*BD-BA*BD*FSD
      A(3,2)=BC*BB*FSC-BD*BA*FSC
      A(4,2)=UC*BD*BA*FSA-UC*BC*BB*FSA
      A(1,3)=0.0
      A(2,3)=-BD
      A(3,3)=AA*BD+BA*BD*FSB-AB*BC-BB*BC*FSB
      A(4,3)=BD*BA*FSA-BC*BB*FSA
      A(1,4)=-BD
```

```
      A(2,4)=AA*BD-BB*BC*FSB-AB*BC+BA*BD*FSB
      A(3,4)=BD*BA*FSA-BC*BB*FSA
      A(4,4)=0.0
      B(1)=X+AA+BA*FSB+AD+BB*FSD
      B(2)=Y-AA*AD+AC*AB-BA*FSB*AD-AA*BB*FSD+AB*BA*FSD
     1+AC*BB*FSB+BA*FSA+BB*FSC
      B(3)=Z-AD*BA*FSA-UC*BB*FSA+AC*BB*FSA-AA*BB*FSC+AB*BA*FSC
      B(4)=W-UC*AB*BA*FSA+UC*AA*BB*FSA
      CALL SIMQ(A,B,N,KS)
      WRITE(6,200)B
      WRITE(6,300)KS
      WRITE (6,200) FSA,FSB,FSC,FSD
      WRITE (6,200) X,Y,Z,W
      STOP
      END
//GO.SYSIN     DD *



C
C PROGRAM 8
C
C PREPARATION FOR ROOT LOCUS
   50 FORMAT(I2)
  100 FORMAT(4E12.5)
  200 FORMAT(4E16.5)
C READ INPUT DATA
    4 READ(5,50)N
      IF(N.EQ.0)STOP
      READ (5,100) SOA,SOB,SOC,SOD
      UC=25.33
      AA=-0.43769E-01
      AB=0.47727E-03
      AC=-6.2278
      AD=-0.15443
      BA=-0.42783
      BB=-0.94019E-02
      A=-BA*SOB-BB*SOD
      B=-BA*SOA-AC*BB*SOB-BB*SOC+AA*BB*SOD+AD*BA*SOB-AB*BA*SOD
      C=SOA*(BA*AD-BB*AC+BB*UC)+SOC*(BB*AA-BA*AB)
      D=SOA*(UC*BA*AB-UC*AA*BB)
      WRITE (6,200) A,B,C,D
      GO TO 4
      END
# DATA
```

```
// EXEC FORTCLG,REGION.GO=126K
//FORT.SYSIN    DD *
C
C  PROGRAM 9
C
C  PROGRAM TO CALCULATE VALUES OF K MINIMIZING S.S.ERROR TO STEP INPUT
C            K=(0.436/DFCOM)**X
C  ACCEPTABLE VALUES OF K MUST BE REAL POSITIVE AND BETWEENO-1
   100 FORMAT(4E12.5)
   200 FORMAT(5(5X,E12.5))
C  READ INPUT DATA
     1 READ (5,100) SCA,SCB,SCC,SCD
       READ (5,100) SOA,SOB,SOC,SOD
       READ (5,100)FRC,TRQ,UC,P
       AA=-0.43769E-01
       AB=0.47727E-03
       AC=-6.2278
       AD=-0.15443
       BA=-0.42783
       BB=-0.94019E-02
       D=TRQ*BA-FRC*BB
       E=TRQ*AA-FRC*AB
       F=BA*AB-BB*AA
       R1=D*((SCC-SOC)+(SCB-SOB)*UC)
       R2=(D*(SOC+SOB*UC)+E*UC)
       R3=F*UC*(SCA-SOA)
       R4=F*UC*SOA
       R5=D*(SOA-SCA)
       R6=-D*SOA
       A=1.0
       B=(R4*R1**2+R1*R2*R3-R3*R5*R6*2.*P**2
      1)/(R3*R1**2-R3*(P*R5)**2)
       C=(R1*R2*R4-R3*(P*R6)**2)/(R3*R1**2-R3*(R5*P)**2)
C  TEST SIGN  OF DISCRIMINANT
     5 IF(B**2-4.*A*C) 30,20,10
C  REAL DISTINCT ROOTS
    10 DISC=(B**2-4.*A*C)**0.5
       X1=(-B+DISC)/(2.*A)
       X2=(-B-DISC)/(2.*A)
       GO TO 50
C  REAL REPEATED ROOTS
    20 X1=-B/(2.*A)
       X2=X1
       GC TO 50
C  COMPLEX ROOTS
    30 ROOT=(4.*A*C-B**2)**0.5
       XREAL=-B/(2.*A)
       XIMAG=ROOT/(2.*A)
```

```
      GC TO 60
C WRITE OUTPUT
   50 WRITE(6,200)A,B,C,X1,X2
      WRITE(6,200)
      GO TO  1
   60 WRITE(6,200)A,B,C
      WRITE(6,200)XREAL,XIMAG
      WRITE (6,200)
      GC TO 1
      END
//GO.SYSIN      DD *



// EXEC FORTCLG,REGION.GO=126K
//FORT.SYSIN    DD *
C
C PROGRAM 10
C
C PROGRAM TO CALCULATE EQUILIBRIUM ORDER OF BOW PLANES IN RESPONSE
C TO STEP INPUT AND LIMITS ON THE EXPONENT OF K
  100 FCRMAT(4E12.5)
  200 FORMAT(4E16.5)
C READ INPUT DATA
    1 READ (5,100) SCA,SCB,SCC,SCD
      READ (5,100) SOA,SOB,SOC,SOD
      READ (5,100)FRC,TRQ,UC,P
      READ (5,100)BCA,BCC,XK
      AA=-0.43769E-01
      AB=0.47727E-03
      AC=-6.2278
      AD=-0.15443
      BA=-0.42783
      BB=-0.94019E-02
      D=TRQ*BA-FRC*BB
      E=TRQ*AA-FRC*AB
      F=BA*AB-BB*AA
      R1=D*((SCC-SOC)+(SCB-SOB)*UC)
      R2=(D*(SOC+SOB*UC)+E*UC)
      R3=F*UC*(SCA-SOA)
      R4=F*UC*SOA
      R5=D*(SOA-SCA)
      R6=-D*SOA
      X1SS=(R1*XK+R2)/(R3*XK+R4)
      X3SS=(R5*XK+R6)/(R3*XK+R4)
      DFSS=BCA*X1SS+BCC*X3SS
      WRITE(6,200) X1SS,X3SS,DFSS
      XH=DFSS*(SCA+SCC/P-SOA-SOC/P)/(BCA+BCC/P)
```

```
      IF(XH.EQ.0.0) GO TO 21
      T1=(-0.436-DFSS*(SOA+SOC/P)/(BCA+BCC/P))/XH
      T2=(0.436-DFSS*(SOA+SOC/P)/(BCA+BCC/P))/XH
      T3=ABS(0.436/DFSS)
      IF(T1.LE.0.0) GO TO 22
      IF(T2.LE.0.0) GO TO 23
      XL1=ALOG10(T1)/ALOG10(T3)
      XL2=ALOG10(T2)/ALOG10(T3)
      IF(XH.GT.0.0)XL1=XLUP
      IF(XH.LT.0.0) XL2=XLUP
      WRITE(6,200) XH,XL1,XL2,XLUP
      GO TO 24
   21 FLAG=1.0
      WRITE(6,200) FLAG
      GO TO 24
   22 FLAG =2.0
      WRITE (6,200) FLAG
   23 FLAG=3.0
      WRITE (6,200) FLAG
   24 STOP
      END
//GO.SYSIN      DD *


// EXEC FORTCLG,REGION.GO=126K
//FCRT.SYSIN    DD *
C
C PROGRAM 11
C
C PROGRAM TO SET LIMITS ON EXPONENT WHEN INITIAL S.S.ERROR IS CONSIDERED
C THE INPUT IS A STEP ENOUGH TO SATURATE THE BOW PLANES
  100 FCRMAT(4E12.5)
  200 FCRMAT(4E16.5)
C READ INPUT DATA
      READ (5,100) SCA,SCB,SCC,SCD
      READ (5,100) BCA,BCB,BCC,BCD
      READ (5,100) UC,DFMAX
      AA=-0.43769E-01
      AB=0.47727E-03
      AC=-6.2278
      AD=-0.15443
      BA=-0.42783
      BB=-C.94019E-02
      BC=-2.4839E-01
      BD=2.0465E-03
      DEN=(BCC*SCA-BCA*SCC+UC*SCA*BCB-UC*SCB*BCA)*
     1(BA*BD-BC*BB)+(AB*BA-AA*BB)*UC*SCA+
```

```
     1(AB*BC-AA*BD)*UC*BCA
      X1SS=(DFMAX/DEN)*((SCC+UC*SCB)*(-BB*BC-BA*BD)-
     12.0*(BCC+UC*BCB)*BD*BC+UC*(-AB*BC-AA*BD))
      X2SS=(DFMAX/DEN)*(SCA*(BB*BC+BA*BD)+2.0*BD*BC*BCA)
      WRITE(6,200) X1SS,X2SS
      STOP
      END
//GO.SYSIN     DD *


// EXEC FORTCLG,REGION.GO=126K
//FORT.SYSIN   DD *
C
C  PROGRAM 12
C
C  PROGRAM TO CALCULATE VALUES OF K MINIMIZING S.S.ERROR TO STEP INPUT
C/2ND CASE  K=(0.436/DFCOM)**X
C ACCEPTABLE VALUES OF K MUST BE REAL POSITIVE AND BETWEEN0-1
  100 FORMAT(4E12.5)
  200 FORMAT(4E16.5)
C READ INPUT DATA
    1 READ (5,100) SCA,SCB,SCC,SCD
      READ (5,100) SOA,SOB,SOC,SOD
      READ (5,100)FRC,TRQ,UC,P
      AA=-0.43769E-01
      AB=0.47727E-03
      AC=-6.2278
      AD=-0.15443
      BA=-0.42783
      BB=-0.94019E-02
      BC=-2.4839E-01
      BD=2.0465E-03
      FRC=FRC-0.437*BC
      TRQ=TRQ-0.437*BD
      D=TRQ*BA-FRC*BB
      E=TRQ*AA-FRC*AB
      F=BA*AB-BB*AA
      R1=D*((SCC-SOC)+(SCB-SOB)*UC)
      R2=(D*(SOC+SOB*UC)+E*UC)
      R3=F*UC*(SCA-SOA)
      R4=F*UC*SOA
      R5=D*(SOA-SCA)
      R6=-D*SOA
      A=1.0
      B=(R4*R1**2+R1*R2*R3-R3*R5*R6*2.*P**2
     1)/(R3*R1**2-R3*(P*R5)**2)
      C=(R1*R2*R4-R3*(P*R6)**2)/(R3*R1**2-R3*(R5*P)**2)
```

```
C TEST SIGN  OF DISCRIMINANT
    5 IF(B**2-4.*A*C) 30,20,10
C REAL DISTINCT ROOTS
   10 DISC=(B**2-4.*A*C)**0.5
      X1=(-B+DISC)/(2.*A)
      X2=(-B-DISC)/(2.*A)
      GC TO 50
C REAL REPEATED ROOTS
   20 X1=-B/(2.*A)
      X2=X1
      GO TO 50
C COMPLEX ROOTS
   30 ROOT=(4.*A*C-B**2)**0.5
      XREAL=-B/(2.*A)
      XIMAG=ROOT/(2.*A)
      GC TO 60
C WRITE OUTPUT
   50 WRITE(6,200)A,B,C,X1,X2
      WRITE(6,200)
      GC TO  1
   60 WRITE(6,200)A,B,C
      WRITE(6,200)XREAL,XIMAG
      WRITE (6,200)
      GO TO 1
      END
//GO.SYSIN     DD *


// EXEC FORTCLG,REGION.GO=126K
//FORT.SYSIN   DD *
C
C PROGRAM 13
C
C PROGRAM TO CALCULATE EQUILIBRIUM ORDER OF BOW PLANES IN RESPONSE
C TO STEP INPUT AND LIMITS ON THE EXPONENT OF K   /2ND CASE
  100 FCRMAT(4E12.5)
  200 FORMAT(4E16.5)
C READ INPUT DATA
    1 READ (5,100) SCA,SCB,SCC,SCD
      READ (5,100) SOA,SOB,SOC,SOD
      READ (5,100)FRC,TRQ,UC,P
      READ (5,100)BCA,BCC,XK
      READ (5,100) C1,C2
      AA=-0.43769E-01
      AB=0.47727E-03
      AC=-6.2278
      AD=-0.15443
```

```
      BA=-0.42783
      BB=-0.94019E-02
      BC=-2.4839E-01
      BD=2.0465E-03
      FRC=FRC-0.437*BC
      TRQ=TRQ-0.437*BD
      D=TRQ*BA-FRC*BB
      E=TRQ*AA-FRC*AB
      F=BA*AB-BB*AA
      R1=D*((SCC-SOC)+(SCB-SOB)*UC)
      R2=(D*(SOC+SOB*UC)+E*UC)
      R3=F*UC*(SCA-SOA)
      R4=F*UC*SOA
      R5=D*(SOA-SCA)
      R6=-D*SOA
      X1SS=(R1*XK+R2)/(R3*XK+R4)
      X3SS=(R5*XK+R6)/(R3*XK+R4)
      X1SS=X1SS+C1
      X3SS=X3SS+C2
      DFSS=BCA*X1SS+BCC*X3SS
      WRITE(6,200) X1SS,X3SS,DFSS
      CA=-BCA*C1-BCC*C2
      CB=SCA*C1+SCC*C2
      CC=SOA*C1+SOC*C2
      XH=(DFSS+CA)*(SCA+SCC/P-SOA-SOC/P)/(BCA+BCC/P)+CB-CC
      IF(XH.EQ.0.0) GO TO 21
      T1=-0.436-(DFSS+CA)*(SOA+SOC/P)/(BCA+BCC/P)-CC
      T2=0.436-( DFSS+CA)*(SOA+SOC/P)/(BCA+BCC/P)-CC
      T3=ABS(0.436/DFSS)
      IF(T1.LE.0.0) GO TO 22
      IF(T2.LE.0.0) GO TO 23
      XL1=ALOG10(T1)/ALOG10(T3)
      XL2=ALOG10(T2)/ALOG10(T3)
      IF(XH.GT.0.0)XL1=XLUP
      IF(XH.LT.0.0) XL2=XLUP
      WRITE(6,200) XH,XL1,XL2,XLUP
      GO TO 24
   21 FLAG=1.0
      WRITE(6,200) FLAG
      GO TO 24
   22 FLAG =2.0
      WRITE (6,200) FLAG
      GO TO 24
   23 FLAG=3.0
      WRITE (6,200) FLAG
   24 STOP
      END
//GO.SYSIN     DD *
```

```
// EXEC FORTCLG,REGION.GO=126K
//FORT.SYSIN   DD *
C
C PROGRAM 14
C
C PROGRAM TO GIVE VALUES OF K AS SOLUTIONS OF A QUADRATIC EQN.,WITH
C Z AS A PARAMETER.A SECOND ORDER APPROXIMATING SYSTEM IS USED
  100 FORMAT(4E12.5)
  200 FORMAT(4E16.5)
C READ INPUT DATA
    1 READ (5,100) SCB,SCD,SOB,SOD
      READ (5,100) Z
      AA=-0.43769E-01
      AB=0.47727E-03
      AC=-6.2278
      AD=-0.15443
      BA=-0.42783
      BB=-0.94019E-02
      A=(BA*(SCB-SOB))**2+(BB*(SCD-SOD))**2+2.0*BA*BB*(SCB-SOB)*
     1(SCD-SOD)
      B=(SCB-SOB)*(2.0*AA*BA+2.0*AD*BA+4.0*AC*BB*Z**2-BA*AD*4.0*Z**2)
     1+(SCD-SOD)*(2.0*AA*BB+2.0*AD*BB+4.0*AB*BA*Z**2-4.0*BB*AA*Z**2)+
     1((SCB-SOB)*SOD+(SCD-SOD)*SOB)*2.0*BA*BB+(SCB-SOB)*SOB*2.0*BA**2
     1+(SCD-SOD)*SOD*2.0*BB**2
      C=AA**2+2.0*AA*AD+AD**2-4.0*AA*AD*Z**2+4.0*AC*AB*Z**2+SOB*2.0*AA
     1*BA+SOB*2.0*AD*BA+SOB*4.0*AC*BB*Z**2-SOB*BA*AD*4.*Z**2+SOD*2.*AA*
     1BB+SOD*2.0*AD*BB-SOD*BB*AA*4.0*Z**2+SOD*AB*BA*4.0*Z**2
     1+(BA*SOB)**2+(BB*SOD)**2+2.0*BA*BB*SOB*SOD
C TEST SIGN  OF DISCRIMINANT
    5 IF(B**2-4.*A*C) 30,20,10
C REAL DISTINCT ROOTS
   10 DISC=(B**2-4.*A*C)**0.5
      X1=(-B+DISC)/(2.*A)
      X2=(-B-DISC)/(2.*A)
      GO TO 50
C REAL REPEATED ROOTS
   20 X1=-B/(2.*A)
      X2=X1
      GO TO 50
C COMPLEX ROOTS
   30 ROOT=(4.*A*C-B**2)**0.5
      XREAL=-B/(2.*A)
      XIMAG=ROOT/(2.*A)
      GO TO 60
C WRITE OUTPUT
```

```
   50 WRITE(6,200)A,B,C,X1,X2
      WRITE(6,200)
      GO TO  1
   60 WRITE(6,200)A,B,C
      WRITE(6,200)XREAL,XIMAG
      WRITE (6,200)
      GO TO 1
      END
//GO.SYSIN     DD *
SKIP003
// EXEC FORTCLG,REGION.GO=126K
//FORT.SYSIN   DD *
C
C PRCGRAM 15
C
C PROGRAM TO GIVE VARIATION OF K AS FUNCTION OF DFCOM,WITH PARAMETER Z AND X2
  100 FORMAT(4E12.5)
  200 FCRMAT(4E16.5)
C READ INPUT DATA
    1 READ (5,100) SCA,SCB,SCC,SCD
      READ (5,100) SOA,SOB,SOC,SOD
      READ (5,100) BCA,BCB,BCC,BCD
      READ (5,100)Z,P
      AA=-0.43769E-01
      AB=0.47727E-03
      AC=-6.2278
      AD=-0.15443
      BA=-0.42783
      BB=-0.94019E-02
      DO 70 I=1,11
      READ (5,100) XX
      DC 70 J=1,56
      DFCOM=-0.436-0.015*J
      W1=(SCA-SOA)*(DFCOM-XX*(BCB+BCD/P))/(XX*(BCA+BCC/P))+(SCB-SOB)
      W2=(SCC-SOC)*(DFCOM-XX*(BCB+BCD/P))/(XX*(BCA+BCC/P))+(SCD-SOD)
      W3=         SOA*(DFCOM-XX*(BCB+BCD/P))/(XX*(BCA+BCC/P))+SOB
      W4=         SOB*(DFCOM-XX*(BCB+BCD/P))/(XX*(BCA+BCC/P))+SOD
      W5=2.*AA*BA+2.*AD*BA+4.*AC*BB*Z**2-4.*BA*AD*Z**2
      W6=2.*AA*BB+2.*AD*BB+4.*AB*BA*Z**2-4.*BB*AA*Z**2
      A=(W1*BA)**2+(W2*BB)**2+2.*BA*BB*W1*W2
      B=2.*(BA**2)*W1*W3+2.*(BB**2)*W2*W4+
     12.*BA*BB*(W1*W4+W2*W3)+W1*W5+W2*W6
      C=(BA*W3)**2+(BB*W4)**2+W3*W4+W3*W5+W4*W6+(AA+AD)**2+
     14.*AC*AB*Z**2-4.*AA*AD*Z**2
C TEST SIGN  OF DISCRIMINANT
    5 IF(B**2-4.*A*C) 30,20,10
C REAL DISTINCT ROOTS
   10 DISC=(B**2-4.*A*C)**0.5
```

```
      X1=(-B+DISC)/(2.*A)
      X2=(-B-DISC)/(2.*A)
      GO TO 50
C REAL REPEATED ROOTS
   20 X1=-B/(2.*A)
      X2=X1
      GO TO 50
C COMPLEX ROOTS
   30 ROOT=(4.*A*C-B**2)**0.5
      XREAL=-B/(2.*A)
      XIMAG=ROOT/(2.*A)
      GO TO 60
C WRITE OUTPUT
   50 WRITE(6,200)A,B,C,X1,X2
      WRITE(6,200)
      IF(J.EQ.56) WRITE (6,200) Z,XX,P
      GO TO  70
   60 WRITE(6,200)A,B,C
      WRITE(6,200)XREAL,XIMAG
      WRITE (6,200)
      IF(J.EQ.56) WRITE (6,200) Z,XX,P
   70 CCNTINUE
      STOP
      ENC
//GO.SYSIN     DD *


// EXEC FORTCLGP,REGION.GO=154K
//FORT.SYSIN   DD *
C
C PRCGRAM 16
C
C PROGRAM TO SHOW FUNCTIONAL RELATIONSHIP BETWEEN K AND DFCOM
      INTEGER*4 ITB(12)/12*0/
      REAL*4  RTB(28)/28*0.0/
      DIMENSIONX(45),Y(45),ITB(12),RTB(28)
      DATA RTB(3)/'0.3'/
      NUMPTS=45
      ITB(1)=1
      ITB(2)=1
      ITB(12)=1
  100 FCRMAT (6E12.5)
  200 FORMAT (6E16.5)
      READ (5,100) X
      READ (5,100) Y
      CALL DRAWP(NUMPTS,X,Y,ITB,RTB)
      ITB(1)=3
```

```
      ITB(2)=0
      ITB(12)=1
      DO 1 I=1,45
      X(I)=0.436+0.01*I
      Y(I)=(0.436/X(I))**0.3
    1 CONTINUE
      CALL DRAWP(NUMPTS,X,Y,ITB,RTB)
      STOP
      END
//GO.SYSIN     DD *
```

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